ON THE SCATTERING OF COMETS BY SATURN

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Abstract. *The study of the swing-by maneuver is very important to understand the gravitational effects in small bodies like comets, for example, caused by a planet. In the present research the planet considered is Saturn, in a system formed by Saturn and the Sun. With the comprehension of this mechanism, it is possible to generate the results that are shown in this paper to understand the motion of comets, meteors and gas clouds.*

Keywords: swing-by maneuvers, comets, solar system, restricted three-body problem.

1. INTRODUCTION

The comets are among the smallest bodies known in the Solar System, but they are also one of the most important and interesting topics of research in today's celestial mechanics. Comets are believed to carry material from the time the Solar System was formed, which means that a detailed study of that material could help us to answer many basic questions about this important process. There are also some speculations about connections between the impact of the comets and the origin of life on Earth.

The comets are so small, that they are only detected when they are passing near their pericenter (or periapse). This fact makes the estimation of their quantity a very difficult and unsolved problem. The literature almost agrees that a good estimate for a lower limit for their population is about 10^{11} comets related to the Solar System, but several researchers believe that they are much more numerous. The actual observations show that three or four new comets are observed in the interior Solar System every year.

The origin of the comets is still not explained. They can be formed in the interstellar space and then they are captured by the Solar System or they can be formed in the Solar System and then they are expelled to the interstellar space. One of the most popular ideas about the origin of the comets is the existence of a large cloud of comets around the Solar System, called the “Oort Cloud”, with a total of 10^{11} comets at distances up to 10^5 AU.

The present research has the goal of giving a contribution to the problem of capture and escape of comets caused by a close encounter with the planet Saturn. The topics of escape and capture of comets have been discussed in the literature for a long time. In the majority of the papers, a close approach with a large planet is the core of the mechanism of capture and escape. The dynamics are explained in papers as old as Russell (1920) and Woerkom (1948), who derived an expression to calculate the effects of the close approach between Jupiter and one comet.

On the basis of the number of comets detected up to now, the Solar System seems to have more short period comets (period less than 200 years) than it should have, based in the actual flux of near-parabolic comets coming to the Solar System and the mechanism of capture known. Several theories are developed to explain this fact. Hills (1981) suggests that we can be living in a period of a large intensity of comets coming from the Oort Cloud (Oort, 1950) to the Solar System, that is known as "The Comet Shower Theory". Another possible explanation is the fragmentation of an original large body that would generate a large number of smaller bodies. A third option is the existence of another source of comets in the Solar System and/or another mechanism of capture, but there is no strong evidence of any of those new hypotheses. A summary of orbital evolution and terminal stages of a comet is represented in the Fig. (1).

In the present paper, we address this problem by numerical integration of a large number of trajectories of possible comets. We assume that the Solar System is composed by the Sun, a planet (Saturn) and one comet. The swing-by maneuvers are studied under the model given by the three-dimensional circular restricted three-body problem. This maneuver can be identified by five independent parameters: V_p, the magnitude of the velocity of the spacecraft at periapsis; γ, the angle between the velocity vector at periapsis and the intersection between the horizontal plane that passes by the periapsis and the plane perpendicular to the periapsis that holds $\vec{V}_p$; r_p, the distance between the spacecraft and the celestial body during the closest approach; α, the angle between the projection of the periapsis line in the x-y plane and the line that connects the two primaries; β, the angle between the periapsis line and the x-y plane.

With the initial conditions defined, we integrated the trajectory backward and forward in time until we can reach a point that can be considered far from Saturn. At those points we can consider the problem as modeled by the two-body (Sun and Comet) dynamics. Then we calculated the energy and the angular momentum of the comet before and after the encounter and we classify the orbits accordingly to the effects of the close approach. Each part of the orbit (before and after the close encounter) can be elliptic or hyperbolic and the close encounter can increase or decrease the energy, what give us several possibilities for the close encounter. We represent them with letter-plots, where each letter corresponds to one of the classes, describing thus the effect of the close encounter in a two-dimensional diagram.

2. MATHEMATICAL MODEL AND ALGORITHM

The present section gives a small example that illustrates the basic principle of energy increase or decrease during a flyby of a moving body. It justifies the following fundamental conclusions: When a particle has a close approach with a receding massive body, it gains energy, while, if the massive body is approaching, the particle loses energy. It is assumed that Q is for a massive planet and P is the comet with negligible mass. In other words P has no effect on the motion of Q. The comet P has a close approach with the planet Q. A correct treatment of the problem requires the restricted three-body problem, that is described later. The present treatment is somewhat simplified, but it illustrates the idea of energy increase or decrease during a planetary flyby, and it leads to the correct conclusions.

Q is at the location ($x_1(t)$, $y_1(t)$). The force is the usual inverse square attraction with potential function $U = \mu/r$ or with potential energy $V = -\mu/r$. The whole system is referred to an inertial system of reference with origin O, as shown in Fig. (2).

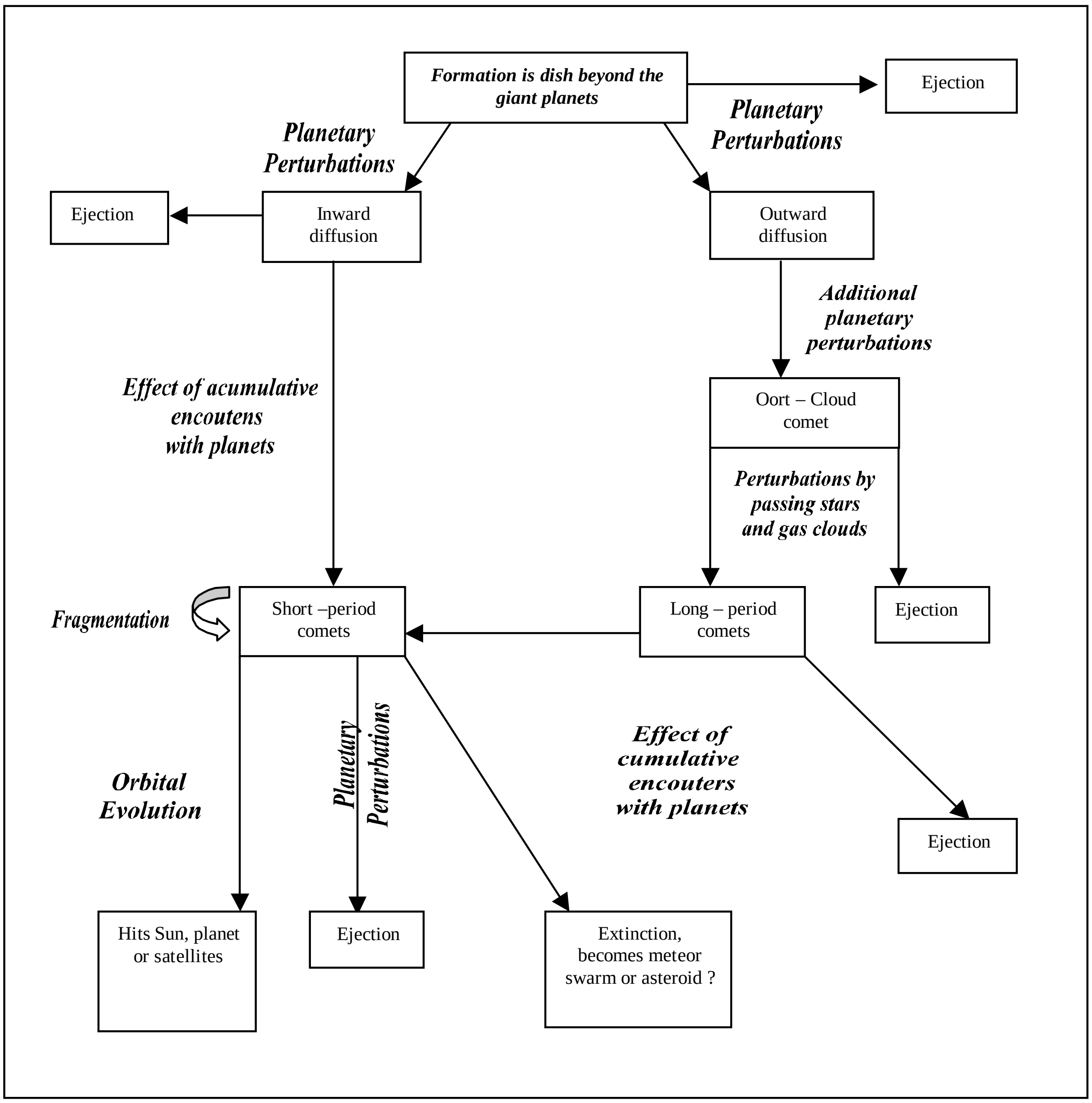


Figure 1. The comet’s orbital evolution.

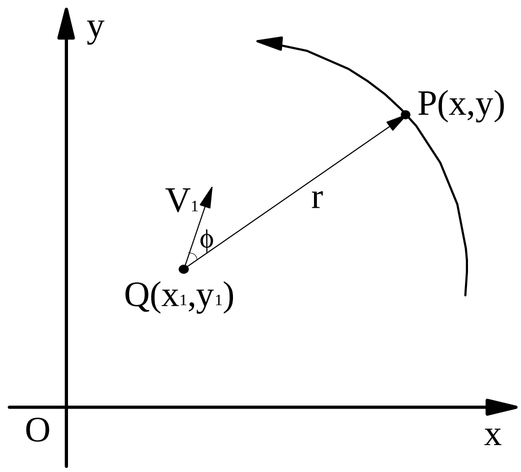


Figure 2 - A Close Approach between P and Q.

The equations of motion of P are thus:

$$\ddot{x} = \frac{\partial U}{\partial x} = -\mu \frac{x}{r^3};\ \ddot{y} = \frac{\partial U}{\partial y} = -\mu \frac{y}{r^3} \tag{1}$$

where

$$r^2 = (x - x_1)^2 + (y - y_1)^2 \tag{2}$$

The question is now the variation of the energy of the particle at P. This is obtained with the use of the energy equation

$$E = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - U\left(r(x, y, t)\right) \tag{3}$$

$$\frac{dE}{dt} = \frac{\partial E}{\partial t} = \frac{\partial V}{\partial t} = \frac{\mu}{r^2}\frac{\partial r}{\partial t} \tag{4}$$

However, we have

$$r\frac{\partial r}{\partial t} = -(x - x_1)\frac{\partial x_1}{\partial t} - (y - y_1)\frac{\partial y_1}{\partial t} = -(x - x_1)\dot{x}_1 - (y - y_1)\dot{y}_1 \tag{5}$$

Therefore, the energy equation is:

$$\frac{dE}{dt} = -\frac{\mu}{r^3}\left[(x - x_1)\dot{x}_1 + (y - y_1)\dot{y}_1\right] \tag{6}$$

In the brackets it is recognized the dot product of the relative position $\vec{r} = \overline{PQ}$ of P with the absolute velocity vector $V_1\left(\dot{x}_1, \dot{y}_1\right)$ of the perturbing body Q. The energy equation can then be written as:

$$\frac{dE}{dt} = -\frac{\mu}{r^3} r V_1 Cos(\phi) = -\frac{\mu V_1}{r^2} Cos(\phi) \tag{7}$$

From the equation (7) is possible to come to an important conclusion about the increase or decrease of the energy of the particle P. This essentially depends only on the factor Cos(ϕ) in the above equation: when ϕ is below 90°, the energy decreases and when ϕ is above 90°, the energy increases. This result can be summarized as follows: When Q approaches: E decreases, When Q recedes: E increases.

3. THREE-DIMENSIONAL CIRCULAR RESTRICTED PROBLEM

The primary problem that is studied in this research is to find under what conditions a comet coming from outside the Solar System is captured. To solve this problem, it is assumed that the Solar System is formed by three bodies: the Sun (body M_1), a planet (in the case considered here Saturn, body M_2) and a third particle of negligible mass (the comet, called M_3). The three dimensional swing-by maneuver consists of using a close encounter with a planet to change the velocity, energy, and angular momentum of a smaller body (comet). This maneuver can be identified by four independent parameters: *1)* V_p is the magnitude of the velocity of the comet at periapse. For the most general case, one would need to give information about the direction of the velocity; *2)* r_p is the distance between the comet and the planet during the closest approach; *3)* α is the angle between the projection of the periapse line in the x-y plane and the line that connects the two primaries; *4)* β is the angle between the periapse line and the x-y plane.

The Fig. (3) shows the geometry involved in the close approach and defines the basic variables used in this research.

To study the swing-by maneuver, the comet needs to be near Saturn, because when the comet is far the system is governed by a two-body (Sun + comet) problem dynamics that does not allow any change in energy. In particular, this research is looking for the energy of the comet before and after the swing-by maneuver, to detect under what conditions a comet is captured (change its energy from positive to negative). Since the initial conditions for the comet vary in a very large range, it is also possible to detect under what conditions a comet is expelled from the Solar System (change its energy from negative to positive) or have a modification in its energy without changing its type of orbit (a change in its energy that is not large enough to modify the sign of the energy).

The standard dimensionless canonical system of units is used in the development of this research, which implies that the unit of distance is the distance between M_1 and M_2; the mean angular velocity ω of the motion of M_1 and M_2 is assumed to be one, the mass of the planet M_2 is given by $\mu = m_2/(m_1+m_2)$, where m_1 and m_2 are the real masses of M_1 and M_2, respectively; and the mass of M_2 is (1-μ); and the unit of time is defined such that the period of the motion of the two primaries is 2π and the gravitational constant is one.

In the rotational system of reference, the origin is the center of mass of the two massive primaries. The horizontal axis x is the line that connects the two primaries at any time. The vertical axis y is perpendicular to the x axis. In this research the Sun and Saturn's positions in the x axis are $x_1=-\mu$, $x_2=1-\mu$, *and* $y_1=y_2=0$, respectively.

Following the theory described by Szebehely (1968), the equations of motion for the massless particle can be written by:

$$\ddot{x} - 2\dot{y} = x - (1-\mu)\frac{x+\mu}{r^3{}_1} - \mu\frac{x-1+\mu}{r^3{}_2} \tag{8}$$

$$\ddot{y} + 2\dot{x} = y - (1-\mu)(y/r^3{}_1) - \mu(y/r^3{}_2) \tag{9}$$

$$\ddot{z} = -(1-\mu)(z/r^3{}_1) - \mu(z/r^3{}_2) \tag{10}$$

where r_1 and r_2 are the distances from M_1 and M_2.

A numerical algorithm to solve the problem has the following steps:

1. Arbitrary values for the parameters r_p, V_p, α, β and γ are given;

2. With these values the initial conditions in the rotating system are computed. The initial position is the point (X_i, Y_i, Z_i) and the initial velocity is (V_{xi}, V_{yi}, V_{zi}), where:

$$X_i = 1 - \mu + r_p\cos(\beta)\cos(\alpha) \tag{11}$$

$$Y_i = r_p\cos(\beta)\sin(\alpha) \tag{12}$$

$$Z_i = r_p\sin(\beta) \tag{13}$$

$$V_{Xi} = -V_p\sin(\gamma)\sin(\beta)\cos(\alpha) - V_p\cos(\gamma)\sin(\alpha) + r_p\cos(\beta)\sin(\alpha) \tag{14}$$

$$V_{Yi} = -V_p\sin(\gamma)\sin(\beta)\sin(\alpha) + V_p\cos(\gamma)\cos(\alpha) - r_p\cos(\beta)\cos(\alpha) \tag{15}$$

$$V_{Zi} = V_p\cos(\beta)\sin(\gamma) \tag{16}$$

3. With these initial conditions, the equations of motion are integrated forward in time until the distance between M_2 and the spacecraft is larger than a specified limit d. At this point the numerical integration is stopped and the energy (E_+) and the angular momentum (C_+) after the encounter are calculated;

4. Then, the particle goes back to its initial conditions at the point P, and the equations of motion are integrated backward in time, until the distance d is reached again. Then the energy (E_-) and the angular momentum (C_-) before the encounter are calculated.

For all of the simulations shown, a fourth-order Runge-Kutta method with step size control and a Runge-Kutta of 8-th order were used for numerical integration. The result of this comparison is that there is no distinction in the plots obtained. The constant value for the Jacobian constant also is a proof that both numerical integration methods worked very well. The criteria to stop numerical integration is the distance between the spacecraft and M_2. When this distance reaches the value d = 0.5 (half of the semimajor axis of the two primaries) the numerical integration is stopped. The value 0.5 is a lot larger than the sphere of influence of M_2 for the Earth-Moon system, that is used here (which is, 0.00077 in canonical units), which avoids any important effects of M_2 at these points. Simulations using larger values for this distance were performed, and it increased the integration time, but did not significantly change the results. To study the effects of numerical accuracy, several cases were simulated using different integration methods and/or different values for the accuracy required with no effects in the results. All of the calculations were performed with an IBM-PC computer using the Microsoft Fortran Power Station 4.0 Compiler.

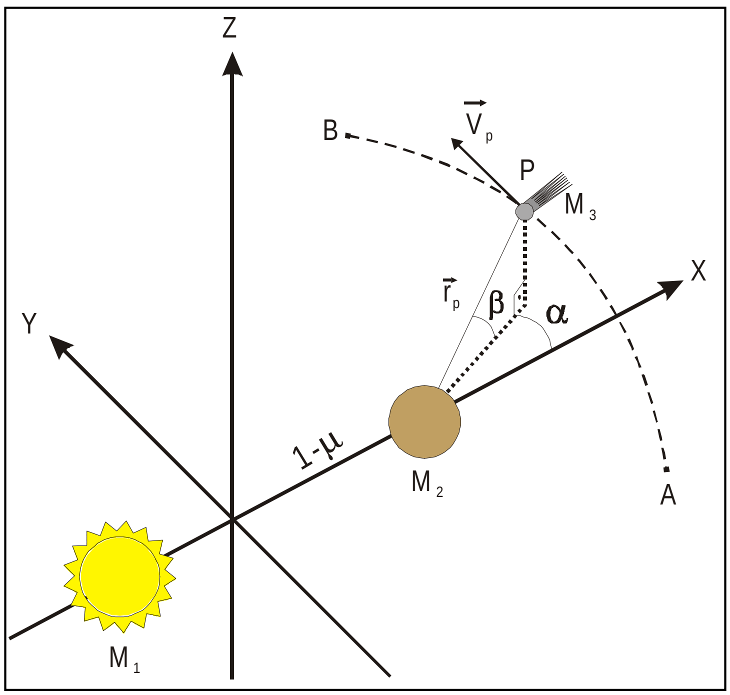


Figure 3. Trajectory of the Comet during the Swing-by.

4. CHANGES AND CLASSIFICATION OF THE ORBITS

The results consist of plots that show the change of the orbit of the comet caused by the close encounter with a planet (Saturn). The Sun-planet system of primaries is useful to study missions using a swing-by with a planet. This system was used in the Voyager, Ulysses and others missions. To develop new technical methods to study the swing-by maneuver, it is necessary to analyze the comet's behavior. The results consist of plots that show what happens to the comet after the close encounter with the planet for a large range of given initial conditions. First of all it is necessary to classify all the close encounters between the planet and the comet, according to the change obtained in the orbit of the comet. The letters A-P are used for this classification in the same way it was used in Felipe and Prado (1999) and Prado (1993). They are assigned to the orbits according to the rules showed in Table 1.

Table 1- Rules to assign letters to orbits.

Before	*After*			
	Direct ellipse	*Retrograde ellipse*	*Direct hyperbola*	*Retrograde Hyperbola*
Direct ellipse	A	E	I	M
Retrograde ellipse	B	F	J	N
Direct hyperbola	C	G	K	O
Retrograde hyperbola	D	H	L	P

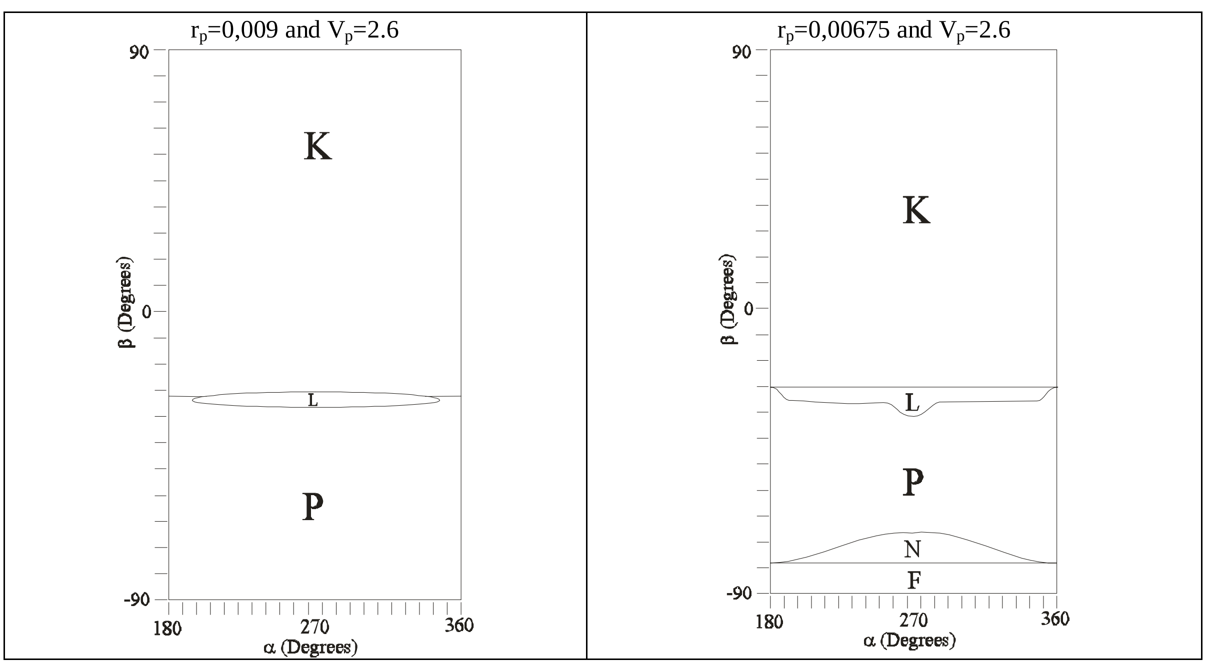
5. RESULTS

After defining the meaning of the letters, the results consist of assigning one of those letters to a position in a two-dimensional diagram that has the parameter α (in degrees) in the horizontal axis and the parameter β (in degrees also) in the vertical axis. This type of diagram is called a "letter-plot", and it was used before in Broucke (1988). For each plot a total of 961 trajectories were generated, dividing each axis in 31 segments. The interval plotted for α is $180° < \alpha < 360°$ because there is a symmetry with respect to the vertical line $\alpha = 180$ °.

Looking at the simulations, it is possible to verify the influence caused by the angles that the maneuver is realized. To perform the simulations the Microsoft Fortran was used, where a integration routine was created to execute all the calculations. The results obtained are represented in the Fig. (4), where the values for the angle α are between 180 and 360 degrees and β is between -90 and 90 degrees. The values to r_p and V_p were chosen to represent the most significant results, after a large number of simulations were realized.

By examining Fig. (4), it is possible to identify the following families of orbits: a) Orbits that result in an escape (transfer from elliptic to hyperbolic), that are represented by the letters I, J, M, N and that appear in horizontal stripes close to the bottom of the plots; b) Orbits that result in a capture (transfer from hyperbolic to elliptic), that are represented by the letters C, D, G, H that does not appear in plots shown, but that exist in the symmetric parts not shown; c) Elliptic orbits (transfer from elliptic to elliptic), that are represented by the letters A, B, E, F and are at the bottom of plots; d) Hyperbolic orbits (transfer from hyperbolic to hyperbolic), that are represented by the letters K, L, O, P that are at the upper part of the plots; e) Orbits that change the direction of motion from retrograde to direct, that are represented by the letters B, D, J, L, that appear in the center of the plots; f) Retrograde orbits that are represented by the letters F, H, N, P, that appear in the lower parts of the plots; g) Direct orbits that are represented by the letters A, C, I, K that appear in the upper part of the plots.

The border lines between those families are also interesting families of orbits. The borders that separate elliptic from hyperbolic orbits are made by parabolic orbits. An example of a border that has parabolic orbits after the close approach is F-N. An example of a border that have parabolic orbits before the close approach is N-L. The borders that separate direct from retrograde orbits represent orbits with zero angular momentum. In this case, position and velocity are parallel (rectilinear orbits). An example of a border that has zero angular momentum after the close approach is L-P. An example of a border that has zero momentum before the close approach is K-L.



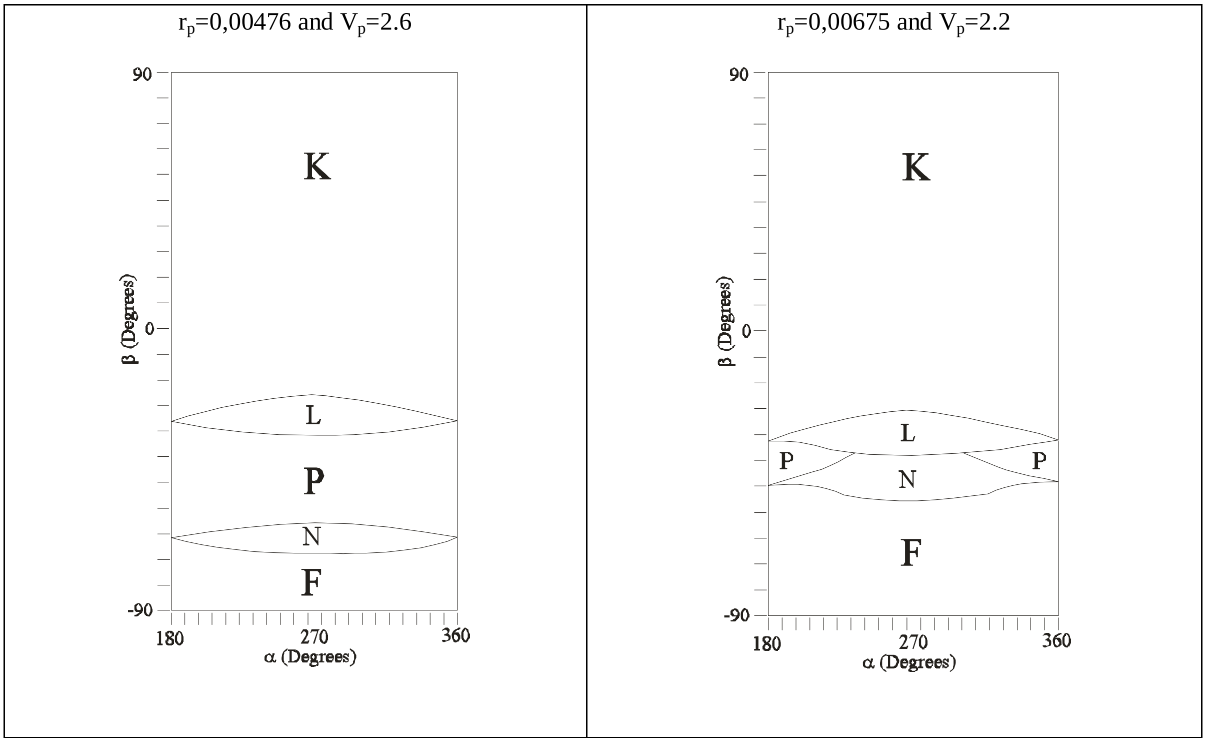


Figure 4. Some results of a swing-by maneuver.

It is easy to see those families of orbits by examining the figures. In a general way, a typical plot can be divided in three regions with respect to the energy (elliptic to elliptic, elliptic to hyperbolic, hyperbolic to hyperbolic) and in three regions with respect to the angular momentum (retrograde to retrograde, direct to retrograde, direct to direct). The final format is a result of the intersections of those regions.

In the literature, there are not many papers producing similar results to compare with ours. One of them is Broucke (1988) that generated similar results for the planar case. Another one is Felipe and Prado (1999), which studied simulations and the solution of optimal problems using the three-dimensional case. Considering the study of comets, there is also Prado (1993) that studied the close approach between a comet and the planet Jupiter, in the planar case, including statistics about the orbital distribution of the orbits after the passage. The results found in the present paper are complementary to the ones found in the literature, but they are consistent, which allow us to believe that the numerical algorithm is working very well.

5. CONCLUSIONS

In this paper the circular three-dimensional restricted three-body problem is described and used to study the swing-by maneuver. Several letter-plot type of graphics are made to represent the effect of a close approach in the orbit of a comet.

A large number of simulations were realized, allowing us to verify the relations between the parameters α, β, r_p, V_p and their influences in the orbital trajectory of a comet. The gravitational captures happens when the comet pass from a hyperbolic orbit to an elliptic orbit. Those cases are represented for the letters C-D-G-H. The results showed that the captures usually happens for lower values of r_p (distance between Saturn and the Comet), because the orbits tend to be elliptic after the passage.

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