

DIDACTIC ANALYSIS OF THE EPICYCLICAL GEAR TRAIN REPRESENTATION

Danilo Amaral
Federal University of Minas Gerais - School of Engineering, Department of Engineering Mechanics, Av. Antonio Carlos 6627, CEP 31970-001, Belo Horizonte, MG, Brazil
e-mail: danilo@demec.ufmg.br

Antonio Eustáquio de Melo Pertence
Federal University of Minas Gerais - School of Engineering, Department of Engineering Mechanics, Av. Antonio Carlos 6627, CEP 31970-001, Belo Horizonte, MG, Brazil
e-mail: pertence@demec.ufmg.br

Marcelo Becker
Pontifical University Catholic of Minas Gerais - Polytechnical Institute, Av. Dom Jose Gaspar, 500, CEP 30535-610, Belo Horizonte, MG, Brazil
e-mail: marcelo.becker@pucminas.br

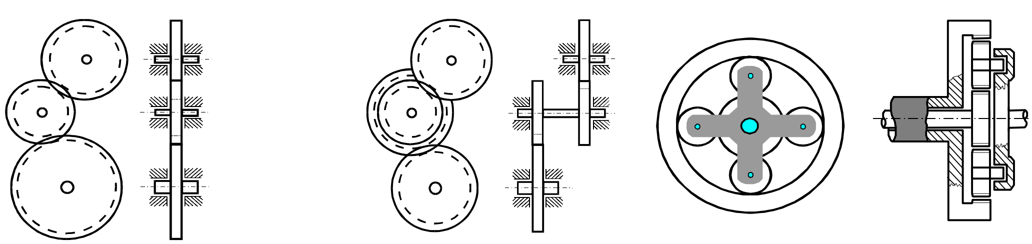
Franco Giuseppe Dedini
University of Campinas, Faculty of Mechanical Engineering, Department of Machine Design, P.Box - 6051, CEP 13089-970, Campinas, SP, Brazil
e-mail: dedini@fem.unicamp.br

Abstract: The epicyclical gear trains or planetary of gear trains (PGTs) are transmission systems of high complexity kinematics and difficult visualization. However, its advantages are many, since they are: compact, light, allowing high reductions of speed. They are also permanently connected which gives them high trustworthiness and finally, their capacity of bifurcation and addition of power, allow multiple transmission relations. Its main application is the automatic transmission of the modern vehicles. The establishment of analysis of representation of this system is very important for its understanding because of the great variety of possibilities of configurations in the union of some PGTs and of basic types. This paper has the objective systemize and also present the diverse analysis of representation adopted by some authors and showing comparisons between them and its applications in the didactic analysis of epicyclical gear trains representations

Keyword : Planetary gear train, Epicyclical gear train representation

1. Introduction

Gear train is destined to transmit power and rotation. According to Albuquerque (1980), three assemblies are possible: simple train, compound train and epicyclical train. The simple train is a system of gears where exists only one gear in each axis (Fig. 1a). The gear train is called as a compound, when exists one or more axes with two gears or more (Fig. 1b). In these two cases, the support of the axes of the gears is fixed. When a support exists, of at least an axis, endowed with rotation movement, the train is called of epicyclical (Fig. 1c). It is observed that the axes that support the intermediate gears between the central gear and the external (this last one with internal teeth), they are mounted in a support that rotates around the central axis of the group. That possibility of the axis of a gear also to rotate around of another axis, besides rotating around itself is that it characterizes an epicyclical train. That nomenclature is due to the fact of a point, belonging to the gear that possesses movable axis, to describe an epicyclical curve.



(a) Simple train (b) Compound train (c) Epicyclical train

Figure 1. Types of the gear trains.

2. Planetary Gear Train

Due to the analogy with our solar system, this type of epicyclical train is frequently called of planetary train or planetary gear train or PGT. Because of that, the central gear is called of sun gear. The gears that rotate around her are called of planet gear. It is almost always used, also, a gear of internal teeth around PGT. This is called ring gear. The element that supports the movable axis of the planet gears and that rotate around the main axis of PGT is called support or arm. Figure (2) show these elements.

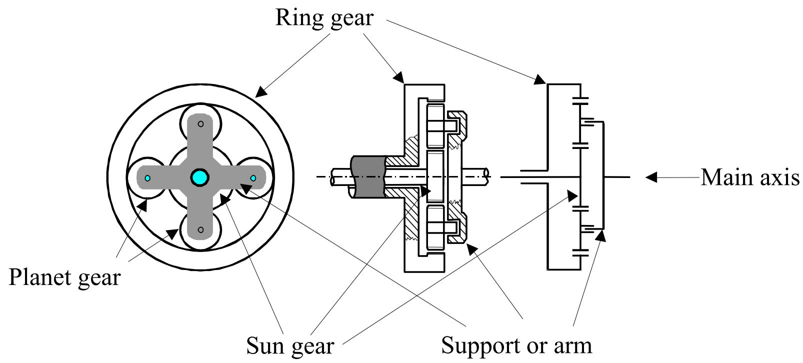


Figure 2. Nomenclature of the PGT elements.

Several authors defined what is a planetary gear train. Dubbel (1944) wrote that simple planetary gears are characterized with a fixed wheel and other piece rotates around of the fixed and it is geared with it. Some systems of gears differ of the common ones, for the fact of they possess one or more gears with possibility of rotating around of the own axis and simultaneously around another axis (Lima, 1980). Shigley (1984) wrote that, in a type of gear train can be obtained surprising effects, being done with that some of the axes rotate in relation to the others. Olson et al (1987) defined that the planetary gears trains consist of one or more central gears with planet gears geared and that they rotate around them, in such a way that the points of the planet gears describe epicyclical curves. Brasil (1988) defined PGTs as gear trains in which some axes are movable, rotating not only around themselves, but also around other axis of the gear train. The planet gears are linked by an arm in such a way that the distance among the centers of the gears stays constant. PGTs are systems of transmission of high cinematic complexity and of difficult visualization. For this, several authors were devoted to the study in their representation ways to facilitate its understanding (Amaral and Dedini, 2000; Olson, 1987; and Buchsbaum and Freudstein, 1970). However, their advantages are must: compact, light, high reduction of speed, high reliability, high potency density, bifurcation capacity and potency addition, differential capacity, systems of multiple transmission relationships and permanent geared, still allowing the minimization of the efforts limps us and alignment of the axes. These are some of the characteristics that turn PGTs systems of great potential of applications, although still no so much studied and researched, in such a way to allow its use more application (Dedini, 1985). Their advantages turned them preferable for the military use, where multiple geared reduce the stop risk. The soft operation also turns them appropriate for use in submarines and the great reduction capacity turns possible its application in turbines. PGTs are also used in aerospace applications and in helicopters, besides the automotive use as differential and automatic transmission (Amaral, 2000).

It can increase the complexity of PGT, losing temper the arrangement of the configuration of the planetary gears. A PGT can also possess more than one planet gear among the two central gears. That doesn't change the cinematic character of PGT. An increase of the number of planetary gear results in a larger division of the load transmitted among the planet gears. That is one of the great advantages of PGTs, where the effort in the bearings is quite diminished due to the symmetry of the application of the force by the planet gears, in the teeth of the sun gear. Therefore is always to avoid the use of a single TGP because, in this case, it would not be possible the compensation of the efforts. In practice, usually two or three planets are used. Important classes of trains of epicyclical gears are that possesses bevel gears (Fig. 3). Only with the use of that gear type it cam to use a sun gear with the same number of teeth of the ring gear. It is impossible if all gears were cylindrical. The use of bevel gears doesn't change the cinematic character of the PGTs and the great applications are the differential ones automotive. This present work is restricts to study of PGTs with cylindrical gears.

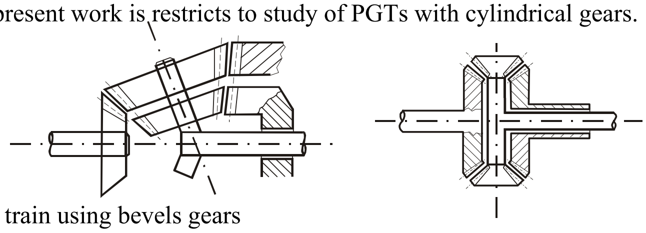


Figure 3 Planetary gear train using bevels gears

3. Types of Planetary Gears Trains

Lévai (1968) identified in his work, four types of PGTs: Elementary PGT; Simple PGT; Linked PGT (Incorporated PGT) and Planet PGT. The Simple and Linked PGTs types are the most important in practice for they be more commercially applied. Linked PGT is characterized for the fact that possesses more than two central gears and it can be separate in two or more simple PGT. The main application of Linked PGT is the automatic transmission used in automotive vehicles.

3.1. Elementary PGT

They are those that just possess a central gear. The central gear that whose rotation axis is the main axis of PGT. Like this, the sun and ring gears are central gears, according to display the Fig. (4).

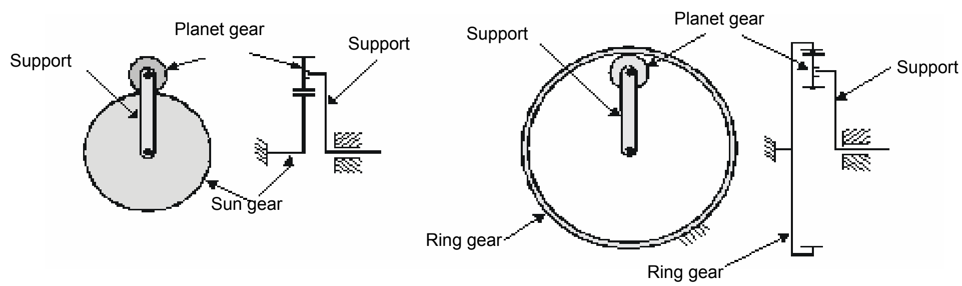


Figure 4. Example of the elementary PGT.

3.2. Simple PGT

They are those that possess two central gears, one or more planets gears and one support (Fig. 5). The alteration of the amount of planetary gears doesn't uncharacterized its framing as simple PGT. The use of a larger number of planets contributes to an adult and better distribution of the active loads. This type of is called of Basic PGT.

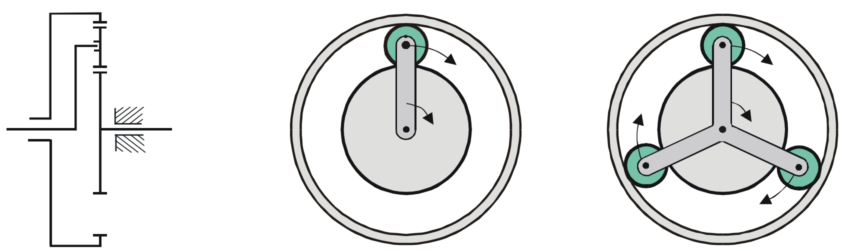
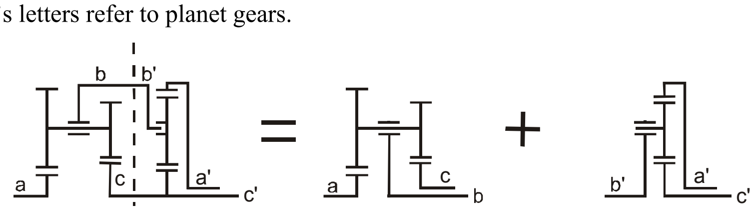


Figure 5. Example of the simple PGT

3.3. Linked PGT

Linked PGTs characterize for the fact that its possess more than two central gears and they can be separate in two or more simple PGTs. The separation of a linked PGT won't be very obvious if an embedded exists among planet gears. In this case the PGT is classified as Incorporate PGT. It is treated therefore, of a case peculiar of linked PGT. In the Fig. (6), P for gears of external teeth (positive), N for gears of internal teeth (negative) and the fence's letters refer to planet gears.



Type P(PP)P + Type P (P)N

Figure 6. Example of the linked PGT

3.4. Planet PGT

This PGT is classified as planet PGT when a secondary PGT is introduced in the planetary axis of the primary PGT, according to display the Fig. (7).

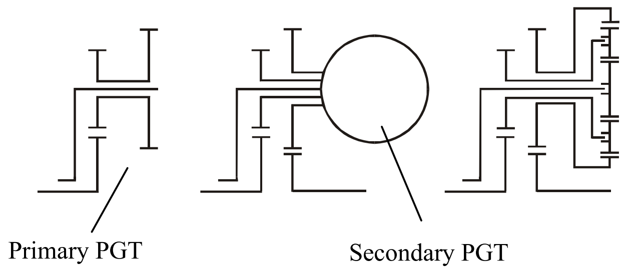


Figure 7. Example of the Planet PGT.

4. PGTs Representations

As a great variety of possibilities of configurations of PGTs exist, it is necessary to establish representation forms, to define and to identify the system in study. Several forms of representation of topology were already developed. Basically three representation types exist: functional, schematic and graph. The functional representation can still be presented in the three-dimensional form, in plant or conventional. The schematic representation can be presented under the outline form to define a PGT and under the form of schematic diagram of blocks to define the connections among several PGTs. The graph representation, more recently used to represent PGTs has the advantage of facilitating the use of computers for the solution of relative problems to this mechanism. However to solve difficulties of the same forms observed in the original graph representation, it is proposed a new representation form, besides the canonical form. The Fig. (8) available representation forms of the PGTs.

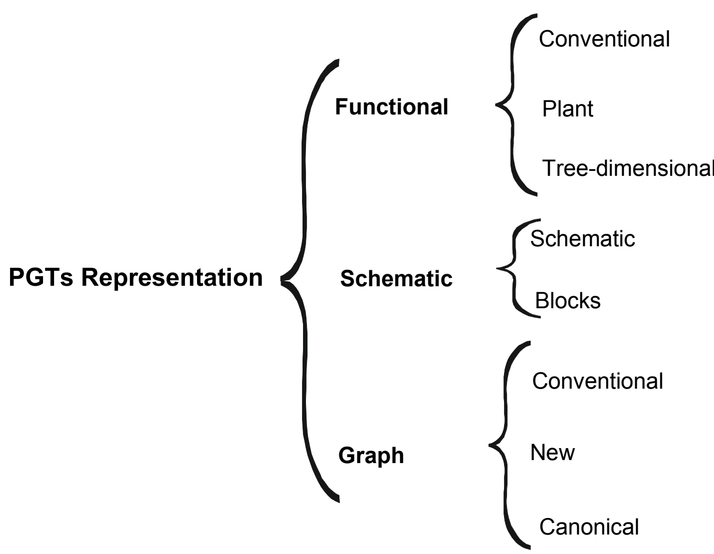
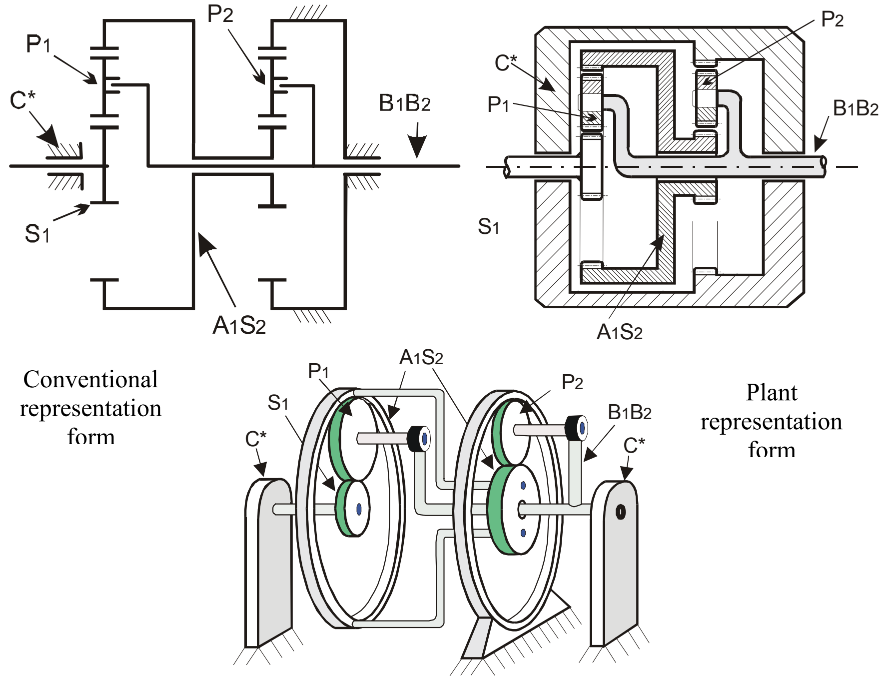


Figure 8. PGTs representation forms

4.1. Fuctional representation

The functional representation was the first to be used to identify a PGT. Its advantage is that there is parity between the model and the representation. Is can be introduced under the conventional form (outline), in plant and three-dimensional. The Figure (9) display the three forms of functional representation of a same PGT, where the gears are represented by their primitive circles that indicate in that points they touch each other. Due to symmetry of the planetary mechanisms, it is usual the representation of just half of the mechanism. The PGT functional representation is made including the carcass or support of the mechanism. When it is not included the carcass or support, they are obtained the cinematic structure.



Three-dimensional representation form

Figure 9. Functional representation form

4.2. Schematic Representation

There are two different forms from schematic representation of the PGT: through schematic or blocks representations. The first form has for purpose to identify the cinematic structure and second has for objective the cinematic study and of the potency flow in a system of several linked PGTs.

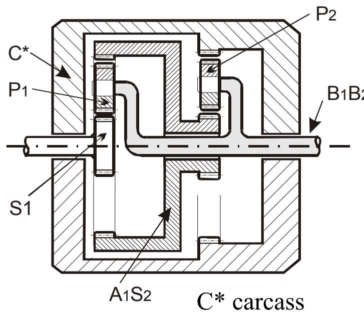
4.2.1. Schematic Representation Form

The schematic representation form was published in 1969 by Buchsbaum and Freudenstein (1970) to identify the cinematic structure. A vertex, acted by a white circle, it means a support of an axis or the own axis. A vertex, acted by a black circle, it means that there is a geared. Like this, each element (group of gears and/or axes that are embed amongst themselves) of PGT forms a polygon hatched whose number of vertexes will depend on the number of bonds (geared, supports, axes) of this element. The smallest number of bonds that an element separately can have is two (example: a gear has the bond with other gear and the support around an axis). In this case, the shaded polygon reduces to a line that unites the two vertexes - a white and the other black circles. A two-supported gear or a three-supported axis will be represented by a polygon of three vertexes (a geared and two supports or the three supports). Three embed gears with a central hole for the axis will be represented by a polygon of four vertexes. The Fig. (10) display an example.

The sum of the number of hatched areas and the isolated lines it represents the number of elements of the epicyclical train, there included the carcass or support. The PGT of the Fig. (10) has 6 elements. The representation schematic will be 5 hatched areas and one more segment than it represents the sun gear S1 with the geared bond of (black circle) in planetary P1 and a support bond and bearing in the carcass C * (white circle).

4.2.2. Block Representation Form

Macmillan (1961) it proposed a representation form for the study of the potency flow in differential mechanisms. Sanger, (1972), used this representation form in a system of several planetary connected amongst themselves. This assembly type is quite used in systems of transmission of multiple speeds. The basic element is simple PGT, with two degrees of freedom, that it can be coupled with other PGTs. The external points of PGT can be used as entrance, exit or control member. Therefore, for a schematic block representation is necessary knows the number of the PGTs and their respective external points. The PGTs are represented by blocks and the external points represented by circles, as it demonstrates the Fig. (11).



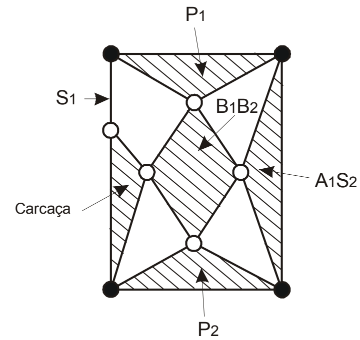


Figure 10. Schematic representation form.

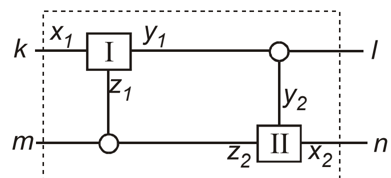


Figure 11. Block representation form

4.3. Graph Representation

Wilson et al (1990) wrote that a graph is a diagram of points called vertexes, united for lines called borders, in such a way that each border unites two vertexes. The representation theory for graph has a several applications, from studies on flexible systems of manufacture, administration of the production, urban traffic and until the representation of mechanisms. The graph representation has been used as model of mechanisms since 1960 (Olson et al.). Its use has the advantage of facilitating the use of computers and the main characteristic is that there is a correspondence between the vertexes, the elements of the PGT and the borders with the connections among the elements. The graph representation can also be presented under the matrix form, from where she can use algebraic techniques for the analysis of the PGT.

4.3.1. Conventional Representation Form

The conventional representation form is made in the following way. Each element is represented by a vertex. Gears different and embeds are represented by a single vertex. It is treated of a single element. Same for an embed gear with an axis. The arm also an element is considered as well as the gears. Fixed elements are represented by a circle around the vertex. The borders or sides of dash lines represent a geared among two elements. They are called of geared borders. A border or continuous sided of continuous lines indicate that an element rotates on the other. This side is called of revolution borders. Each border and each vertex is denominated with the respective symbol used in the other representation forms.

This representation is called commonly of bicolor in function of the need of distinguishing the difference among a connection of elements through geared. To illustrate this type, they are had in the Fig. (12) the conventional functional representation and the respective graph representation. Some authors use continuous borders for both cases, differentiating them for the thickness of the line.

However, the conventional representation form can result in that same PGT can have more than a representation (Hsu, 1992). According to Hsieh (1987), when it exists three or more coax elements in a PGT, it can be done a rearrange in the coax elements without to affect and to alter the functionally of the mechanism. This can result in a problem of the same form. Two representation graph are of the same form when exist a biunique correspondence between their vertexes and borders. For PGTs, two graphs no of the same form or different when can represent mechanisms cinematic equivalent. Such representation graph are called of pseudo the same form.

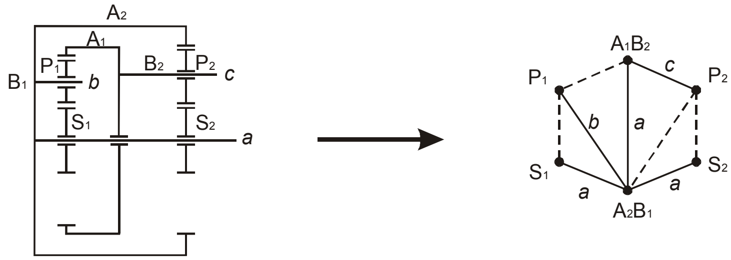


Figure 12. Conventional representation form

4.3.2. New Representation Form

Several authors (Olson et al., 1987, Yan and Hsu, 1992) published, a new proposal of representation of PGTs, without the inconveniences verified in conventional representation form. In this new representation form, a PGT with "n" elements is identified by a complete graph with "n" vertexes. The difference is that, besides vertexes, geared borders (hatched line) and revolution borders (continuous lines) still is had a solid polygon. For this proposal, an only graph (Fig. 13) represents PGT shown in the Figure (12).

Considering that the elements that rotate around a same axis are in a same level, Hsu (1987) proposes a new representation form of PGTs for graph, where these elements of same level form a solid polygon.

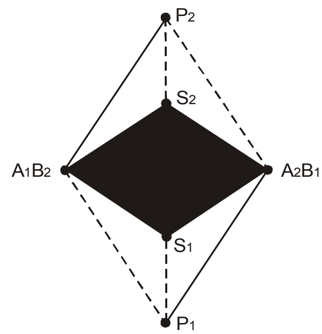


Figure 13. New representation form

4.3.2.1. Matrix Representation

The adjacency matrix is the fundamental and it completes representation of a graph. The different types of matrix characterize a graph. An example is adjacency matrix "element-to-element" or "one-for-one". This matrix to represent a PGT of "n" elements, it is defined as a matrix of n X n whose elements are given by $a_{ij} = 1$ if the element i is adjacent with j and, otherwise, $a_{ij} = 0$ (besides $a_{ii} = 0$).

For the new representation for graph with n vertexes, the adjacency matrix can be defined as a symmetrical matrix of order n in that the element $a_{ij} = 1$ if the vertex i is adjacent to the vertex j for a revolution border (continuous line), $a_{ij} = 2$ if i is adjacent to the vertex j, united for an geared border (hatched line), $a_{ij} = m$ if the vertex i is adjacent to the vertex j with a solid polygon with m vertexes and $a_{ij} = 0$ if the vertexes are not adjacent. Still a_{ii} is had = 0. For instance, the Fig. (13) presents a graph with 6 vertexes, bicolor and with a solid polygon of 4 vertexes. The correspondent connective matrix using the following order of the elements (P1, P2, S1, S2, A1B2 and A2B1) is display in the Fig. (14).

4.3. Canonical Representation Form

To avoid the problem of the pseudo the same form in the conventional representation for graph, several authors (Tsai, 1988, Chatterjee, 1995) proposed another alternative in function of the existence of at least three coax elements: the canonical representation for graph. In this representation, there is a distinction among the borders and the vertexes that are divided in several levels. In the first level, they are the representations of the sun gear, ring gears and the arm. In the second level, they are the planet gears. Below all it is the level base, that represents the axis common of the group. The Fig. (15) display an example of canonical representation for graph of the PGT.

$$\text{Matrix} = \begin{vmatrix} 0 & 4 & 4 & 4 & 2 & 0 \\ 4 & 0 & 4 & 4 & 2 & 1 \\ 4 & 4 & 0 & 4 & 0 & 2 \\ 4 & 4 & 4 & 0 & 1 & 2 \\ 2 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 0 & 0 \end{vmatrix}$$

Figure 14. Matrix representation form

The canonical representation for graph has typical application in of the epicyclical transmission called automatic transmission used thoroughly in automobiles. These systems usually work with a single common axis, supported in a carcass for ball bearings. The reduction of the group is the relationship between the rotation of the exit axis and the rotation of the entrance axis. With the use of brakes and clutches, several reduction are obtained.

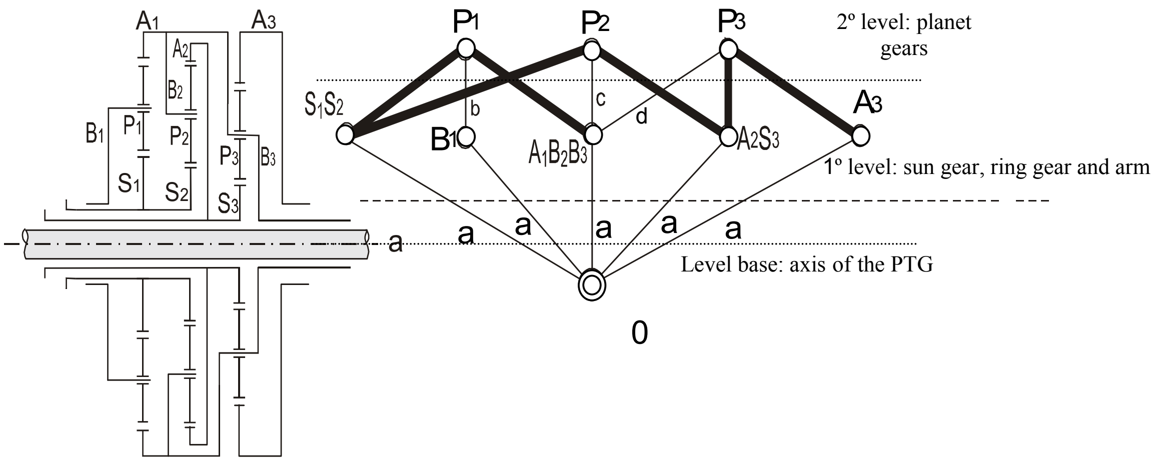


Figure 15. Canonical representation form

5. Application Example

In the Fig. (16), an example of the several representations of a transmission system called "Wilson Transmission" of 4 speeds. It is a series of four TGTs linked amongst themselves with 10 elements and a carcass. For the block representation, it can be observed that there are two direct connections to a PGT (S1 and A4), three connects uniting two exits (B1A2, S2S3, B3B4) and a connect uniting four exits that it results in a system with two degrees of freedom. For use of this system as transmission, is necessary to act in the members of control in such a way that it results in degree of freedom equal to 1. Being applied a brake of every time, they are obtained this result, altering the transmission relationship, without need that there is disconnection of the elements.

6. Conclusion

This work presents several forms of representation of PGTs adopted by several authors showed, through the example of application of “Wilson Transmission”. This work aims at to be consultation source for the Machine Design discipline of the course of Mechanical Engineering for condensing, in a single text, several available representation forms facilitating plenty the understanding for the students, the operation of the mechanical systems used in planetary gears trains.

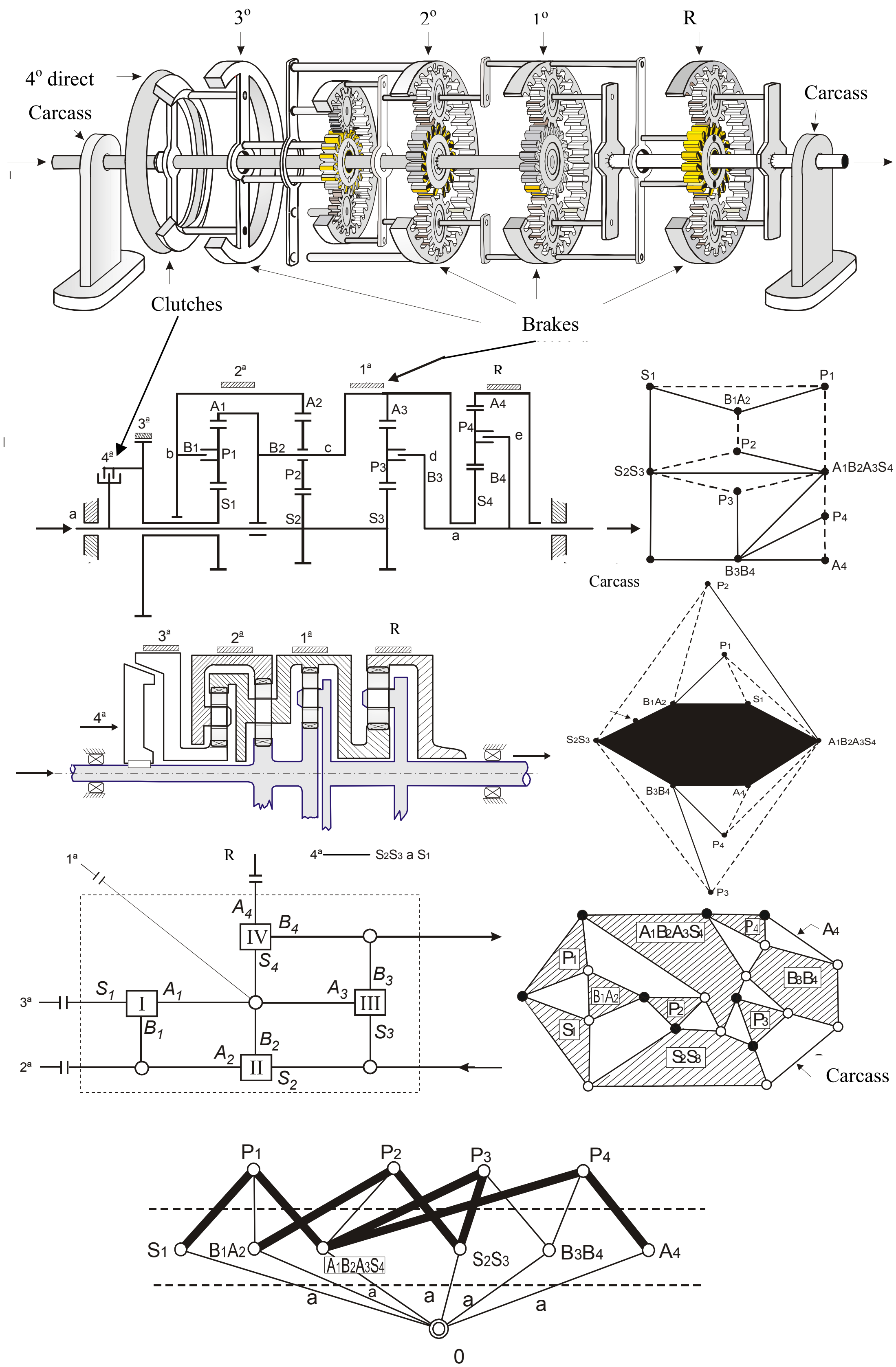


Figure 16. Example of application of the PGTs representation forms using “Wilson Transmission”

7. References

Amaral, D., Metodologia, Sistematização e Seleção de Projetos de Transmissão Epicicloidais com Um e Dois TEPs Ligados, Tese de Doutorado, Unicamp, 180p, 2000

Brasil, H. V., Máquinas de Levantamento, *Editora Guanabara S.A,* Rio de Janeiro, 230 p., 1988

Buchsbaum, F., Freudenstein, F., Syntesis of Kinematic Structure of Geared Kinematic Chains and other Mechanisms, *Journal of Mechanisms*, v. 5, p. 357-392, 1970.

Chartterjee, G., Tsai, L. W., Computer-Aided Sketching of Epicyclic-Type Automatic Transmission Gear Trains, *Tecnical Research Report of ISR*, paper TR 95-92, 7 p., 1995.

Dedini, F. G., Projeto e Otimização de uma Transmissão Planetária por Rolos de Tração, *Dissertação de Mestrado,* UNICAMP, 150 p. , 1985.

Dubbel, H., Manual Del Constructor de Maquinas, 2ª Edição, Editorial Labor S.ª, Barcelona, Madri, 1951 p., 1944

Hsieh, H-I., Enumeration and Selection of Clutching Sequences Associate with Epicyclic-Type Transmission Mechanisms, Tese (Doutorado), 212 p., 1997

Hsu, C. H., A Graph Representation for the Structural Synthesis of Geared Kinematic Chains, *Journal of Franklin Institute,* v. 330, nº. 1, p. 131-143, janeiro 1993.

Hsu, C. H., Lam, K. T., A New Graph Representation for the Automatic Kinematic Analysis of Planetary Spur-Gear Trains, *ASME Journal of Mechanical Design*, v. 114, p. 196-200, 1992.

Lévai, Z., Structure and Analysis of Planetary Gear Trains, *Journal of Mechanism*, v. 3, p. 131-148, 1968

Lima, C. S., Trem de Engrenagens Planetários: Análise, Síntese e Aplicação em Veículo Híbrido, *Dissertação de Mestrado*, Faculdade de Engenharia Mecânica da UNICAMP, 1980.

Macmillan, R. H., Power Flow and Loss in Differential Mechanisms*, Journal of Mechanical Engineering Science*, v. 3, n. 37, p. 45-53, 1961

Olson, D. J., Riley, D. R., A New Graph Theory Representation for the Topological Analysis of Planetary Gear Trains, *Proceedings of the 7th Word Congress on the Theory of Machines and Mechanisms*, v. 3, p. 1421-1425, 1987.

Pires e Albuquerque, O. A. L., Elementos de Máquinas, Editora Guanabara Dois S. A. , Rio de Janeiro, 440 p., 1980

Sanger, D. J., Syntesis of Multiple-Speed Transmissions of the Planetary – Gear Type, *Journal of Mechanical Engineering Science*, v. 14, p. 93-101, 1972

Shigley, J. E., Elementos de Máquinas, Livros Técnicos e Científicos Editora S. A, Rio de Janeiro, 700 p., 1984.

Tsai, L. W., Lin, C. C., The Creation of True Two-Degree-of-Freedom Epicyclic Gear Trains, *Technical Research Report of ISR*, paper TR 88-21, 1988

Wilson, R. J., Watkins, J. J., Graphs – An Introductory Approach, *John Wiley & Sons*, 340 p., 1990.