COMPLIANCE OPTIMIZATION OF LAMINATED SHELL STRUCTURES

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Abstract. *This paper presents different approaches for solving problems of topology and orientation optimization of laminated shell structures. The objective of the design is the minimization of volume under compliance constraints. The design variables are the relative densities and the principal material direction orientation of each layer in an element. A two-level strategy is used, optimizing sequentially the orientation and then the density, aiming reducing the computational effort during each iteration. Sequential Linear Programming method is used to solve both optimization problems. Mathematical algorithms were derived for the solution of the problem. These algorithms were coded for single and multiple loading cases. The topology optimization can be considered as an extension of Cardoso (2000) and Sant'Anna (2002) works for laminated shell structures. An eight node degenerated shell finite element with explicit integration on the thickness direction, as in Kumar et al., is used to solve the equilibrium equations for laminated composites. Some illustrative examples are presented and discussed to show the applicability of the proposed optimization approache.*

Keywords*. laminated shells, topology optimization, orientation optimization, two-level strategy, compliance.*

1. Introduction

Laminated shell structures are currently being used in many applications, from high technology aircraft to simple handmade surf boards. However, design methodologies for such structures were developed only in the last few decades. Nowadays, simple structural analysis is not enough to obtain a good design. For example, in aircraft, an aerodynamic surface structure is not only expected to resist to all kind of loads it is subject to, but is also expected to weight the least possible, increasing the aircraft power/weight ratio and thus increasing performance. This and other examples show the importance of optimization methods in the laminated shell design.

Structural optimization is an important tool for the engineer, because it involves at the same time the structural analysis and the search for the best design under certain objectives and constraints. These objectives depend on the whole project, but the most common are the minimization of total mass or total volume of the structure, under compliance, stress, strain or failure criteria constraints, among others. In a general point of view, all objectives are related to cost minimization.

The characteristics and properties of laminated shells made of orthotropic layers give the engineer many design parameters that can be used to achieve the desired operation conditions, such as displacements, compliance, stress, strain, etc. Some of these parameters are layer thickness, layer or fiber orientation, layer material, number of plies, stacking sequence, etc. Fiber and layer orientation are important design parameters because they have influence on the structure behavior when in service life, and are often present in laminated composite structure optimization. It was searched in this work to combine orientation optimization with topology optimization methods applied to continuum isotropic structures, aiming volume minimization under compliance constraints. For multi-layer shells, the design obtained is similar to definition of stiffeners.

Structural optimization requires the solution of static or dynamic equilibrium equations. The most used method is the Finite Element Method (FEM). Finite elements for solution of equilibrium equations of laminated shells were derived from isotropic single layer shell finite elements. One example is the degenerated shell finite element with explicit trough-thickness integration presented by Kumar and Palaninathan (1997), used in the present work.

1.1. Laminated composite material

Composite material are those made of two or more different materials or phases, with different physical and mechanical properties. These combinations are made in order to obtain a material with a resulting behavior that could not be achieved by conventional materials (Agarwal and Broutman, 1990). Different classifications of composite material can be found. Dietz (1969,apud Cardoso and Fonseca, 2000), divides them in three groups: fibrous, particulated and laminated. Jones (1975) presented a nomenclature definition for the study of fiber reinforced laminated composite material. A ply can be considered as a plane arrangement of unidirectional fibers or woven. A laminated is a sequence of plies with different material principal direction orientation. Laminates can be composed of layers made of different material or made of different fiber reinforced plies. Fiber and matrix can be even metallic or non-metallic. The most used fibers are metals such aluminum, cooper, iron, steel and titanium or organic material such as glass, carbon, boron and graphite (Reddy, 1997). The methodology here presented can be used to optimize shells made of any of these materials.

2. Laminated Shell Finite Element

The laminated shell finite element used in the present work is derived from an eight node degenerated shell FE. To consider the contributions of each layer of the structure, an explicit through-thickness integration is used. Reminding that the objective of the work is optimization of laminated shell structures, the element formulation must allow an easy and fast calculation of stiffness matrix derivatives.

The element kinematics are similar to what is presented, for example, by Hughes (1987) or Zienkiewicz and Taylor (1991). Kumar and Palaninathan (1997) presented an explicit integration method for the thickness direction, using three models. The difference between these models is the way the inverse Jacobian matrix elements vary through the thickness. The model chosen for this work consider it constant. This leads to the following element stiffness matrix:

$$\mathbf{K}_e = \int_{-1}^{1}\int_{-1}^{1}\left(\mathbf{B}_1^T\mathbf{C}_1\mathbf{B}_1 + \mathbf{B}_1^T\mathbf{C}_2\mathbf{B}_2 + \mathbf{B}_2^T\mathbf{C}_2\mathbf{B}_1 + \mathbf{B}_2^T\mathbf{C}_3\mathbf{B}_2\right)\frac{2}{t}\left|\mathbf{J}\right| d\xi_1 d\xi_2 \ . \tag{1}$$

where matrices $\mathbf{B}_1$ and $\mathbf{B}_2$ are the strain-displacement matrices as presented by Kumar and Palaninathan (1997), t is the total element thickness and $\mathbf{J}$ is the Jacobian. The constitutive tensors above are defined here as:

$$\mathbf{C}_1 = \sum_{k=1}^{nl}\rho_k\mathbf{C}_k^{lc}\left(z_t - z_b\right)_k, \quad \mathbf{C}_2 = \sum_{k=1}^{nl}\rho_k\mathbf{C}_k^{lc}\left(z_t^2 - z_b^2\right)_k, \quad \mathbf{C}_3 = \sum_{k=1}^{nl}\rho_k\mathbf{C}_k^{lc}\left(z_t^3 - z_b^3\right)_k, \qquad (\text{no sum on k}). \tag{2}$$

In these equations, nl is the element number of plies and z_t and z_b are the layer top and bottom coordinates. The relative density ρ_k is inserted in this work because it is used as design variable for the topology optimization, explained in the section 3.4. The layer constitutive tensor $\mathbf{C}_k^{lc}$ is function of the layer orientation, and is obtained trough a plane rotation of $\mathbf{C}_k^{lm}$, which is the modified layer constitutive tensor accounting for transversal shear effects (Jones, 1975) and expressed in the layer coordinate system:

$$\mathbf{C}_k^{lc} = \mathbf{Q}_{pl}^T\mathbf{C}_k^{ts}\mathbf{Q}_{pl} \ . \tag{3}$$

The rotation matrix $\mathbf{Q}_{pl}$ is given by:

$$\mathbf{Q}_{pl} = \begin{bmatrix} \cos^2(\theta) & \sin^2(\theta) & 0 & 0 & 0 & \cos(\theta)\sin(\theta) \\ \sin^2(\theta) & \cos^2(\theta) & 0 & 0 & 0 & -\cos(\theta)\sin(\theta)c \\ 0 & 0 & 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & 0 & 0 & \sin(\theta) & \cos(\theta) & 0 \\ -2\cos(\theta)\sin(\theta) & 2\cos(\theta)\sin(\theta) & 0 & 0 & 0 & \cos^2(\theta)-\sin^2(\theta) \end{bmatrix}. \tag{4}$$

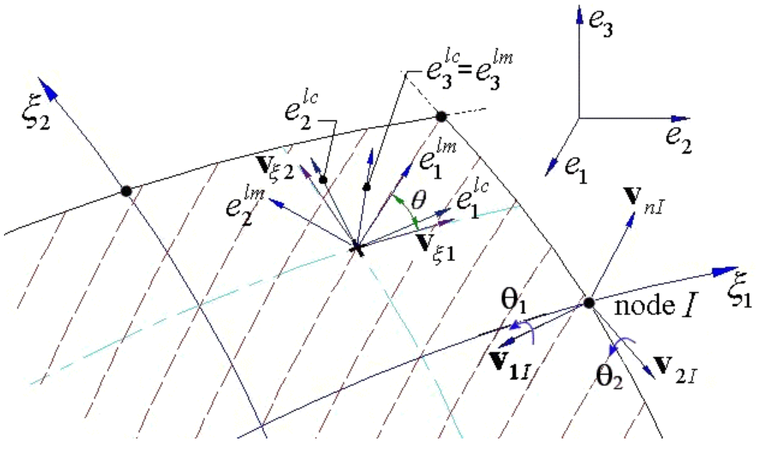

Figure 1. Coordinate systems for laminated shell finite element.

Figure (1) presents these rotations and different systems. The $\mathbf{C}^{lm}$ constitutive tensor is expressed in the layer coordinate system, defined by vectors $\mathbf{e}_1^{lm}$, $\mathbf{e}_2^{lm}$ and $\mathbf{e}_3^{lm}$. The vector $\mathbf{e}_1^{lm}$ is parallel to the larger material elasticity modulus direction. The tensor $\mathbf{C}^{lc}$ is defined in the local coordinate system, defined by $\mathbf{e}_1^{lc}, \mathbf{e}_2^{lc}$ and $\mathbf{e}_3^{lc}$. This system lies on the same plane of the layer coordinate system, and they share the same perpendicular vector ($\mathbf{e}_3^{lm} = \mathbf{e}_3^{lc}$). Vectors $\mathbf{v}_{\xi 1}$ and $\mathbf{v}_{\xi 2}$ are parallel to the parametric coordinate system and are calculated at each integration point, being necessary to define the local coordinate system. To calculate the rotations degrees of freedom at each node, a nodal coordinate system is defined by vectors $\mathbf{v}_{1I}$, $\mathbf{v}_{2I}$ and $\mathbf{v}_{nI}$, being this last one a vector normal to the reference surface at the node. The global coordinate system, defined by $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$, is the reference one and used to express nodal displacements.

3. Optimization Procedure

Earlier works in laminated structures optimization appeared only in the late 1960's. Foye (1968) studied minimum weight of laminates, searching for strength and stiffness optimum design, for in-plane multiple loads. Waddoups (1969) obtained minimum weight designs using strength constraints under distinct multiple load cases, considering maximum strain or Tsai-Hill failure criteria. The design variable was the ply orientation, but the search for the optimum was exhaustive. Schmit and Farshi (1973) presented a method to obtain the minimum weight optimum design of symmetric composite laminates under multiple in-plane loads, using the layer thickness as design variables. The optimization algorithm was an adaptation of the so called inscribed hyperspheres and consisted in a sequence of linear programming with fast convergence.

An important work on fiber orientation optimization of composite material structures was made by Pedersen (1989). He searched for the maximum and minimum energy densities on orthotropic material structures, working only with fiber orientation. Solving plane elasticity problems, he concluded that the optimum orientation depends only of the relation between the two principal strain directions and non-dimensional invariant material parameters. In 1990, Pedersen returned to this subject, considering now FEM analysis. For materials with relative low in-plane shear stiffness , the maximum stiffness is obtained aligning the biggest material elasticity modulus direction with the biggest principal strain direction. Cheng (1994) has discussed Pedersen’s results for the use of principal strain directions to update the orientation and compared methods presented by Suzuki e Kikuchi (1991) and Díaz and Bendsøe (1992), who used principal stress directions. Pedersen’s method presents coupling between principal strain direction and the design variable θ, while using methods based on stress, this coupling becomes weaker. Díaz and Bendsøe could also solve multiple loads problems. Cheng presented then a stress based improved method, using a formulation with generalized stress, and not only principal stress.

Conceição António et al. (1995) solved laminated plate and shell problems using a two-level strategy. Their objective was to obtain a minimum weight structure that could support a set of external static loads without failure. The domain was splitted in macro-elements with different stacking sequence on each one. In the first level it was maximized the structure efficiency, the ply orientation being the design variable and using mathematical programming. In the second level, the weight of the structure was minimized, working with ply thickness as design variable.

Mota Soares et al. (1995) presented an model for the optimization of thin composite laminated plates, using also a two-level approach. In the first level it was minimized the maximum displacement or maximized certain vibration mode frequency, using as design variables the orientation angles of certain ply, under boundary constraints. The objective of the second level was the volume minimization under displacement, stress and/or Tsai-Hill failure criteria, specific vibration mode frequency or boundary constraints.

Maute and Ramm (1997) presented a work in adaptive topology optimization of shell structures. They made topology optimization to find the basic configuration, inserting or removing holes in the ground structure, and then made a shape optimization. Maximum stiffness problems with mass constraints were discussed, using optimally criteria. It was verified that, for thin shells, the transversal shear energy caused checkerboards when shear locking occur.

Considering the works cited above, the approaches to be used in this work were defined. The option for a two-level strategy is obvious. The idea is to find a structure with the minimum volume for a certain compliance, specified *a priori*. For laminated structures made of orthotropic layers, it is possible to optimize the orientation, minimizing the compliance in a first level. In a second level, each layer can have its topology optimized, reducing the structure volume. The use of two levels leads also to the number of design variables reduction during each iteration, and gives an easier and more robust way to obtain each of the objective functions and all sensitivities. The topology optimization level is based on the work of Cardoso (2000) and Sant’Anna (2002). When optimizing the topology of only some layers of the structure, this approach is similar to the design of shell stiffeners. The chosen methods and optimization formulas are described below.

3.1. Two level approach

In this work a two-level approach is used, each one being:

- 1st level - compliance minimization, with the ply orientation on each element as design variable, without constraints (size optimization);

- 2nd level - volume minimization, with relative densities of the plies of each element as design variable, and considering the compliance as constraint (topology optimization).

On Fig. (2) it can be seen a flow chart for the strategy used. As it is presented here, the process passes through each level just once. Another strategy would be to come back to first level after each step of the second level, but tests showed no improvement in this strategy.

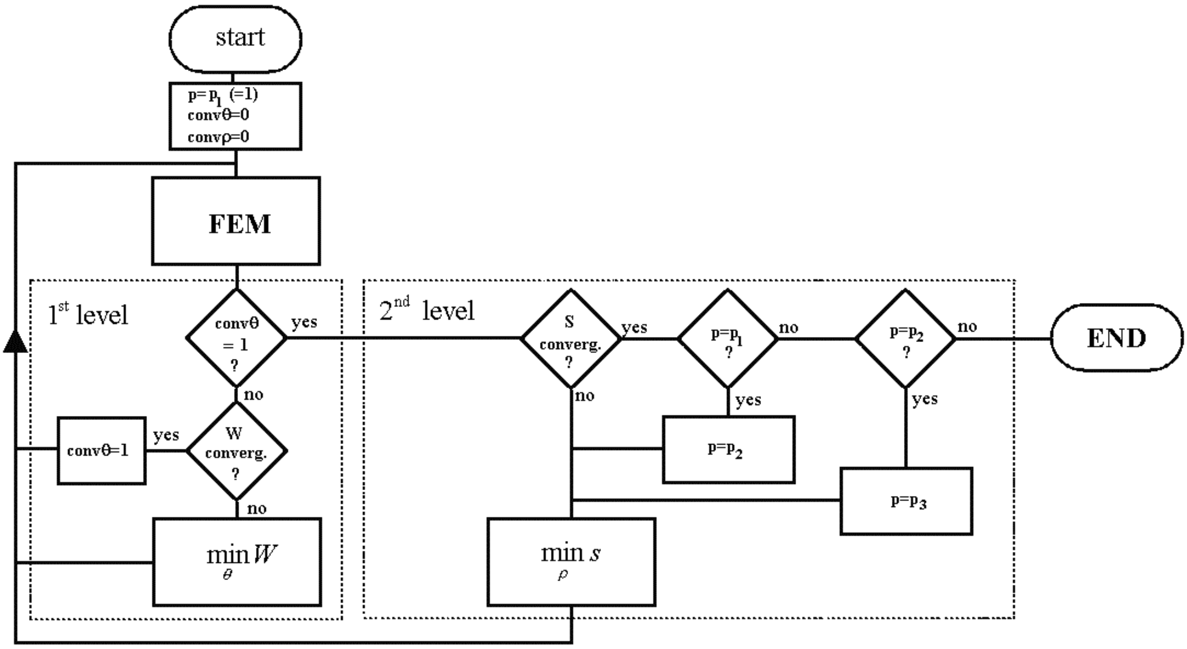

Figure 2. Two-level strategy procedure.

After the convergence of the first level, the flag $conv\theta$ is turned on, what deviates the algorithm to the second level. At this level it is shown the passage through the Continuation Method (CM) steps (see Topology Optimization section). The algorithm verifies the convergence of the objective function *s*, and in the positive case, verify to which CM step it must go. Tests showed that there is practically no difference between three or more steps, and in the most cases only two are enough.

Sequential Linear Programming (SLP) is used in both levels. This method requires the linearization of each objective function and constraints. First order Taylor series are used, what leads to differentiation of the related functions with respect to each level design variable. A first order approximation, however, is good only in the neighborhood of the optimum point and then it is necessary to use move limits. A similar strategy used by Sant'Anna (2002), based on the iteration history and on design variable behavior, was applied in both levels to update the move limits. Theory and details about SLP and Mathematical Programming in general can be found in Haftka and Gürdal (1992) and Cheng (1992). The mathematical algorithms derived below were coded using MATLAB, what allowed the use of some of its functions. One example is the "linprog", used to solve the Linear Programming step.

3.2. Orientation Optimization

The orientation optimization problem, in the first level, can be written as:

$$\min_{\theta} W(\theta). \tag{5}$$

This is an unconstrained optimization, since the stiffest structure is searched in this optimization level, allowing more volume reduction in the second level. The structure compliance is given by:

$$W = \mathbf{q}^T \mathbf{K} \mathbf{q}, \tag{6}$$

where $\mathbf{q}$ is the global displacement vector and $\mathbf{K}$ is the global stiffness matrix. The differentiation of Eq. (6) with respect to the design variable θ is given by:

$$\frac{\partial W}{\partial \theta} = \mathbf{q}^T \frac{\partial \mathbf{K}}{\partial \theta} \mathbf{q} + 2\mathbf{q}^T \mathbf{K} \frac{\partial \mathbf{q}}{\partial \theta}. \tag{7}$$

The design variable θ is defined on each element layer, and thus the stiffness matrix derivative must be calculated at each layer. So, $\partial W/\partial \theta$ is a vector with dimension n_{desvar} (number of design variables, equal to element number multiplied by the number of layers to be optimized). The derivative of $\mathbf{K}$ with respect to θ is then given by:

$$\frac{d\mathbf{K}_n}{d\theta}=\int_{-1}^{1}\int_{-1}^{1}\left(\mathbf{B}_1^T\frac{d\mathbf{C}_{1n}}{d\theta}\mathbf{B}_1+\mathbf{B}_1^T\frac{d\mathbf{C}_{2n}}{d\theta}\mathbf{B}_2+\mathbf{B}_2^T\frac{d\mathbf{C}_{2n}}{d\theta}\mathbf{B}_1+\mathbf{B}_2^T\frac{d\mathbf{C}_{3n}}{d\theta}\mathbf{B}_2\right)\frac{2}{t}\left|\mathbf{J}\right|d\xi_1 d\xi_2 \,, \tag{8}$$

where n varies from 1 to the number of n_{desvar} and

$$\frac{d\mathbf{C}_{1n}}{d\theta}=\rho_n\left(z_t-z_b\right)_n\frac{d\mathbf{C}_n^{lc}}{d\theta},\quad \frac{d\mathbf{C}_{2n}}{d\theta}=\rho_n\left(z_t^2-z_b^2\right)_n\frac{d\mathbf{C}_n^{lc}}{d\theta},\quad \frac{d\mathbf{C}_{3n}}{d\theta}=\rho_n\left(z_t^3-z_b^3\right)_n\frac{d\mathbf{C}_n^{lc}}{d\theta}. \tag{9}$$

In the expression above, ρ_n is the relative density of the element layer being optimized, z_b and z_t are the bottom and top coordinate of the layer in the thickness direction. .

For the differentiation of $\mathbf{q}$ with respect to θ, it should be used the expression $\mathbf{Kq}=\mathbf{f}$. Since the derivative of $\mathbf{f}$ with respect to θ is null, this equation leads to:

$$\mathbf{K}\frac{\partial\mathbf{q}}{\partial\theta}=-\frac{\partial\mathbf{K}}{\partial\theta}\mathbf{q}\,. \tag{10}$$

Finally, inserting Eq. (10) into Eq. (7):

$$\frac{\partial W}{\partial\theta}=\mathbf{q}^T\frac{\partial\mathbf{K}}{\partial\theta}\mathbf{q}+2\mathbf{q}^T\left(-\frac{\partial\mathbf{K}}{\partial\theta}\mathbf{q}\right)=-\mathbf{q}^T\frac{\partial\mathbf{K}}{\partial\theta}\mathbf{q}\,. \tag{11}$$

This means that the differentiation of W with respect to θ is actually given only by the differentiation of the constitutive tensor on Eq. (8) and Eq. (9).

To extend this problem to multiple load cases, a strategy suggest by Krog and Olhoff (1997) is used. The idea is to consider a weighted sum of the compliance associated to each load case. This approach maintains the formulation obtained above, keeping all advantages of the use of a SLP method, its robustness and simplicity, not being necessary to reformulate the problem for different type of structures. The optimization problem on Eq. (5) is simply modified to:

$$\min_{\theta} w_j W_j\left(\theta\right),\qquad j=1..n_{cases}\,, \tag{12}$$

where w_j and W_j are the weight and compliance associated to each j load case, and n_{cases} is the number of load cases applied to the structure. To obtain all weights w_j, each compliance W_j is calculated after the first FEM step, as:

$$W_j=\left(\mathbf{q}^{\mathrm{T}}\mathbf{f}\right)_{\mathrm{j}}\,. \tag{13}$$

Each weight is then calculated in the following way:

$$w_j=\left|W_j\right| \Big/ \sum_{j=1}^{n_{casos}}\left|W_j\right|\,. \tag{14}$$

3.3. Topology optimization

For continuum structures, the topology optimization introduces an universe of possible structural elements, in a certain design domain, called ground structure, and search for the best material distribution inside this universe (Fonseca, 1997). Bendsøe (1995) presented a revision of the earlier works, including works of Bendsøe and Kikuchi (1988), Suzuki and Kikuchi (1991) and Diaz and Bendsøe (1992), for example. The use of microstructures has been introduced, leading to optimal solutions for many cases and problems. Microstructure, however, brings the problem of structures with intermediate densities, what may be very difficult to build, if not impossible. To solve this problem, different procedures have been developed (for example, Suzuki and Kikuchi, 1991, Haber et al., 1996, Beckers, 1997, among others). Here a cost function penalization (Rozvany, 1997) has been chosen, as in Sant'Anna (2002), and the optimization problem is defined as:

$$\begin{aligned}&\min_{\rho}\quad s\left(\rho\right)=\sum_{n=1}^{n_{des\,var}}\left[\rho_n^p+\alpha\rho_n\left(1-\rho_n\right)\right]V_n\\&\text{s. t.}\quad W_j\left(\rho\right)-W_{\lim}\le 0,\qquad j=1..n_{cases,}\\&\qquad 0<\rho\le 1\end{aligned} \tag{15}$$

where $s(\rho)$ is the objective function, ρ is the relative density of the cell material (Bendsøe , 1995), defined on each layer being optimized, p and α are the penalization parameters, $W_{\lim}$ is the limit compliance specified for the final solution and V_n is the volume of each layer to be optimized inside each element. This formulation includes multiple load cases through the constraints, with $W_j(\rho)$ being the linearized compliance associated to each load case.

The use of this penalized formulation brings a new problem: the optimization problem becomes non-convex, what leads to the non-uniqueness of the discrete problem solution. To overcome it, a Continuation Method (CM) is used (Cardoso, 2000): the optimization problem is solved in two or more steps. The problem is initially solved with a linear approach for ρ, using $p = 1$ and $\alpha = 0$. This is a convex problem, what guarantees the solution unicity, even obtaining many intermediate densities. The solution obtained is then used to start new optimization steps, with $p < 1$ and $\alpha > 0$, what changes the problem to a non-convex one. Intermediate densities become more "expensive", and the algorithm searches for solution containing only minimum or maximum densities areas. The behavior of the penalized function is seen on Fig. (3).

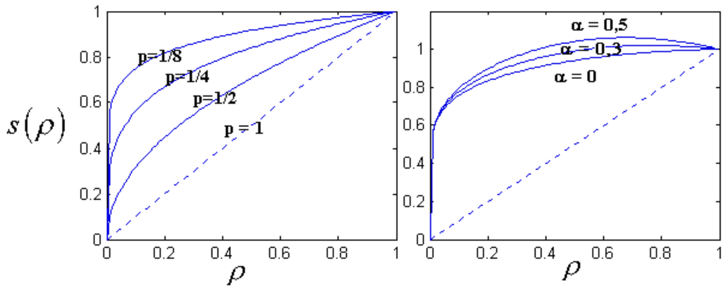

Figure 3. Penalization of $s(\rho)$ as in Eq. (15) for $\alpha = 0$ at left, and for p = 1/8 and various values of α. at right.

The volume Vn is calculated as:

$$V_n = \int_{-1}^{1}\int_{-1}^{1}\left(z_t - z_b\right)_n \left|\mathbf{J}\right| d\xi_1 d\xi_2 \,. \tag{16}$$

The differentiation of $s(\rho)$ with respect to ρ on each "n" element layer is then given by:

$$\frac{\partial s_n}{\partial \rho} = \left[p\rho_n^{p-1} + \alpha\left(1 - 2\rho_n\right)\right] V_n \,. \tag{17}$$

As it is used SLP, it is necessary also to obtain a linear expression for the constraint. To obtain the compliance derivative for each "j" load case, it is used the Eq. (13), leading to:

$$\frac{\partial W_j}{\partial \rho} = \frac{\partial}{\partial \rho}\left(\mathbf{q}_j{}^T \mathbf{K} \mathbf{q}_j\right) = \mathbf{q}_j^T \frac{\partial \mathbf{K}}{\partial \rho} \mathbf{q}_j \,. \tag{18}$$

The stiffness matrix derivative for each n design variable is given by:

$$\frac{d\mathbf{K}_n}{d\rho} = \int_{-1}^{1}\int_{-1}^{1}\left(\mathbf{B}_1^T \frac{d\mathbf{C}_{1n}}{d\rho}\mathbf{B}_1 + \mathbf{B}_1^T \frac{d\mathbf{C}_{2n}}{d\rho}\mathbf{B}_2 + \mathbf{B}_2^T \frac{d\mathbf{C}_{2n}}{d\rho}\mathbf{B}_1 + \mathbf{B}_2^T \frac{d\mathbf{C}_{3n}}{d\rho}\mathbf{B}_2\right)\frac{2}{t}\left|\mathbf{J}\right| d\xi_1 d\xi_2 \,. \tag{19}$$

The derivatives of the constitutive tensors are calculated from (2) as:

$$\frac{d\mathbf{C}_{1n}}{d\rho} = \left[\mathbf{C}^{lc}\left(z_t - z_b\right)\right]_n, \quad \frac{d\mathbf{C}_{2n}}{d\rho} = \left[\mathbf{C}^{lc}\left(z_t^2 - z_b^2\right)\right]_n, \quad \frac{d\mathbf{C}_{3n}}{d\rho} = \left[\mathbf{C}^{lc}\left(z_t^3 - z_b^3\right)\right]_n . \tag{20}$$

4. Numerical Results

The two-level optimization strategy was applied in 2D elasticity, plates, cylindrical and spherical shells problems. Isotropic problems were also tested, to compare results, and stacking sequence is varied, to compare orientation and topology solutions. In this paper, some of the obtained results are presented, including multiple loads cases, using both orientation and topology optimization. The structure limit compliance (W_{lim}) has always been defined as a factor of the original structure compliance (W_0), which is calculated after the first FEM step. Volume reduction results are expressed as function of the original volume (V_0). In the tests made, it was used a homogeneous density distribution, always equal to one, and it was not investigated the influence of different initial distributions. The material properties used are given by Reddy (1997).

Problems of 2D elasticity are common in the literature. Due to its simplicity, they were used to evaluate the algorithm during its implementation. The fixed beam problem is useful to show the advantage of the orientation optimization before the topological one. On Fig. (4), results are presented for beams made of one Graphite – Epoxy layer, with the principal material direction parallel to the x_1 direction, using an W_{lim} of $2W_0$.

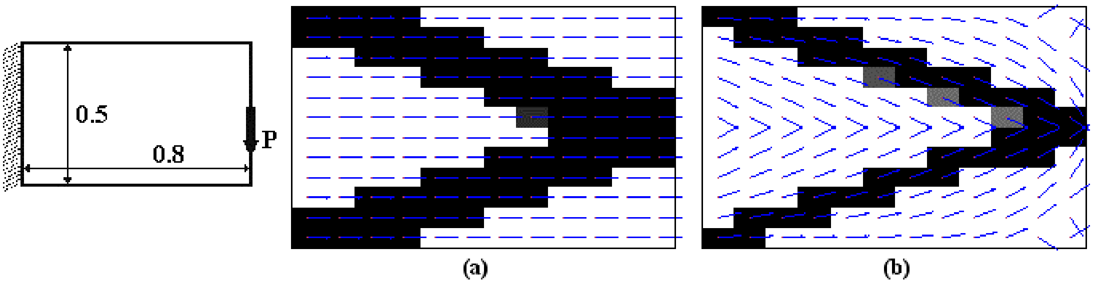

Figure 4. Final topology and orientation of a fixed boundary beam: (a) only topology optimization (V_f = 0,45V_0) and (b) optimizing also the orientation with PSD-SLP (V_f = 0,27V_0). Material: Graphite-Epoxy.

Figure (5) shows the graphics of the objective function, structure volume and compliance behavior during the iteration history for both cases cited above. At left, there is only topology optimization, in two steps. In the first step, the structure total volume is equal to the objective function $s(\rho)$. After its convergence, the penalization factor is changed to 1/8, what is noticed by the modification in the objective function behavior. The structure compliance reaches its maximum values (W_{lim}) in few iterations, and the final volume obtained when applying only topology optimization is $0.45V_0$. The vertical dashed line divides the two CM steps. At right, there are two optimization levels. First, the orientation one, with minimization of structure compliance, and $s(\rho)$ and V remaining constant. The second level has two CM steps also, with similar behavior of all curves. It may be noticed that the structure compliance now starts with less than 50% of its original value, what allows more volume reduction: as the algorithm tries to obtain a structure compliance of $2W_0$, it can remove more material, and the final volume decreases to $0.27V_0$.

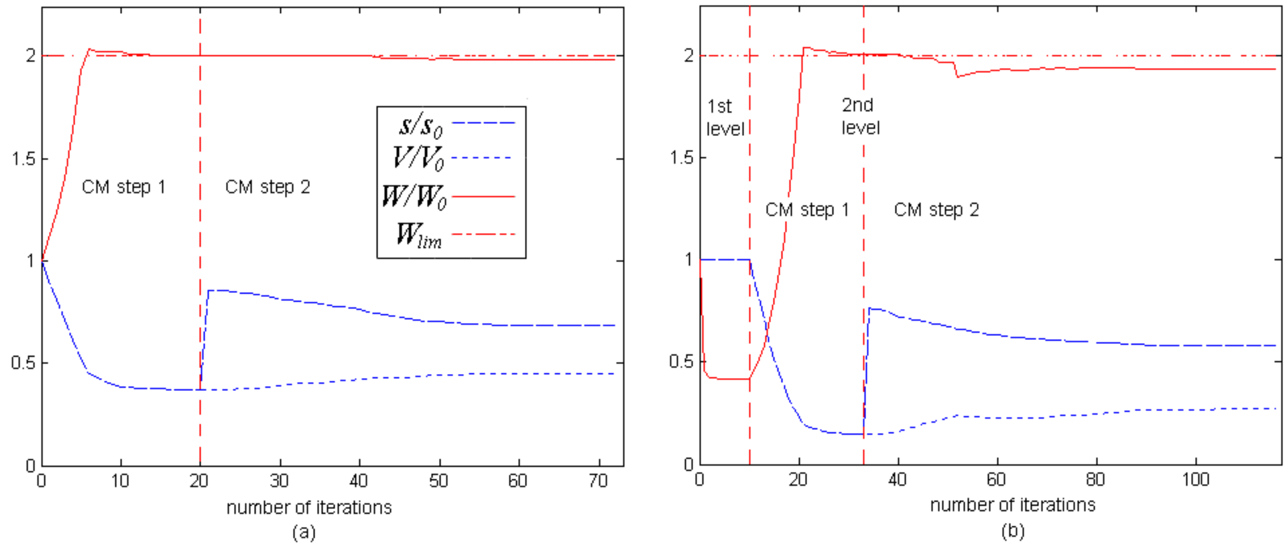

Figure 5. Final topology and orientation of a fixed boundary beam: (a) only topology optimization (V_f = 0,45V_0) and (b) optimizing also the orientation with PSD-SLP (V_f = 0,27V_0). Material: Graphite-Epoxy.

The solution of a single layer orthotropic shell problem is shown on Fig. (6). The original layer orientation was 45º on the quadrant, and W_{lim} = 0.5, what leads to a final volume of $0.75V_0$. Comparing with the previous result, it can be seen the importance of the definition of this limit compliance. The first optimization level can reduce excessively the compliance, what may lead to an excessive volume reduction in the topology optimization level, sometimes obtaining a statically underterminated structure. In this problem, three steps were used in the CM: p = 1 and α = 0, p = 1/8 and α = 0.3 and finally p = 1/12 and α = 0.5. Even with three steps, instead of only two used in the most cases tested, not all intermediate densities were eliminated. The remaining, meanwhile, are low, near minimum, and could be eliminated in a topology interpreting post-processing step (not performed here).

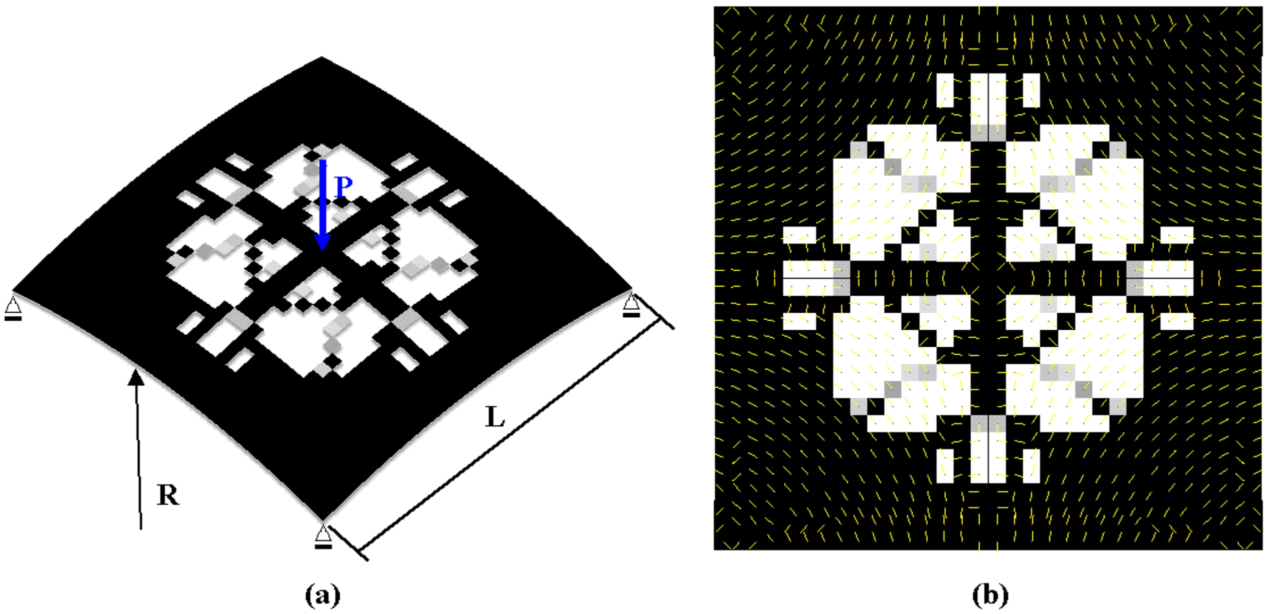

Figure 6. On (a), a 3D topology representation of the single orthotropic layer optimized shell with initial orientation at 45º on each quadrant and W_{lim} = 0.5 W_0 (b) topology and orientation solution. Here, R = 10, L = 5, thickness 0.1.

For two layers and same boundary and load conditions of the previous problem, the solution obtained is shown on Fig. (7). The bottom layer had a orientation perpendicular to the upper one, and both were made by the same material (Gr-Ep). The limit compliance was $0.5W_0$, with two steps in the CM: p = 1 and p = 1/8, for α = 0. These two steps were enough to eliminate almost all intermediate densities, and the final volume obtained was 0.55 V_0. The construction of such structures is not trivial, however, since some areas of the upper layer is not in contact with the lower. When optimizing all layers of multi-layer structures, holes can appear inside the structures, or a layer can even disappear. This avoid this, the optimization should be applied to only the external layers, as in the following problem.

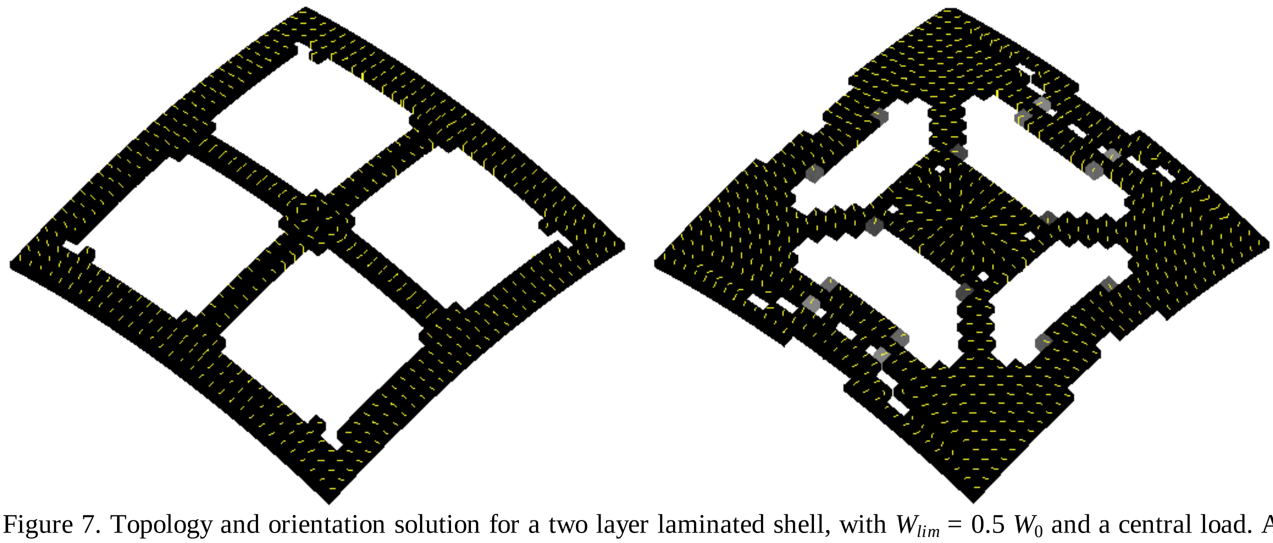

Figure 7. Topology and orientation solution for a two layer laminated shell, with W_{lim} = 0.5 W_0 and a central load. At left, lower layer and at right the upper one.

The solution of a multiple load and three layers problem is presented on Fig. (8). One applied load (Load 1) is composed by a central force (F_1) and the other load (Load 2) is composed by equal forces (0.7 F_1) applied in the middle of the edges. Only the lower and upper layers were optimized, what is equivalent to the determination of stiffeners for the middle structure. After solving the FEM problem in the first iteration, the weight of the compliance associated to each load case was calculated, being w_1 = 0.642 and w_2 = 0.357. In this problem, the compliance with bigger weight was chosen as W_0, but is not a rule. In this example, the limit compliance was $0.4W_0$, what resulted in a final volume of 48% of the initial one for the two layers. Only two steps of the CM were used, with p = 1/8 and α = 0.3 on the second one.

The iteration history in Fig. (8b) shows the two optimization levels and the two CM steps. In the first level, before the vertical dashed line, it can be seen the minimization of the compliance associated to each load case. In the second level, both compliance values reach their constraint. In the first step of the topology optimization level, the objective function and the volume of the optimized layers are the same, since the penalization factor p is equal to one. In the second CM step, the volume practically does not change, but *s* increases suddenly in the beginning to decrease later, when reaching the convergence. This is the typical behavior of all cases tested.

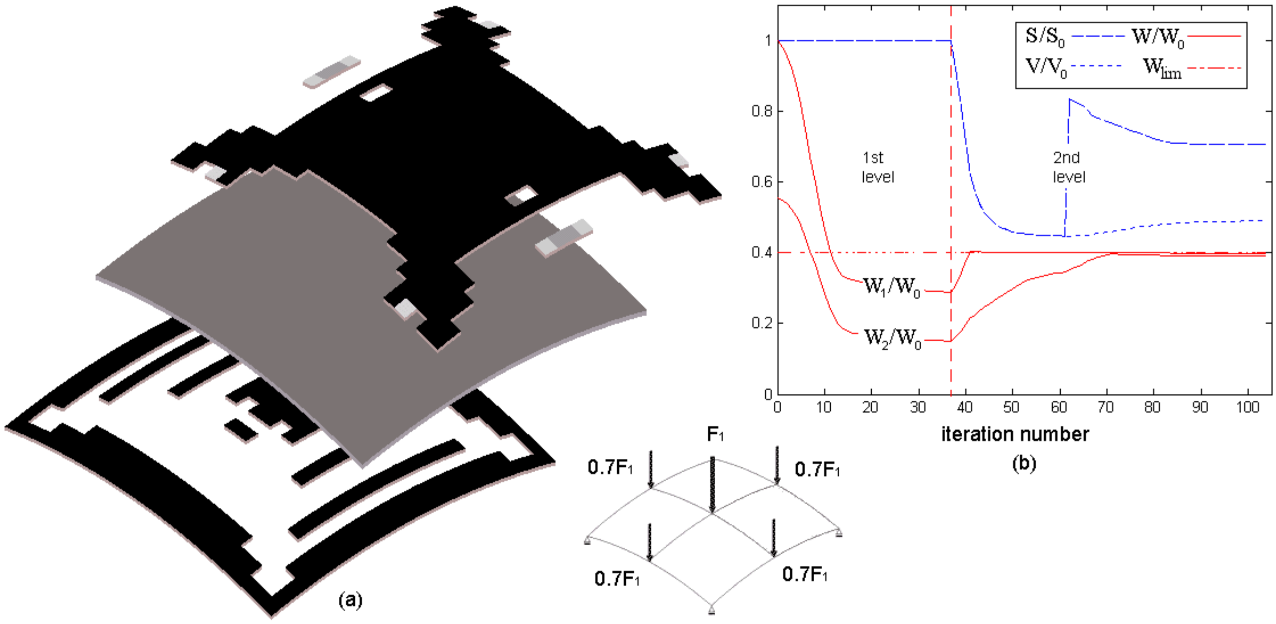

Figure 8. In (a), topology solution for a three layer optimized shell, with W_{lim} = 0.4 W_0 and multiple applied load. In (b), iteration history for this problem..

In the problem above, a variation on the weight of each load case (w_j) is possible only if the loads were modified, or new load cases were taken into account. Thus, when including new load cases in the analysis, new weights for all loads have to be defined. As can be seen in the optimal design obtained, on Fig. (8a), the upper layer resisting mostly to Load 1 and the bottom one to Load 2. Since the compliance associated to Load 2 was lower, and consequently its $w_2 < w_1$, it allowed more volume reduction on the bottom layer. This indicates the influence of each weight w_j on the optimal design.

4. Conclusion

A two-level strategy for the optimization of laminated shells has been presented. The two levels consisted in optimization of principal material orientation on each ply, minimizing the structure compliance, and topology optimization, minimizing each ply volume. In both levels Sequential Linear Programming (SLP) was used. Mathematical algorithms were derived, allowing solution of multiple load cases problems. For the equilibrium solution an eight-node degenerated laminated shell element with exact through thickness integration was implemented.

In the tests made, the method showed efficiency to solve many kinds of problems, from 2D elasticity and plates to spherical and cylindrical shells problems. The example of 2D elasticity presented here showed the advantage of a orientation optimization before the topology one: a larger volume reduction was obtained in comparison to only topology optimization for orthotropic structures. With the shell optimization examples, this advantage was emphasized. It could be seen that the orientation optimization leads to a large compliance reduction in the first optimization level, allowing an efficient volume reduction in the second level, even for low prescribed limit compliance (W_{lim}). Care should be taken to avoid the appearance of holes between layers, and good results were achieved when optimizing the external layers of the structure. The algorithm could solve also efficiently the multiple load problems, using a weighted compliance sum approach.

The methods presented here represent a contribution for the knowledge of laminated shells design methodologies. The use of this kind of structure is spreading from the aerospace industry to other sectors, as the efforts towards weight and cost reduction are universal. Therefore, the research in optimization is important for the development of the whole Brazilian industry.

5. Acknowledgement

This work has been supported by CNPq. Support from Grupo de Mecânica Aplicada, DEMEC-UFRGS and all of their personal is also gratefully acknowledged.

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