NUMERICAL MODEL OF AN INFLATABLE SOLAR COLLECTOR

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Resume: The main objective of this paper is to present a mathematical model to simulate the work of an inflatable solar collector for heat air used in grain drier. The use of inflatable solar collectors as energy solar absorbers for grain dry is a technically and financially workable solution to low the electrical energy consumption in this area. This collector is made by an absorber plate made by flexible black PVC and by one double cover made by flexible and transparent PVC. The solar collector reports a semi-cylindrical format when air is injected in the confined space between the two covers' walls. The main vantages of the equipment studied are the low costs and its easy transportation. The mathematical model of the collector is obtained from the application of the conservation energy law on the four basic parts of the collector, which are, its absorber plate, the two covers and the air flowing inside the equipment. To improve the model accuracy, the solar collector was divided in N control volumes, each one containing the four parts quoted before, so that one system with 4N differentials equations is obtained. The unknown quantities of this system are the absorber plate, the air and the two cover's walls temperatures on each control volume. The equation system is solved using the Runge Kutta of fourth order method. The results through the computational simulations showed that the collector performance depends strongly on some model entrance variables (inlet flow and air temperature) and of geometric equipment parameters (collector length, internal ray of the inferior cover and the space between the two covers). In this way, the model can be used on the solar collector optimization.

***Keywords:** solar collector, numerical model, solar air heaters, thermal performance, mathematical modeling.*

1. Introduction

The electricity is the form of energy more consumed in the world. Traditionally in our country, the electric energy was cheap because the majority of its generation to be proceeding from hydroelectric plants. Such plants use the force of waters of rivers for the production of electric energy. This kind of energy is considered a natural, renewable, of low cost and high ambient impact.

In accordance with the Electric Energy Conservation Program, PROCEL 2001, the Brazilian industrial sector consumes 46% of all electric energy produced in the country, or either, approximately 142,000,000 of megawatts. The Brazilian residential sector, that consumes about 24% of the electric energy produced in the country, is divided in the following way: 26% for the water heating for electric resistance, 32% for refrigeration, 24% for illumination and 18% for the remain. The electric shower is responsible for about 96% of the hot water production only in the Southeastern region, or either, electric resistances consume almost the totality of the used energy for bath. It is important to remember that this type of heating produces great overload for the generation of electricity in the schedules of consumption peak, besides being little efficient, since the electric resistance is a low efficiency equipment.

The economic growth of Brazil, in particular in the last years, has resulted in a bigger demand for energy. As the investments carried through for the government has not followed this growth, in 2001 the country suffered a serious crisis of electric energy supplying. This situation has lead people to reflect more about electric energy waste and, consequently, to search for other kind of energy for day-by-day use.

The present work is a theoretical study of a device for solar energy utilization. The use of solar collectors can reduce the demand of electric energy, since; in this case, the solar radiation is the only source necessary. The use of electric energy is only necessary when it does not have the sun direct radiation for a long period. The use of solar energy is economically viable because the only costs occur in the construction and maintenance of the equipment. Moreover, Brazil, for being a country almost totally located in the tropical region of the planet, it presents relatively high taxes of sunstroke during the year. This is another attractive in the equipment that makes use of this form of energy.

The inflatable solar collector numerical model was developed in order to simulate the air heating to be used in drying of grains. The inflatable solar collectors are constituted of a black absorber plate and a double transparent covering. The use of double covering is justified for the presence of air inside it and, also, to serve as sustentation for the collector. The plate and the covering are made by flexible plastic. The inflatable solar collectors can be used in the industrial sector, for drying of grains, wood and in food factories. For grains drying, these are put outside the collector and in the extremity of the air outlet for the best use of the heating air and the attainment of a faster drying. The objective of this work is to model and to simulate the functioning of inflatable solar collectors.

2. Mathematical modeling of the inflatable solar collector

Generally, heating systems that use solar collectors present a low thermal efficiency. For this, the satisfactory performance of solar collectors depends basically on an adequate project. In the last two decades, hundreds of studies in this area have shown a good development of conventional solar collectors, in particular, plan collectors. On the other hand, there are few works that treat on inflatable solar collectors and its analysis is restrict to the steady state thermal behavior of the collector (KOURY et al., 1982).

The inflatable solar collector used in the mathematical simulation of this work has a plan absorber plate in black and flexible PVC and two cylindrical coverings in transparent and flexible PVC.

The collector was divided in *N* control volumes, all of them with the same size. Each control volume presents four regions of different temperatures: absorb plate (T_p), portion of air (T_f), inferior cover (T_{ci}) and superior cover (T_{cs}).

Through the application of the laws of energy in the four components of the collector, for each control volume, four distinguishing equations are obtained:

$$\frac{dT_p}{dt}=\frac{1}{d_p\cdot c_{pp}\cdot 2r_1\cdot\Delta x\cdot e_p}\left(\frac{S\cdot 2r_1\cdot\Delta x\cdot\tau_c\cdot\alpha_p}{1-(1-\alpha_p)\cdot\rho_c}-h_{cpf}\cdot 2r_1\cdot\Delta x\cdot(T_p-T_f)-\frac{K_i\cdot 2r_1\cdot\Delta x}{e_i}\cdot(T_p-T_s)-h_{rpci}\cdot 2r_1\cdot\Delta x\cdot(T_p-\overline{T_{ci}})\right) \quad (1)$$

$$\frac{dT_f}{dt}=\frac{1}{d_f\cdot c_{vf}\cdot\frac{\pi r_1^2}{2}\Delta x}\left(h_{cpf}\cdot 2r_1\cdot\Delta x\cdot(T_p-T_f)+\dot{m}_f\cdot c_{pf}\cdot(T_{fe}-T_{fs})-h_{cfci}\cdot\pi r_1\cdot\Delta x\cdot(T_f-T_{ci})\right) \quad (2)$$

$$\frac{dT_{ci}}{dt}=\frac{1}{d_c\cdot c_{pc}\cdot\frac{\pi}{2}(r_2^2-r_1^2)\cdot\Delta x}\Big(\frac{S\cdot(1-\rho_c-\tau_c)\cdot 2r_1\cdot\Delta x}{2}+h_{cfci}\cdot\pi r_1\cdot\Delta x\cdot(T_f-T_{ci})+$$
$$h_{rcip}\cdot\pi r_1\cdot\Delta x\cdot(\overline{T}_p-T_{ci})-h_{rcics}\cdot\pi r_2\cdot\Delta x\cdot(T_{ci}-\overline{T}_{cs})-\frac{K_f\cdot\pi r_2\cdot\Delta x}{d_{ec}}\cdot(T_{ci}-T_{cs})\Big) \quad (3)$$

$$\frac{dT_{cs}}{dt}=\frac{1}{d_c\cdot c_{pc}\cdot\frac{\pi}{2}(r_4^2-r_3^2)\cdot\Delta x}\Big(\frac{S\cdot(1-\rho_c-\tau_c)\cdot 2r_1\cdot\Delta x}{2}+h_{rcsci}\cdot\pi r_3\cdot\Delta x\cdot(\overline{T}_{ci}-T_{cs})-$$
$$h_{ccsa}\cdot\pi r_4\cdot\Delta x\cdot(T_{cs}-T_a)-h_{rcsa}\cdot\pi r_4\cdot\Delta x\cdot(T_{cs}-T_{ceu})+\frac{K_f\cdot\pi r_3\cdot\Delta x}{d_{ec}}\cdot(T_{ci}-T_{cs})\Big) \quad (4)$$

Equations 1, 2, 3 and 4 refer to the absorb plate, the fluid, the inferior and superior covers, respectively. In the equations above subscripts *p*, *f*, *ci*, *cs*, *i*, *c*, *a* and the numbers *1*, *2*, *3* and *4* indicate the absorb plate, the fluid, the inferior and superior covers, the insulator, the double cover, the environment, internal part of the inferior cover, external part of the inferior cover, internal part of the superior cover and external part of the superior cover, respectively. Symbols *T* and *d* represent temperature and density. The symbols *cp* and *cv* represent the specific heat to the constant pressure and the volume, respectively. The radius, the thicknesses, the sunstroke and the thermal conductivity are symbolized by letters *r*, *e*, *S* and *K*, respectively. The coefficient of heat transfer of conduction and radiation are represented by h_c and h_r. The length of a control volume, the distance between the covers and the flow are symbolized by Δx, d_{ec} e $\dot{m}_f$. Finally, the transmissivity, the absorptive and the reflectivity are represented for τ, α and ρ, respectively.

The assumptions and other considerations used in the modeling of inflatable solar collector had been:

i. Collector located on the ground and guided horizontally,
ii. The collector was divided in *N* control volumes,
iii. The axial conduction in the plate and in the covers was neglected because the small thickness and low thermal conductivity of them,
iv. The heat exchange between the covers occurs by means of thermal conduction since the distance between them is small,
v. The solar radiation reaches the absorb plate perpendicularly,
vi. The air temperature in the center of the control volume is the arithmetic mean between the air inlet and outlet temperatures (T_{fe}, T_{fs}), or either:

$$T_f = \frac{T_{fe} + T_{fs}}{2} \tag{5}$$

vii. The temperature $\overline{T}_{ci}$ is the average temperature throughout all the length of the inferior cover, in a certain instant, or either:

$$\overline{T}_{ci} = \frac{\sum_{i=1}^{N} T_{ci}(i)}{N} \tag{6}$$

Obs. The temperatures $\overline{T}_p$ and $\overline{T}_{cs}$ are calculated in a similar manner to the temperature $\overline{T}_{ci}$.

viii. The expression for the h_{rpci} is given by the equation below:

$$h_{rpci} = \frac{\sigma \cdot (T_p^4 - \overline{T}_{ci}^4)}{T_p - \overline{T}_{ci}} \cdot \frac{\varepsilon_p \cdot F_{pci}}{F_{pci} \cdot (1 - \varepsilon_p) + \varepsilon_p} \tag{7}$$

where σ is the Stefan-Boltzmann constant, ε_p is the plate emissivity and F_{pci} is the form factor of the plate to the inferior cover.

Obs. The calculations of h_{rcip}, h_{rcics}, h_{rcsci} and h_{rcsa} are carried through similarly to the h_{rpci}.

ix. The coefficient h_{rcsa} is simpler to calculate because the form factor of an element of the superior cover for the sky is always unitary, independently of the position x. Its value is:

$$h_{rcsa} = \varepsilon_c \cdot \sigma \cdot \left(\frac{T_{cs}^4 - T_{ceu}^4}{T_{cs} - T_{ceu}} \right) \tag{8}$$

x. The convective coefficients h_{cpf} and h_{cfci} were calculated according to DUFFIE (1974), for the expression:

$$h_c = \frac{0{,}0158 \cdot k_f}{\mu_f^{0{,}8}} \cdot \frac{G^{0{,}8}}{D_H^{0{,}2}} \tag{9}$$

where k_f is the fluid thermal conductivity, μ_f is the fluid dynamic viscosity, G is the mass velocity and D_h is the fluid hydraulic diameter. The expressions below show how G and D_h are calculated.

$$G = \frac{\dot{m}_f}{A_f} \tag{10}$$

$$D_H = \frac{4 \cdot A_f}{r(\pi + 2)} \tag{11}$$

$$A_f = \frac{\pi r_1^2}{2} \tag{12}$$

xi. The convective coefficient h_{ccsa} was calculated, according to DUFFIE (1974), for the expression:

$$h_{ccsa} = 3{,}8 \cdot v + 5{,}7 \tag{13}$$

In the expression above, v is the speed of the wind outside the collector.

xii. For the calculation of the form factor, was used the expression below suggested for KOURY et al. (1991). This equation shows a reasonable variation of the form factor to the long of the collector,

$$F_{pci}(x) = e^{\left(-\frac{\ln 16}{a^2}\right) \cdot x^2} \tag{14}$$

This equation shows what can be easily verified intuitionally, or either, the form factor is 1 when the control volume analyzed is located in the center of the collector. This means that the inferior cover absorbs all the energy emitted for this control volume of the plate. The form factor is 0.5 for the two control volumes of the extremities, or either, half of the energy emitted for them is absorbed by the inferior cover and the other half it is absorbed by the environment. The figure below shows graphically the behavior of the equation above.

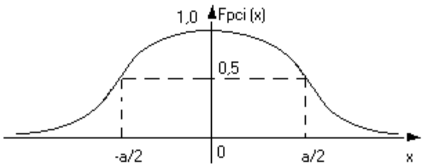

Figure 1: Variation of the form factor along the collector.

xiii. According to KOURY et al. (1991), the thermal efficiency in the interface between two control volumes of the collector, η, is given by:

$$\eta_{x,i+1} = \frac{\dot{m}_f \cdot c_{pf} \cdot \left(T_f(i+1) - T_f(i)\right)}{S \cdot A_p} \quad (15)$$

The mathematical modeling produced, for *N* control volumes, a system of *4N* differentials equations. The initial conditions of the equation system are the absorb plate, air and covers temperatures that are equal to the environment temperature. The methodology of solution of this equation system involves a numerical treatment through the method of Runge-Kutta of fourth order. The solution of the system gives the thermal profile along the collector at each time step Δt. Thus, in the generation of the thermal profile in the instant of time t, all profiles of the previous times are already known. On the other hand, some parameters of the differential equations are functions of temperature, as the radiation and the convective coefficients, the air density, etc. Such values, in the generation of the thermal profile of instant t, can be calculated from the thermal profile of the instant t-Δt. Since that Δt is small enough, a small error will be obtained to T_p, T_f, T_{ci} and T_{cs}.

3. Input data of the program

The simulation of the collector behavior of made through a computational program created in programming language C.

The input data of the program are:

K_i = 0,52W/m·K	e_c = 0,001m	e_i = 0,10m
N = 35	τ_c = 0,72	ρ_c = 0,20
$\alpha_c = \varepsilon_c$ = 0,10	T_a = 27 °C	$c_{pp} = c_{pc}$ = 835J/kg·°C
$\alpha_p = \varepsilon_p$ = 0,90	$T_{ceu} = T_a - 2{,}0$	v = 2,0m/s
$d_p = d_c$ = 2010kg/m^3	$T_s = T_a - 2{,}0$	$T_p = T_f = T_{ci} = T_{cs} = T_a$
$\dot{m}_f$ = 10L/s/m^2		

KOURY et al. (1991) use the value of the flow of incident solar radiation in the superior cover, *S*, equal to 800W/m^2, being this value the daily average flow solar radiation in one typical day of summer in Brazil.

The parameters a, d_{ec}, r_1 and $\dot{m}_f$ had been varied in the simulation in order to analyze its influence on the collector operation. The values of r_2, r_3 and r_4 are given by:

$$r_2 = r_1 + e_c \quad (16)$$

$$r_3 = r_2 + d_{ec} \quad (17)$$

$$r_4 = r_3 + e_c \quad (18)$$

4. Results and discussion

Next items present simulation results of the behavior of the inflatable solar collector. The thermal profiles in transient and steady-state regimen, the influence of the length of the collector, of the inferior cover internal radius, of the air flow inside the collector and the distance between the inferior and superior covers on the efficiency of the inflatable solar collector.

The thermal efficiency in the interface of two control volumes is calculated through equation 15.

4.1. Thermal profile in transient regime

The thermal profile in transient regime of the inflatable solar collector was obtained to the last control volume (N = 35) and length of the collector of a = 10,0m, internal radius of the inferior cover r_I = 0,30m, airflow inside the collector $\dot{m}_f$ = 10L/s/m^2 and distance between the covers d_{ec} = 0,005m. The result is:

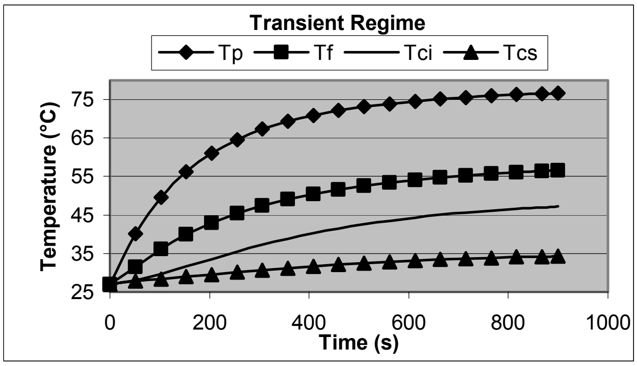

Figure 2: Thermal profile of the collector in transient regime.

The figure above shows that the collector reached the permanent regime around 700s.

Considering that the total time of solar exposition throughout one day is long (about 10 hours) and that the collecting time to enter in permanent regime was about 700s (12min), it can be concluded that the solar collector reaches the operation in permanent regime quickly.

4.2. Thermal profile in steady state regime

The steady-state regime of the inflatable solar collector was considered from 20min. The graph below shows the thermal profile in permanent regime of the absorb plate temperature in function of the position of the collector. The values of a, N, r_I, $\dot{m}_f$ and d_{ec} are the same ones used to originate the thermal profile in transient regime. The result is:

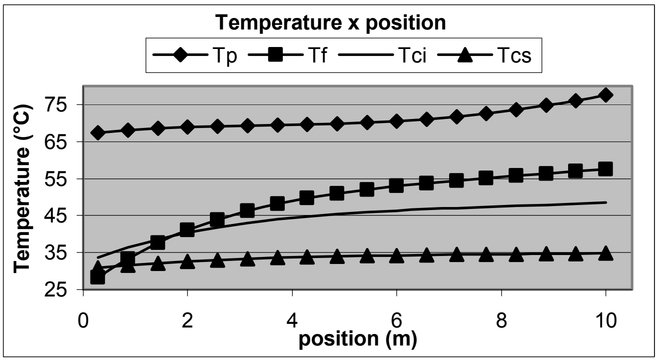

Figure 3: Thermal profile of the collector in permanent regime.

Figure 3 shows that T_p, T_f, T_{ci} and T_{cs} increase with the position of the collector and the temperature of the absorb plate is greater than the air temperature. The air temperature is greater than the inferior cover temperature and, finally, this is greater than the superior cover temperature.

4.3. Influence of the flow on the efficiency on the inflatable solar collector

The airflow inside the collector was varied in order to see its influence on the efficiency and the graph of the figure below was obtained. The other parameters were not modified. The results confirm that to bigger flows the collector thermal efficiency is greater.

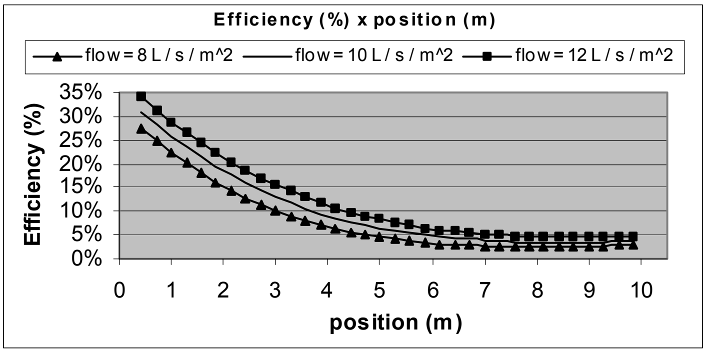

Figure 4: Influence of the flow on the collector efficiency.

4.4. Influence of collector length on the inflatable solar collector efficiency

It is verified, through the analysis of figure 4 that, for any flow, the collector thermal efficiency decreases along the collector, in direction to the air exit. This means that the average collector efficiency decreases if it increases its length. In figure 6, the length of the collector, was varied in order to prove the affirmation above. The other parameters had the following values: $N = 35$, $r_I = 0{,}30\text{m}$, $\dot{m}_f = 10\text{L/s/m}^2$ and $d_{ec} = 0{,}005\text{m}$.

As the collectors have different lengths, the efficiency was located in function of the number of control volume, since this did not vary in any of the tested configurations.

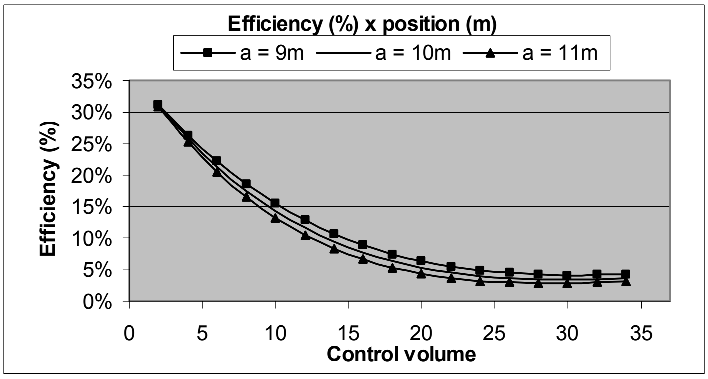

Figure 5: Influence of the length of the collector on the collector efficiency.

4.5. Influence of the internal radius of the inferior cover on the inflatable solar collector efficiency

The internal radius of the inferior cover, r_I, is another very important parameter in the analysis of the construction of the inflatable solar collector. The objective in the construction of the collector is to obtain a large efficiency with a minimum area.

Figure 6 shows the influence of r_I on the efficiency of the collector. The other parameters of the simulation had the following values: $a = 10{,}0\text{m}$, $N = 35$, $\dot{m}_f = 10\text{L/s/m}^2$ and $d_{ec} = 0{,}005\text{m}$.

The results of figure 6 indicate that, for any value of the internal radius of the inferior cover, the collector efficiency decreased along the collector, in direction to the air exit. Another conclusion is that the collector efficiency increases with the increase of r_I. Such fact occurs because bigger internal radius of the inferior cover implies in a bigger flow and since the efficiency is a directly proportional function to the flow, its value increases.

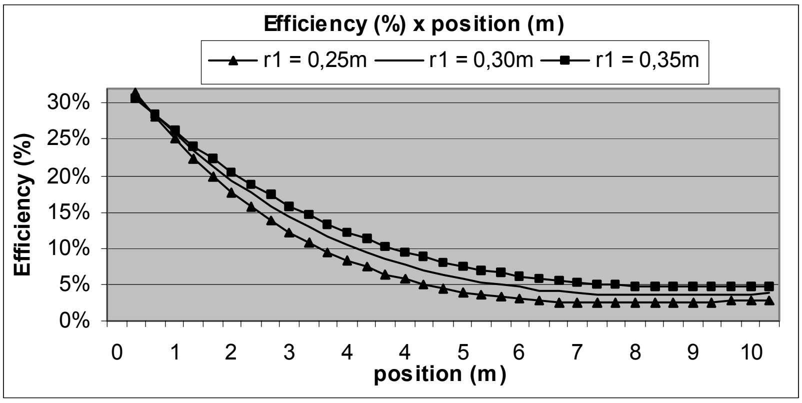

Figure 6: Influence of the internal radium of the inferior cover on the collector efficiency.

4.6. Influence of the distance between the covers on the inflatable solar collector efficiency

The distance between the inferior and superior covers, d_{ec}, is the last parameter of the inflatable solar collector efficiency to be analyzed. The objective, this time, is to determine the best value for d_{ec} in order to get the best thermal insulation with lower covers material spending.

Figure 7 shows the influence of d_{ec} on the collector efficiency. The other parameters of the simulation had the following values: a = 10,0m, N = 35, $\dot{m}_f$ = 10L/s/m^2 and r_1 = 0,30m.

In accordance with figure 7, the collector efficiency increase with the increase of the distance between the covers. The explanation for this fact is that a bigger distance between the covers generates a bigger thermal insulation, with the disadvantage to use more material for the construction of the inflatable solar collector.

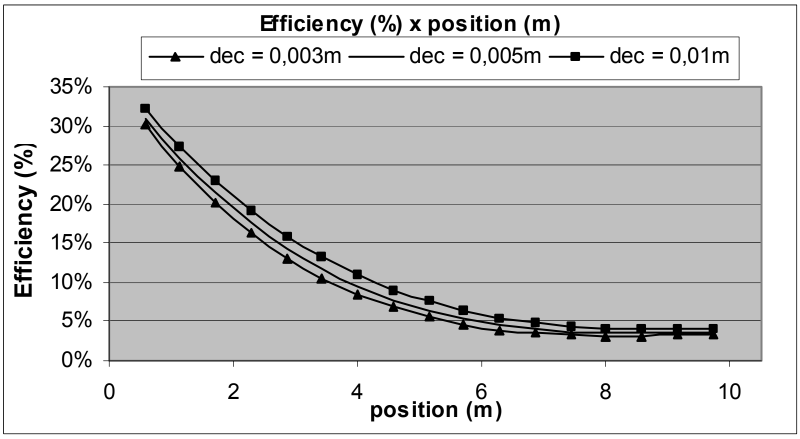

Figure 7: Influence of the distance between the inferior and superior covers on the collector efficiency.

5. Conclusion

This work presents results of the thermal profiles in transient and steady state regimes and the influence of airflow, length of the collector, internal radius of the inferior cover and distance between the covers on the collector efficiency. Beyond these items, for the project of the collector, it must be considered the involved expenses with construction materials. An example is the increase of the distance between the covers. This makes the collector efficiency also increases; however, greater expense with cover material occurs. The results obtained in this work indicate that the use of the inflatable solar collectors is a good solution for the reduction of the consumption of electric energy in Brazil. The results also show that the inflatable solar collectors quickly reach the conditions of steady state regimen; the average collector efficiency decreases if its length increases and the air flow in the collector, the internal radius of the inferior cover and the distance between the covers also influence the efficiency of the collectors. Finally, the conclusions of this work were obtained for fixed conditions, therefore for other environment conditions, material and geometrics the results can be different.

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