BEHAVIOR OF ERA AND ITS VARIANTS IN THE PRESENCE OF NOISE AS FUNCTION OF THE DAMPING AND FLEXIBILITY OF A STRUCTURE

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Abstract. *Several techniques were developed for identification of modal parameters (natural frequencies, damping factors and modes) for linear dynamical systems. The problem consists in identifying a minimum realization of the system in state space using experimental data. If there is no noise all the identification techniques perform well; when there is noise present in the data the techniques differ a lot. Our interest is in the Eigensystem Realization Algorithm technique, ERA, and in its variants: ERA/DC, that uses correlations for the Hankel matrices; OKID/ERA (Observer/Kalman filter identification/ ERA), that introduces a state observer to compress the data and to improve the results of the identification; OKID/ERA/DC, that uses correlations and a state observer. A comparison of the four variants is done using a structure developed by Nasa Langley Research Center and known as Mini-Mast.*
In the simulations done in order to compare the four variants we vary the degree of noise in the data, the flexibility of the structure and the damping. The smaller the damping, the greater the flexibility, and the higher the degree of noise in the data the harder is the identification of the structure. The examples we use separate well the four variants and show that ERA/OKID/DC is the best method.
The results show that ERA is the less efficient variant in the presence of noise and it is not capable of identifying some of the modes, and, even worse, it introduces spurious modes. The methods with observer can discriminate modes associated with frequencies that are close. If the damping is small, even with little noise, the ERA and ERA/DC do not identify well the parameters. In all cases the variants with observer were the most reliable and also required the least amount of computational effort whence they should be preferred.

Keywords: *modal parameter identification, flexible structures, realization algorithms, structure dynamics.*

1. Introduction

Some methods have been developed for identification of modal parameters in the time domain. A group of them, known as realization algorithms and based on the minimum realization theory in linear systems analysis conceived by Ho and Kalman, is studied in this work. The formal application of the minimum realization theory for identification of modal parameters was first introduced for Juang and Pappa, 1985, with the Eigensystem Realization Algorithm (ERA). An extension of the ERA using data correlation gave origin to the ERA/DC Juang et al., 1988. Later, Juang et al., 1993, had presented a new algorithm that calculates the system Markov parameters using the OKID (Observer/Kalman filter identification).

Juang, 1994, made an extensive study of the method ERA and its variants showing many successful practical cases, studying the way to select the number necessary of observer Markov parameters and system Markov parameters for a particular system to get the biggest amount of data with low contamination for noise. Lew et al., 1993, had studied the method ERA and ERA/DC for identification of modal parameters, comparing them with other methods, testing the methods for some levels of noise and damping factors bigger than 1%. Abdelghani et al., 1998, compared the OKID/ERA with other methods, also for some levels of noise, but without varying the damping of the system.

The aim of the present study is to compare the effectiveness of the ERA and its variants to identify the modal parameters in a MIMO system (Mini-Mast), with some modes associated to almost identical natural frequencies, in the presence of low or high noise levels. Also, is studied if the separation of frequencies property is kept in the case to vary in the system the damping factor, studying for low or high damping for different noise levels.

2. Mini-Mast problem

In the numerical example a simulated Mini-Mast model is considered (see Fig. (1)). The Mini-Mast is a deployable space truss of 20 m of length, built for NASA Langley Research Center for research in structural dynamics and active control of vibrations. It is deployed vertically and cantilevered from its base in a rigid foundation. The system has two actuators (torque wheels) and two sensors (Kaman sensors).

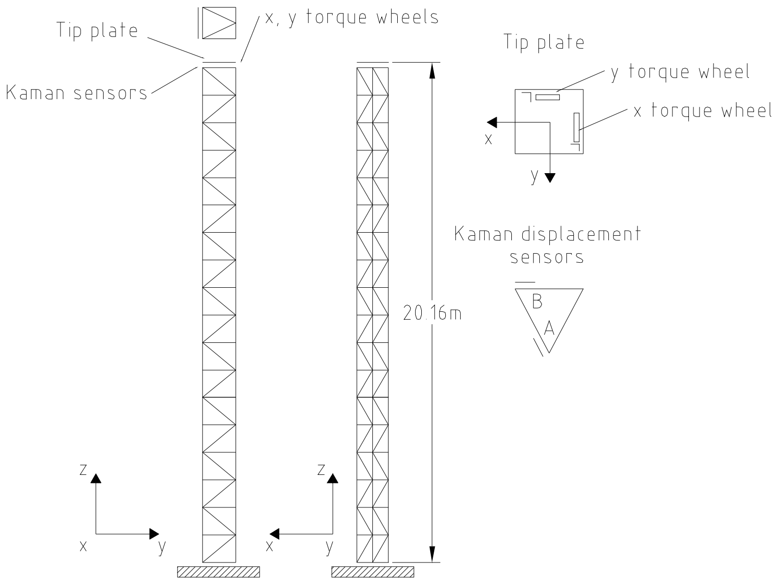

Figure 1: The Mini-Mast structure (Abdelghani et al., 1998).

The mathematical model has 5 modes (10 states) that are listed in Tab. (1), the first two frequencies are close and the associated modes representing the first bending mode in the x and in the y axes, respectively; the third mode is the first torsional mode; the last two frequencies are also close, and the associated modes are the second bending mode; this make with that the model is enough complex to test the identification methods employed. The matrices of the system in space of states are shown in Lew et al., 1993.

Table 1: Modal parameters of the Mini-Mast's mathematical model

mode	ω (Hz)	ζ (%)
1	0.8010	1.8
2	0.8016	1.8
3	4.3644	1.2
4	6.1041	1.0
5	6.1568	1.0

As magnitude of comparison of the exactitude of the methods in the identification, we consider the maximum singular values of the function of transference of the identified system. Only the frequencies identified inside of the band up to 10 Hz are considered. The number of samples is always $l = 1800$ with a sampling time of 0.03 s. For the comparison, the dimension of the Hankel matrix in ERA and OKID/ERA, and the dimension of the Hankel block correlation matrix in ERA/DC and OKID/ERA/DC, is kept the same for each case of study, as suggested for Lew to keep same times of execution of the algorithms.

The inputs used in the simulation are gaussian white noises with average zero and standard deviation 10. In the study are considered 3 levels of noise of 1%, 5% and 10%, for each case of study of the system with low damping ($\zeta < 1\%$), average damping ($1\% < \zeta < 4\%$) and high damping ($\zeta > 4\%$). The process noise is set at $\mathbf{f}\%$ of the input and the measurement noise about $\mathbf{f}\%$ of the maximum output, both as standard deviation ratios, where $\mathbf{f}$ is the level of desired noise. To observe the consistency of the algorithms, the average of 20 runs in each analysis was calculated.

3. System with average damping

The damping factors of the original system is in the band $1\% < \zeta < 4\%$.

In the ERA, were used $k = 250$ ($k + 1$ system Markov parameters calculated).

In the ERA/DC, were used $k = 325$ ($k + 1$ system Markov parameters calculated).

In the OKID/ERA, were used $p = 50$ ($p + 1$ observer Markov parameters calculated) and $k = 250$ ($k + 1$ system Markov parameters calculated).

In the OKID/ERA/DC, were used $p = 50$ ($p + 1$ observer Markov parameters calculated) and $k = 325$ ($k + 1$ system Markov parameters calculated).

3.1. CASE: 1%<ζ<4% with low noise levels (1%)

Table 2: Low noise identification results for system with average damping: natural frequencies (ω) and modal damping factors (ζ).

	mode 1		mode 2		mode 3		mode 4		mode 5	
	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)
TRUE	0.8010	1.800	0.8016	1.800	4.3644	1.200	6.1041	1.000	6.1568	1.000
ERA	0.7917	1.447	0.8034	2.958	4.3655	1.188	6.1034	0.910	6.1556	0.995
ERA/DC	0.7929	1.456	0.8075	2.042	4.3587	1.202	6.1033	1.023	6.1600	1.012
OKID/ERA	0.8009	1.806	0.8016	1.786	4.3659	1.285	6.1063	1.133	6.1584	1.036
OKID/ERA/DC	0.8009	1.806	0.8016	1.787	4.3659	1.284	6.1063	1.132	6.1584	1.036

For average damping and with 1% of noise, all modes are sufficiently well identified by the four variants (fig. (2)). Tab. (2) show good results for the natural frequencies and can be observed a little better approach with OKID/ERA and OKID/ERA/DC in the identification of the damping factors, especially with those associate to modes 1 and 2.

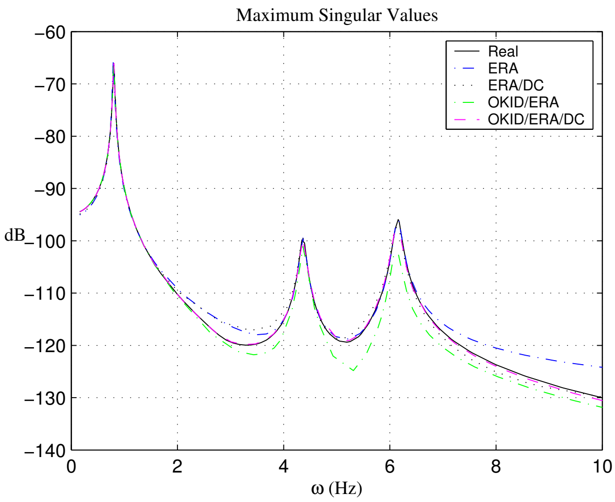

Figure 2: Mini-Mast 1%< ζ <4% (1% of noise).

3.2. CASE: 1%<ζ<4% with medium noise levels (5%)

Table 3: Medium noise identification results for system with average damping: natural frequencies (ω) and modal damping factors (ζ).

	mode 1		mode 2		mode 3		mode 4		mode 5	
	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)
TRUE	0.8010	1.800	0.8016	1.800	4.3644	1.200	6.1041	1.000	6.1568	1.000
ERA	0.8011	2.427	0.8093	1.183	4.3600	1.138	6.0988	1.022	6.1532	1.009
ERA/DC	0.7989	1.180	0.8085	1.495	4.3649	1.078	6.1168	1.024	6.1383	1.174
OKID/ERA	0.8002	1.878	0.8014	1.809	4.3841	2.938	6.1156	2.024	6.1774	2.060
OKID/ERA/DC	0.8005	1.867	0.8016	1.829	4.3774	3.137	6.1063	1.994	6.1733	2.117

For noise levels of 5%, the comparison of the maximum singular values of the transfer functions is shown in fig. (3), the natural frequencies are still good identified by all the methods. The modal damping factors associates to modes 1 and 2, are better identified by OKID/ERA and OKID/ERA/DC (see Tab. (3)), but in the case of the modal damping factors associates to modes 3, 4 and 5, are better identified by ERA and ERA/DC. This happens because the number of Markov parameters used in the ERA (k = 250) and in the ERA/DC (k = 325) is enough to guarantee the decline, but in the case of methods using OKID, the number of Markov parameters employed is bigger of the necessary ones, including dates contaminated after of the complete decline.

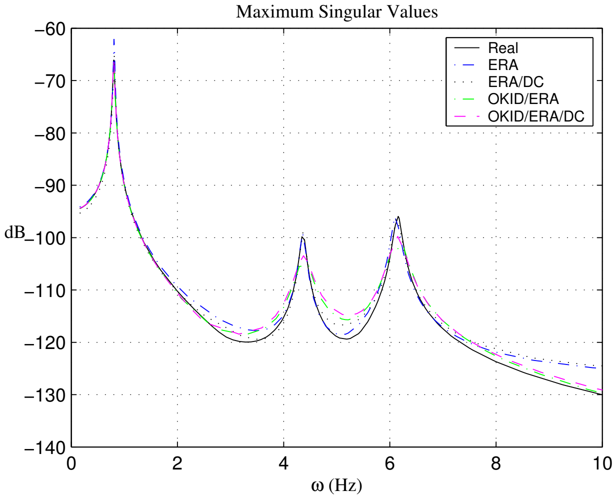

Figure 3: Mini-Mast 1%< ζ <4% (5% of noise).

3.3. CASE: 1%<ζ<4% with high noise levels (10%)

Table 4: High noise identification results for system with average damping: natural frequencies (ω) and modal damping factors (ζ).

	mode 1		mode 2		mode 3		mode 4		mode 5	
	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)
TRUE	0.8010	1.800	0.8016	1.800	4.3644	1.200	6.1041	1.000	6.1568	1.000
ERA	0.8052	1.128	0.8735	10.634	4.0600	0.790	-	-	6.1552	0.977
ERA/DC	0.7977	2.220	0.8811	0.453	5.0567	1.010	-	-	6.1566	1.007
OKID/ERA	0.7972	2.164	0.8026	1.649	4.3820	4.524	6.0937	2.512	6.1604	2.614
OKID/ERA/DC	0.7973	2.159	0.8026	1.656	4.3818	4.533	6.0946	2.516	6.1599	2.620

For 10% of noise, the results in Tab. (4) show that OKID/ERA and OKID/ERA/DC keep a robustness to the noise for the identification of the natural frequencies, obtaining errors smaller than 0.5% and differentiating the two pairs of modes associated to almost identical natural frequencies, but ERA and ERA/DC have problems to identify some modes (mode 4 was not identified) and obtaining errors of 16% approximately as in the case of ω_3 with the ERA/DC. The errors in the identification of the damping factor using OKID is the same with respect to the case of 5% of noise, but using ERA, the errors are between 49% for ζ_2, and 2% for ζ_5, greaters that in the case of 5% of noise; and with the ERA/DC the error of ζ_2 increased for 75%.

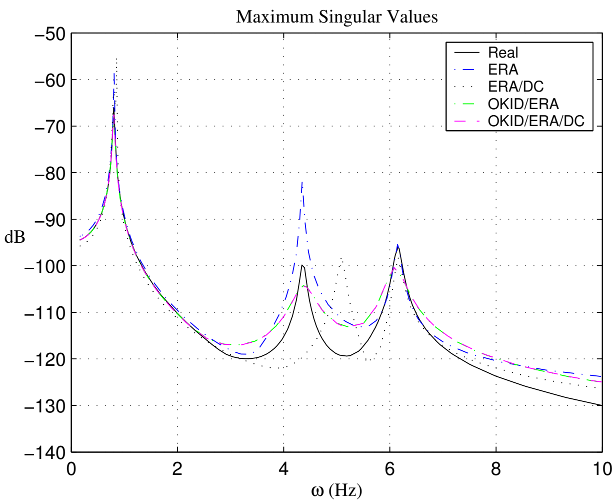

Figure 4: Mini-Mast 1%< ζ <4% (10% of noise).

In the Fig. (4) is observed that with ERA/DC a computacional mode is introduced, because high noises in the data introduce errors in the calculation of the modes of the system and the correlation of data or the criteria of Modal Amplitude Coherence (Juang, 1994), that it was used in this work to distinguish the modes associated with the system of the modes associated with the noise, are not sufficiently effective to eliminate the strange modes.

4. System with high damping

The modal damping factors of the system are modified ($\times 4$) to compare the robustness of the methods when the system has relatively high damping ($\zeta > 4\%$). In this case we will only use $p = 35$ ($p + 1$ observer Markov parameters) in the methods with OKID; $k = 100$ ($k + 1$ system Markov parameters) in ERA and OKID/ERA and $k = 130$ in ERA/DC and OKID/ERA/DC. The Tab. (5) show the modal parameters of the system with high damping.

Table 5: Modal parameters of the system with high damping

mode	ω (Hz)	ζ (%)
1	0.8029	7.2
2	0.8035	7.2
3	4.3691	4.8
4	6.1086	4.0
5	6.1614	4.0

4.1. CASE: ζ>4% with low noise levels (1%)

Table 6: Low noise identification results for system with high damping: natural frequencies (ω) and modal damping factors (ζ).

	mode 1		mode 2		mode 3		mode 4		mode 5	
	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)
TRUE	0.8029	7.200	0.8035	7.200	4.3691	4.800	6.1086	4.000	6.1614	4.000
ERA	0.7981	8.588	0.8172	7.671	4.3811	5.001	6.0979	3.863	6.1831	3.590
ERA/DC	0.8077	8.294	0.8115	7.868	4.3719	4.479	6.0930	3.905	6.1483	4.082
OKID/ERA	0.8033	7.180	0.8035	7.214	4.3656	5.238	6.1031	4.104	6.1735	4.101
OKID/ERA/DC	0.8033	7.193	0.8035	7.206	4.3654	5.241	6.1032	4.104	6.1741	4.106

The comparison of the maximum singular values of the Transfer Functions is shown in the Fig. (5).

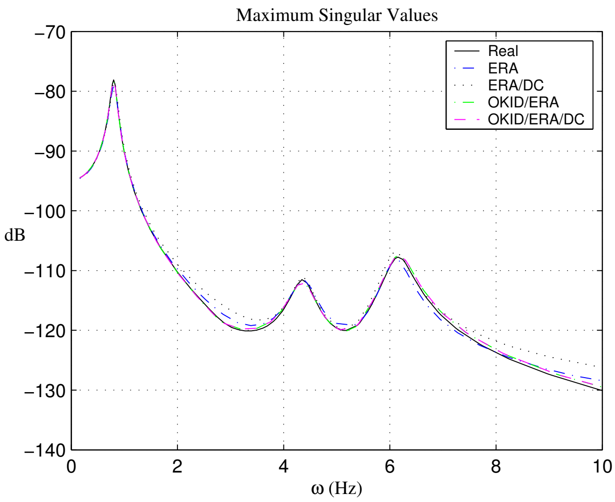

Figure 5: Mini-Mast ζ >4% (1% of noise).

For low levels of noise, the modal parameters of the system with high modal damping factors, can easily be identified by any of the studied methods, the results with ERA and ERA/DC are as good as the ones from the OKID methods because the Markov parameters of the system have fast decline.

4.2. CASE: ζ>4% with medium noise levels (5%)

Table 7: Medium noise identification results for system with high damping: natural frequencies (ω) and modal damping factors (ζ).

	mode 1		mode 2		mode 3		mode 4		mode 5	
	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)
TRUE	0.8029	7.200	0.8035	7.200	4.3691	4.800	6.1086	4.000	6.1614	4.000
ERA	0.8026	7.155	0.8168	7.223	4.3893	3.948	6.0948	4.069	6.1957	3.857
ERA/DC	0.8011	9.106	0.8132	5.953	4.3498	4.272	6.0567	5.064	6.1503	3.382
OKID/ERA	0.8017	7.212	0.8057	7.517	4.4116	7.122	6.0830	3.957	6.1505	5.340
OKID/ERA/DC	0.8018	7.204	0.8058	7.521	4.4079	7.100	6.0816	3.961	6.1417	5.401

Increasing the level of noise to 5%, the results in the identification of the modal parameters of the system are good for all the methods.

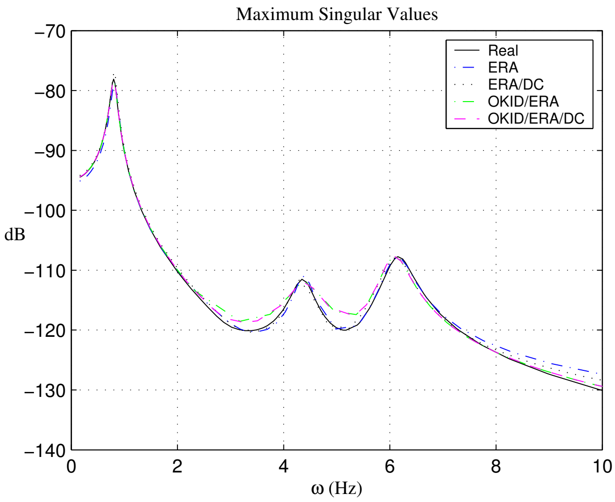

Figure 6: Mini-Mast ζ >4% (5% of noise).

5. System with low damping

To study the robustness of the methods in the presence of noise for the case of the system with low damping ($\zeta < 1\%$), the modal damping factors of the system are reduced ($\times 0.5$). The Tab. (8) show the modal parameters of the system with low damping.

Table 8: Modal parameters of the system with low damping

mode	ω (Hz)	ζ (%)
1	0.8009	0.9
2	0.8015	0.9
3	4.3641	0.6
4	6.1038	0.5
5	6.1566	0.5

In the algorithms, we will use the same number of observer Markov parameters and the system Markov parameters that in the case of average damping because with low damping the decline is slow.

5.1. CASE: ζ<1% with low noise levels (1%)

Table 9: Low noise identification results for system with low damping: natural frequencies (ω) and modal damping factors (ζ).

	mode 1		mode 2		mode 3		mode 4		mode 5	
	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)
TRUE	0.8009	0.900	0.8015	0.900	4.3641	0.600	6.1038	0.500	6.1566	0.500
ERA	0.7958	1.919	0.8156	1.171	4.3654	0.807	6.1020	0.487	6.1599	0.487
ERA/DC	0.7959	2.026	0.8070	1.109	4.3649	0.693	6.1040	0.486	6.1589	0.486
OKID/ERA	0.8009	0.892	0.8014	0.902	4.3656	0.762	6.1049	0.592	6.1559	0.536
OKID/ERA/DC	0.8009	0.892	0.8014	0.903	4.3656	0.761	6.1048	0.594	6.1559	0.537

For 1% of noise, all the methods identified the natural frequencies relatively well and they were able to differentiate the modes associated to almost identical natural frequencies (see Tab. (9) and Fig. (7)), with the ERA and the ERA/DC the errors are smaller than 2%, and with the OKID/ERA and the OKID/ERA/DC the errors are smaller than 0.1%. OKID/ERA and OKID/ERA/DC obtain good results to identify the modal damping factors, the biggest error is 27% for ζ_3 and errors smaller than 1.2% are gotten for ζ_1 and ζ_2; but the ERA and the ERA/DC are not robust in the identification of the modal damping factors when the system has slow damping, even though the noise level is only of 1%, the obtained errors are 113% and 126% for ζ_1 with the ERA and the ERA/DC respectively, however the modal damping factors associates to modes 4 and 5 have errors less than 2%.

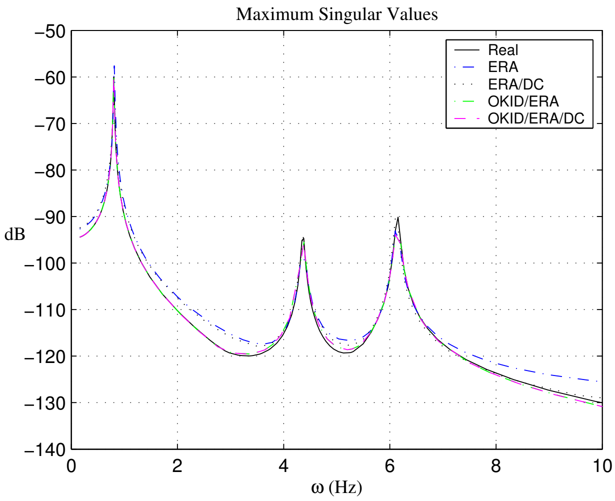

Figure 7: Mini-Mast ζ <1% (1% of noise).

5.2. CASE: ζ<1% with medium noise levels (5%)

Table 10: Medium noise identification results for system with low damping: natural frequencies (ω) and modal damping factors (ζ).

	mode 1		mode 2		mode 3		mode 4		mode 5	
	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)
TRUE	0.8009	0.900	0.8015	0.900	4.3641	0.600	6.1038	0.500	6.1566	0.500
ERA	0.7868	1.792	0.8183	1.633	4.3727	0.752	6.1039	0.651	6.1570	0.524
ERA/DC	0.7802	3.311	0.7872	1.135	4.3727	0.560	-	-	6.1532	0.397
OKID/ERA	0.8009	1.028	0.8012	0.958	4.3843	1.902	6.1192	1.876	6.1484	0.785
OKID/ERA/DC	0.8009	1.023	0.8011	0.961	4.3843	1.903	6.1186	1.880	6.1484	0.789

The results in Tab. (10) for 5% of noise show that ERA/DC does not identify the mode 4 in the system with low damping, one of the modes associated to almost identical natural frequencies. The ERA can differentiate the modes associated to similar natural frequencies and obtain good results identifying the frequencies of the system, but the modal damping factors in modes 1 and 2 have errors of 99% and 81% respectively and the modal damping factors associates to modes 3, 4 and 5 have errors smaller than 30%. The OKID/ERA and OKID/ERA/DC can identify well the natural frequencies and too differentiate the modes associated to almost identical natural frequencies with errors smaller than 0,5%, also the modal damping factors associates to the modes 1 and 2 are good identified with errors of 14% and 7% respectively, but the modal damping factors associates to the modes 3, 4 and 5 have errors of 217%, 276% and 58% respectively.

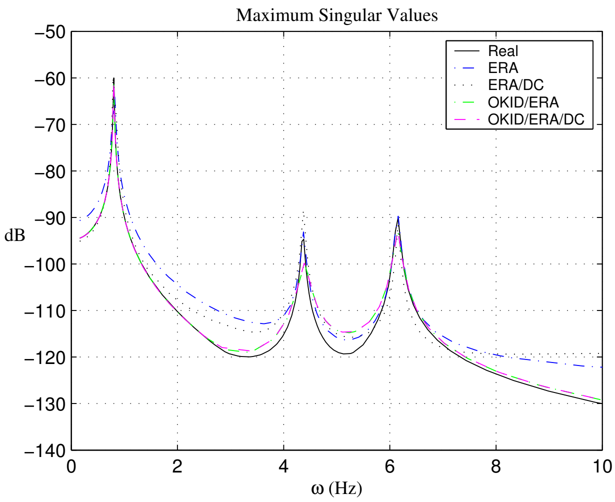

Figure 8: Mini-Mast ζ <1% (5% of noise).

5.3. CASE: ζ<1% with high noise levels (10%)

Table 11: High noise identification results for system with low damping: natural frequencies (ω) and modal damping factors (ζ).

	mode 1		mode 2		mode 3		mode 4		mode 5	
	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)
TRUE	0.8009	0.900	0.8015	0.900	4.3641	0.600	6.1038	0.500	6.1566	0.500
ERA	0.7903	1.164	0.8142	0.378	4.3621	0.460	-	-	6.1524	0.442
ERA/DC	0.7961	1.872	0.8047	0.386	4.3670	0.538	-	-	6.1553	0.440
OKID/ERA	0.7998	1.286	0.8014	0.884	3.7460	4.165	6.0965	2.217	6.1554	2.643
OKID/ERA/DC	0.7987	1.129	0.8017	0.963	4.2615	3.277	6.1457	3.422	6.1774	1.765

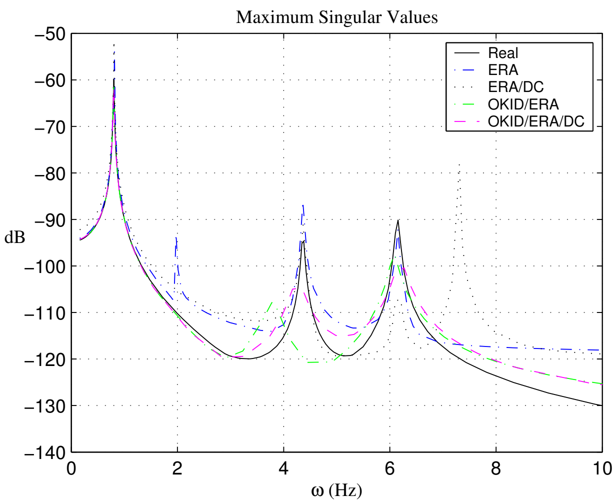

Figure 9: Mini-Mast ζ <1% (10% of noise).

For 10% of noise, the OKID/ERA and OKID/ERA/DC keep its robustness with respect to the case with 5% of noise, and the ERA and ERA/DC do not identify the mode 4 (see Tab. (11)). The OKID/ERA and OKID/ERA/DC differentiate the modes associated to almost identical natural frequencies, but in this case, the results of the identification using the OKID/ERA is better than the results using the OKID/ERA/DC, for example in the natural frequency associate to mode 3 (14% of error with OKID/ERA and 2.4% with the OKID/ERA/DC). In the identification of the modal damping factors, with the ERA the errors are: 29%, 58%, 23% and 12% for ζ_1, ζ_2, ζ_3 and ζ_5 respectively, and with the ERA/DC the errors for the same factors of damping are: 108%, 57%, 10% and 12% respectively. With the OKID/ERA the errors are smaller for ζ_1 and ζ_2: 43% and 2%, but greater for ζ_3, ζ_4 and ζ_5: 595%, 344% and 428% respectively; and with the OKID/ERA/DC the errors are slightly smaller: 26%, 7%, 447%, 584% and 254% for ζ_1, ζ_2, ζ_3, ζ_4 and ζ_5 respectively. The comparison of the maximum singular values of the Transfer Functions is shown in the Fig. (9) and is observed that strange modes to the system was introduced by ERA and ERA/DC.

5.4. CASE: ζ<1% with high noise levels (10%) and p=100

The results of the identification of the natural frequencies using the OKID/ERA/DC and OKID/ERA in the previous cases for system with low damping are good, but in the identification of the modal damping factors in modes 3, 4 and 5 the errors are great for 5% and 10% of noise. It happens because the number of observer Markov parameters used in the methods with OKID is not enough to guarantee the decline, but uses $p = 50$ to keep similar times of execution of the algorithms and to be able to make the comparison. If now we use $p = 100$ observer Markov parameters in the methods with OKID to look the best performance of the OKID/ERA/DC; e $k = 250$ system Markov parameters in ERA and OKID/ERA and OKID/ERA/DC, the news resulted are show in Tab. (12).

Table 12: High noise identification results for system with low damping and p=100: natural frequencies (ω) and modal damping factors (ζ).

	mode 1		mode 2		mode 3		mode 4		mode 5	
	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)	ω(Hz)	ζ(%)
TRUE	0.8009	0.900	0.8015	0.900	4.3641	0.600	6.1038	0.500	6.1566	0.500
ERA	0.7971	1.723	0.8138	1.450	-	-	6.1157	0.431	6.1960	0.552
ERA/DC	0.7962	1.855	0.8162	1.510	-	-	6.1055	0.337	6.1871	0.587
OKID/ERA	0.8020	1.002	0.8021	0.631	4.3442	0.935	6.1131	0.676	6.1723	0.995
OKID/ERA/DC	0.8019	0.995	0.8021	0.617	4.3452	0.955	6.1124	0.634	6.1717	0.946

Is observed that with the ERA and ERA/DC, the mode 3 is not identified and with the OKID/ERA/DC and OKID/ERA all the modal parameters of the system are good identified and improving its performance with respect of the previous case with $p = 50$. The errors in the identification of modal damping factors in modes 3, 4 and 5 with the OKID/ERA/DC and OKID/ERA (with p=100) are minors (58-90%) that in the previous case. But, with ERA and ERA/DC do not improve the results in the presence of high levels of noise, and can be observed in Fig. (10) that strange modes to the system are introduced.

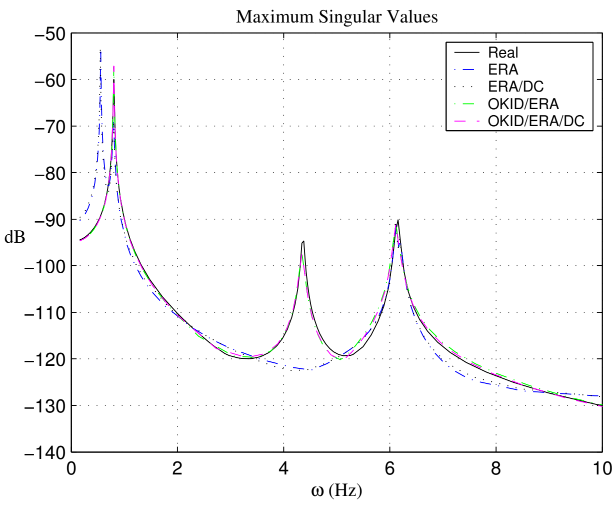

Figure 10: Mini-Mast ζ <1% (10% of noise) with p=100.

6. Conclusions

The results show that the OKID/ERA and OKID/ERA/DC are more robust than the ERA and ERA/DC, performing better identification of the modal parameters of the Mini-Mast structure in all bands of damping studied and with low or high levels of noise. Methods using OKID always can differentiate the modes associated to almost identical natural frequencies.

In the case of the system with low damping, ERA and ERA/DC do not give good results, also when the noise is not very high (5%), some in the modes had not been identified, and when the noise is high (10%) some modes were not identified and also some spurious modes are introduced.

In the case of the system with average damping, all the methods have relatively good results with low levels of noise, but for 10% of noise only the OKID/ERA and the OKID/ERA/DC identify all the modes but with some errors in the values of the modal damping factors.

In the case of the system with high damping, all the methods can identify the modal parameters of the system well and too differentiate the modes associated to almost identical natural frequencies, in the presence of noise in levels of 1% and 5%.

As it was demonstrated in the last analyzed case, ζ <1% and 10% of noise, with $p = 100$, the performance of OKID/ERA/DC and OKID/ERA is better that with $p = 50$, because the decline of the system Markov parameters for the system with low damping is slow.

The disadvantage to use greater number of observer and system Markov parameters is that the greater computational time, but, on the other hand, the OKID/ERA and the OKID/ERA/DC, do not need the same number of system Markov parameters that with the ERA and the ERA/DC when the system has low damping, less number of system Markov parameters are needed with OKID because the observer Markov parameters guarantee the decline.

How much bigger either the contamination of the data with noise, is necessary greater number of samples and greater amount of system Markov parameters to be calculated.

7. References

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