ENTROPY GENERATION IN RECTANGULAR DUCTS WITH NONUNIFORM TEMPERATURE ON THE CONTOUR

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Abstract. Rectangular tubes are widely used in heat transfer devices. The objective of this work is to investigate the effects of the entropy generation associated with heat transfer and fluid friction by using a computational model. This investigation allows to evaluate the entropy generation which is influenced by the temperature distribution in the contour, fluid flow type, properties of the fluid and geometric parameters. The study is made for laminar regime, with developed hydrodynamic and thermal profiles. First, the model calculates the velocity field, then the energy equation is solved obtaining the fluid temperature field for a given distribution imposed in the contour, not necessarily uniform. Known the velocity and the temperature fields, the entropy generation per unit volume is calculated. The entropy generation per length unit is calculated by a double integration in the analysed section. The computational model consistency is verified by the mesh refining and by the tolerance decreasing. The method validations used are done for the velocity and temperature fields calculations.

Keywords. Entropy Generation, Forced Convection, Rectangular Ducts

1. Introduction

Thermal analysis of flows inside rectangular ducts is a very important subject of research in thermal science and engineering. Literature about the subject is extremely wide, but there are comparatively less Second Law based results reported. Second Law analysis is the basis of the so-called "Entropy Minimization Generation Method" (EMGM). This method basically consists in searching situations in which thermal processes could be "less irreversible". Examples of such studies are given in Bejan (1979), Bejan (1982), Bejan (1996), Gerdov (1996) and Saboya (2002). The EMGM, therefore, looks for physical and geometrical configurations in thermal systems that render the inevitable irreversibilities associated to heat transfer and. fluid flow to a minimum. This minimization leads to more efficient equipment design, allowing fuel saving, lesser pollution, etc.

The purpose of this paper is the computation of entropy generation rates in fully thermal and hydrodynamically developed laminar flows in ducts with rectangular cross sections and nonuniform temperature in wall perimeter. This computation gives the necessary information for minimization mentioned above.

Heat exchangers with rectangular cross sections ducts are widely used in chemical processes industries air conditioning and lubrication equipment and other applications, which require compact heat exchangers. The flow, in much of these applications, occurs at low velocities and small dimensions, resulting in low Reynolds numbers. Thus, the flow tends to be laminar and, because of this, laminar flows will be studied in this paper

Figure (1) and Fig. (2) show the schematic of the studied duct. L is the duct height, D is its width and the duct length is considered infinite. Temperature distribution at the wall duct perimeter is variable.

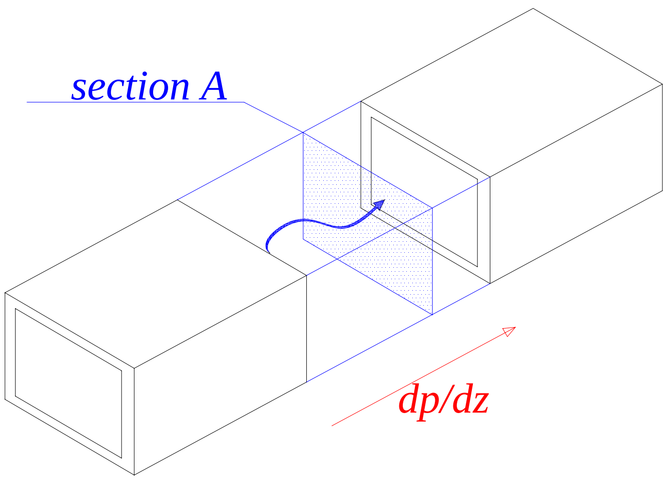

Figure 1. Perspective view of the rectangular duct studied.

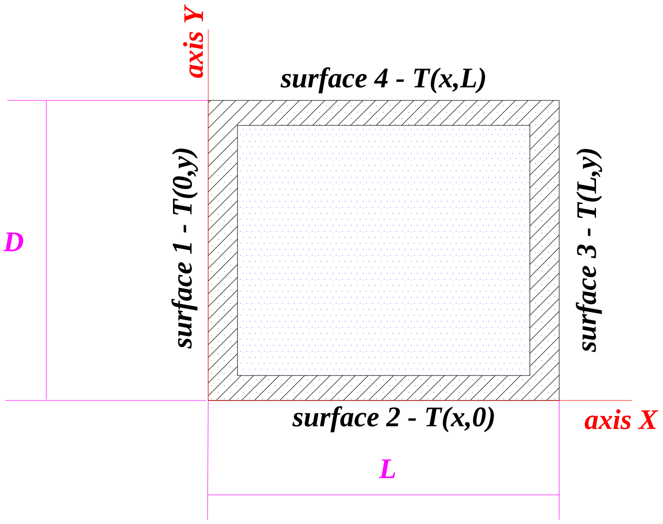

Figure 2. Duct cross-section and variable wall temperature distribution schematics.

2. Mathematical Formulation

2.1. Velocity Field

The flow is considered steady, laminar with a fully developed velocity profile. The work due to viscous tensions is neglected. Therefore, the only nonzero velocity component is in the longitudinal direction. Let "*u*" the longitudinal velocity, "μ" the dynamical viscosity and "*dp/dx*" the axial gradient pressure (constant). Thus, the momentum equation is:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \cdot \frac{dp}{dz} \tag{1}$$

The boundary conditions are zero velocity at the wall (nonslip condition).
It is useful to define the mean velocity at section with area "*A*" (see Fig. (1)):

$$U_B = \frac{1}{A} \cdot \iint u.dx.dy \tag{2}$$

Equation (1), with its boundary conditions, was solved numerically using a finite difference method. Details of the numerical procedures can be found in Matoso (1998) and Garcia (1996).

2.2. Temperature field

The thermal profile is considered developed. Heat conduction in the flow direction is considered much smaller than that in the transversal direction. Natural convection is also neglected. Because the velocity profile is fully developed the transversal flow velocities are zero. Hence, the energy equation is:

$$\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} = \frac{u}{a} \cdot \frac{\partial T_f}{\partial z} \tag{3}$$

where "T_f" and "*a*" are the fluid temperature and diffusivity, respectively.
The fluid bulk temperature is:

$$T_B = \frac{1}{A.U_B} \cdot \iint u.T_f.dx.dy \tag{4}$$

With the purpose of defining a dimensionless variable that will become the thermal profile invariant in the longitudinal direction (characteristic of developed profiles) it is necessary before to define the mean temperature on the duct wall:

$$T_{Wm} = \frac{\frac{1}{L}.\int_0^L T_1(0,y).dy + \frac{1}{D}.\int_0^D T_2(x,0).dx + \frac{1}{L}.\int_0^L T_3(D,y).dy + \frac{1}{D}.\int_0^D T_4(x,L).dx}{4} \tag{5}$$

where "T_1", "T_2", "T_3" e "T_4" are the temperatures on the surfaces 1, 2, 3 and 4, respectively, as shown in Fig. 1.

As the wall temperature distributions are invariant in the longitudinal direction "T_W" is constant in that direction. The fluid will be heated up or will be cooled asymptotically. Therefore the present formulation will be similar to that found in Clark and Kays (1953) that studied rectangular ducts with uniform temperatures both in the duct cross section and in the longitudinal direction. Making a similar development as presented in the quoted reference, Eq. (3) becomes:

$$\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} = \frac{u}{a}.\left(\frac{T_{Wm} - T_f}{T_{Wm} - T_B}\right).\frac{dT_B}{dz} \tag{6}$$

To complete the formulation further definitions of dimensionless variables will be needed and it will be presented now:

$$X = \frac{x}{D_h} \tag{7}$$

$$Y = \frac{y}{D_h} \tag{8}$$

$$\phi = \frac{a.\left[T_{Wm} - T_f\right]}{U_B.D_h^2.\left(dT_B/dz\right)} \tag{9}$$

where "D_h" the hydraulic diameter for the perimeter "P_e". The definition of "D_h" is:

$$D_h = \frac{4A}{P_e} = \frac{2.L.D}{L+D} \tag{10}$$

Substituting Eq. (7) to Eq. (10) into Eq. (6) the dimensionless energy equation is obtained:

$$\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} = -\frac{u}{U_B}.\frac{\phi}{\phi_B} \tag{11}$$

In Equation (11) "ϕ_B" is the value of "ϕ", defined in Eq. (9), for "T_f" replaced by "T_B", that is:

$$\phi_B = \frac{a.\left[T_{Wm} - T_B\right]}{U_B.D_h^2.\left(dT_B/dz\right)} \tag{12}$$

Equation (12) may be rewritten as:

$$\frac{dT_B}{dz} = \frac{a.\left(T_{Wm} - T_B\right)}{U_B.D_h^2.\phi_B} \tag{13}$$

or

$$T_B = T_{Wm} - \frac{D_h^2.U_B}{a}.\frac{dT_B}{dz}.\phi_B \tag{14}$$

Equation (9) gives:

$$T_f = T_{Wm} - \frac{\phi . U_B . D_h^2 . (dT_B / dz)}{a} \tag{15}$$

Substituting Eq. (15) into Eq. (4) results:

$$T_B = T_{Wm} - \frac{D_h^2}{a.A} . \frac{dT_B}{dz} . \iint u.\phi.dx.dy \tag{16}$$

Substituting Eq. (16) in Eq. (12), and using Eq. (7) and Eq. (8), it is obtained the dimensionless form of the bulk temperature:

$$\phi_B = \frac{D_h^2}{AU_B} \iint udXdY \tag{17}$$

The dimensionless boundary conditions are:

$$\phi(0,Y) = \frac{a.\left[T_{Wm} - T_1\right]}{U_B . D_h^2 . (dT_B / dz)} \tag{18}$$

$$\phi(X,0) = \frac{a.\left[T_{Wm} - T_2\right]}{U_B . D_h^2 . (dT_B / dz)} \tag{19}$$

$$\phi(D/D_h, Y) = \frac{a.\left[T_{Wm} - T_3\right]}{U_B . D_h^2 . (dT_B / dz)} \tag{20}$$

$$\phi(X, \tfrac{L}{D_h}) = \frac{a.\left[T_{Wm} - T_4\right]}{U_B . D_h^2 . (dT_B / dz)} \tag{21}$$

These boundary conditions present a special case that it should be noted. If the temperatures on the walls are constant and the same, Eq. (18) to Eq. (21) are equal to zero and they will not be functions of "dT_B / dz", which is an unknown. This simplified case has been studied by Patankar (1991).

Equation (11), Eq. (12) and Eq. (13), with the boundary conditions, form a differential equation system whose unknowns are "ϕ" and "T_B". The determination of these functions allows the computation of the fluid temperature, "T_f" which is needed to calculate the entropy generation.

3. Entropy Generation

The entropy generation rate per unit volume is calculated by means of (Bejan, 1982):

$$\dot{S}'''_{ger} = \frac{k}{T_f^{\,2}} \left[\left(\frac{\partial T_f}{\partial x} \right)^2 + \left(\frac{\partial T_f}{\partial y} \right)^2 + \left(\frac{\partial T_f}{\partial z} \right)^2 \right] + \frac{\mu}{T_f} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 \right] \tag{22}$$

According to Clark and Kays (1953):

$$\frac{dT_f}{dz} = \left[\frac{T_{Wm} - T_f}{T_{Wm} - T_B} \right] \cdot \frac{dT_B}{dz} \tag{23}$$

Hence:

$$\dot{S}'''_{ger} = \frac{k}{T_f^{\,2}} \left[\left(\frac{\partial T_f}{\partial x} \right)^2 + \left(\frac{\partial T_f}{\partial y} \right)^2 + \left[\left(\frac{T_{Wm} - T_f}{T_{Wm} - T_B} \right) \cdot \frac{dT_B}{dz} \right]^2 \right] + \frac{\mu}{T_f} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 \right] \tag{24}$$

Integrating over the duct cross-section the entropy generation rate per unit length:

$$\dot{S}'_{ger} = \iint \dot{S}'''_{ger}\, dxdy \tag{25}$$

4. Numerical Method and Computational Model

The differential equation system described in sections 2 and 3 was solved numerically using a finite difference method. The coupling presented between equations and boundary conditions in the heat transfer section of this system requires an iterative treatment that will be described bellow.

With guessed values of "ϕ_B" and "dT_B /dz" the dimensionless form of energy equation, Eq. (11), is solved. Then, an improved value of "ϕ_B" is calculated using Eq. (17), forming a first iterative loop, as it is shown in Fig. (3). After the convergence of "ϕ_B" a better value of "dT_B /dz" is calculated by Eq. (13). The boundary conditions are then recalculated and the energy equation is again solved, forming a second iterative loop (Fig. (3)).When convergence is achieved, "T_f" is computed using Eq. (15). Hence, the necessary information to determine the entropy generation rates, Eq. 24 and Eq. 25, is obtained. This procedure is schematized in Fig. (3). In this figure "tol" represents the convergence criteria for "dT_B /dz" and for the dimensionless temperature field.

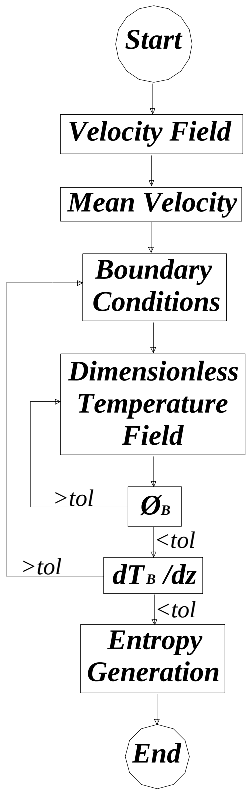

Figure 3 - Computational Model Flowchart.

5. Results and Discussion

Table presents the input parameters of a solution example obtained using the computational model described. The fluid is air.

Table 1. Input parameters

Description	Value	Unity
Duct height	0.01	*m*
Duct width	0.01	*m*
Fluid thermal conductivity	0.0261	W/m^0C
Fluid density	1.1776	Kg/m^3
Fluid thermal diffusivity	$2.22\text{x}10^{-5}$	m^2/s
Pressure gradient	16.0	*Pa/m*
Fluid dynamic viscosity	$1.853\text{x}10^{-5}$	*Pa.s*

Figure (4) shows two variable wall temperature distributions investigated. The temperature has been considered as a sine function of the position on the wall. This function was chosen because it represents well the numerical method capacity to deal with several type boundary conditions.

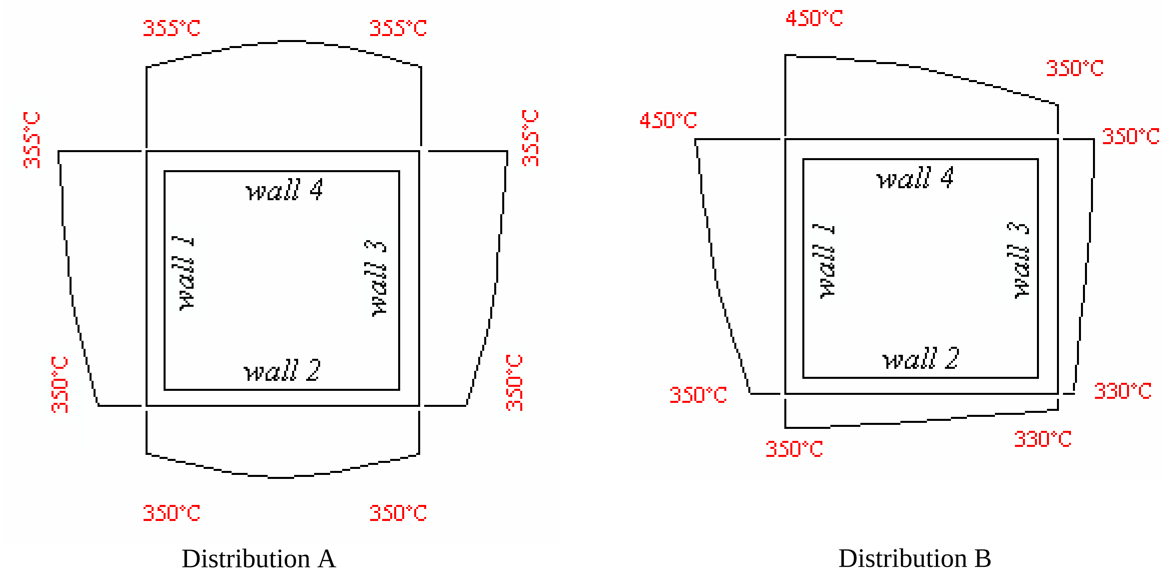

Figure 4. Wall temperature distributions.

Figure (5) shows the velocity field. This field is the same for both temperature distributions because the velocity field does not depend on temperature.

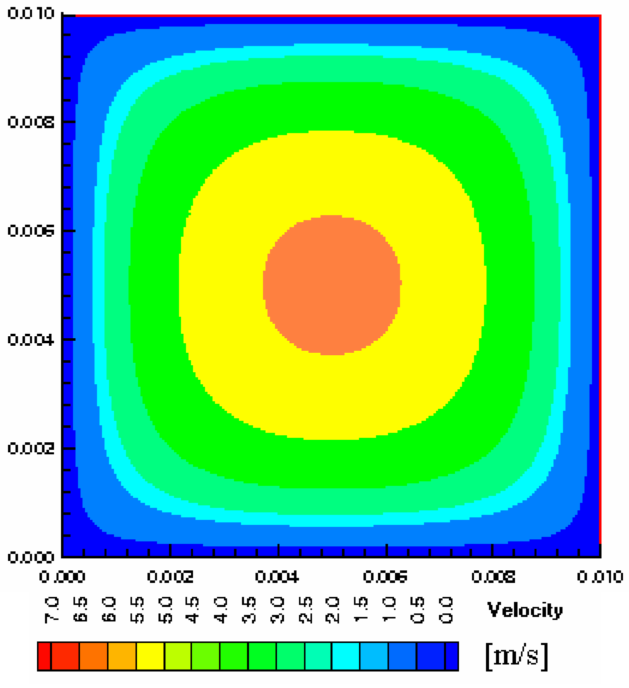

Figure 5 - Velocity Field.

Figure 6 presents the temperature distributions obtained. It is observed the strong thermal boundary condition influence in the temperature distributions.

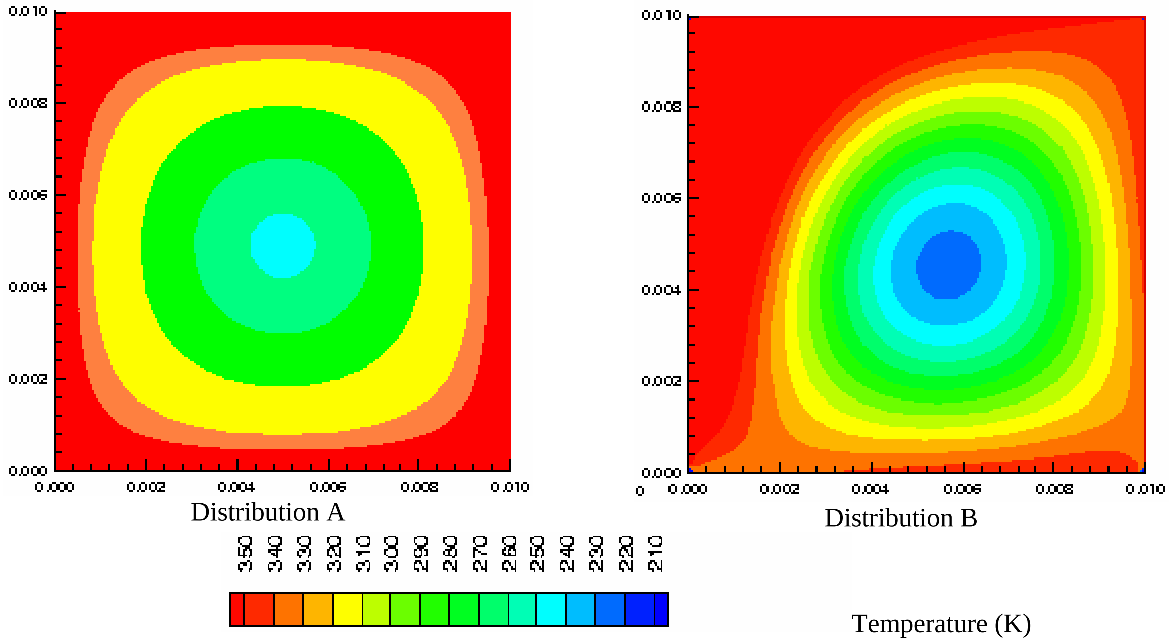

Figure 6 - Temperature Field.

For convenience the entropy generation rate will be presented in two parts. The first one is the entropy generation due to viscous flow. It corresponds to the last bracket in Eq. (24). The other bracket represents the entropy generation rate caused by heat transfer.

The entropy generation results due viscous flow are presented in Fig. (7). It is observed that there is little difference between the profiles resulting from the two distributions.

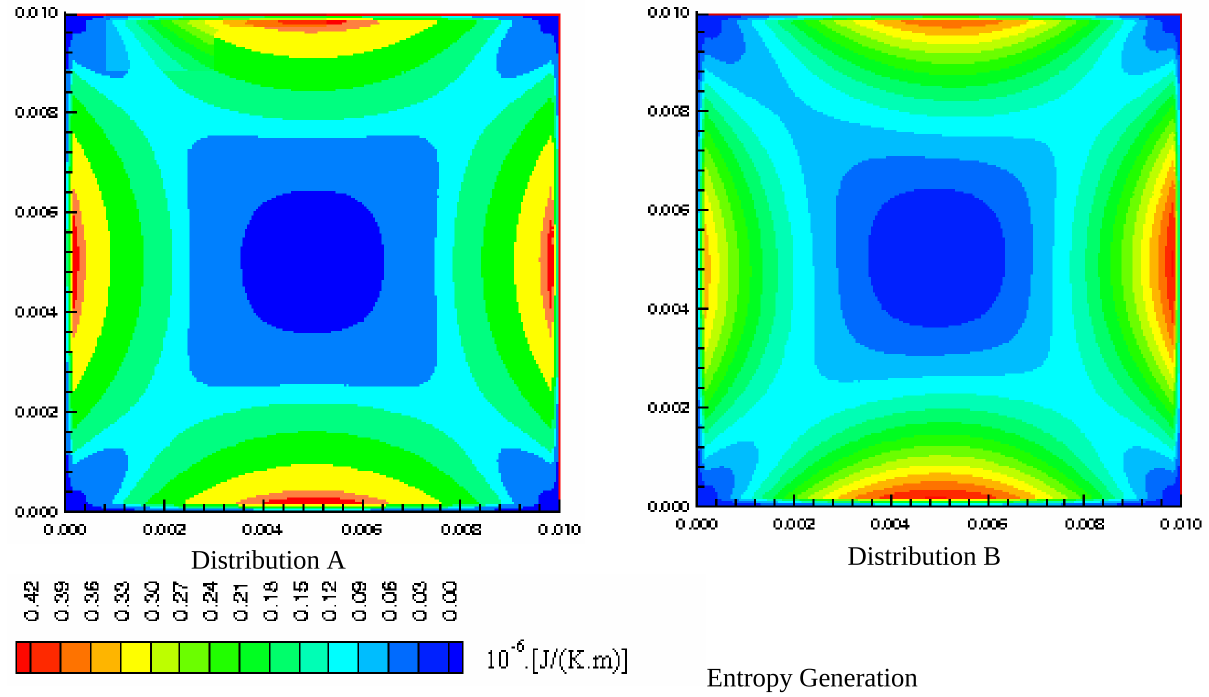

Figure 7. Entropy generation per length unit due to viscous flow.

The corresponding results for entropy generation rate due to heat transfer are in Fig. (8). Here, as it was expected,, there is a strong influence from fluid temperature distributions.

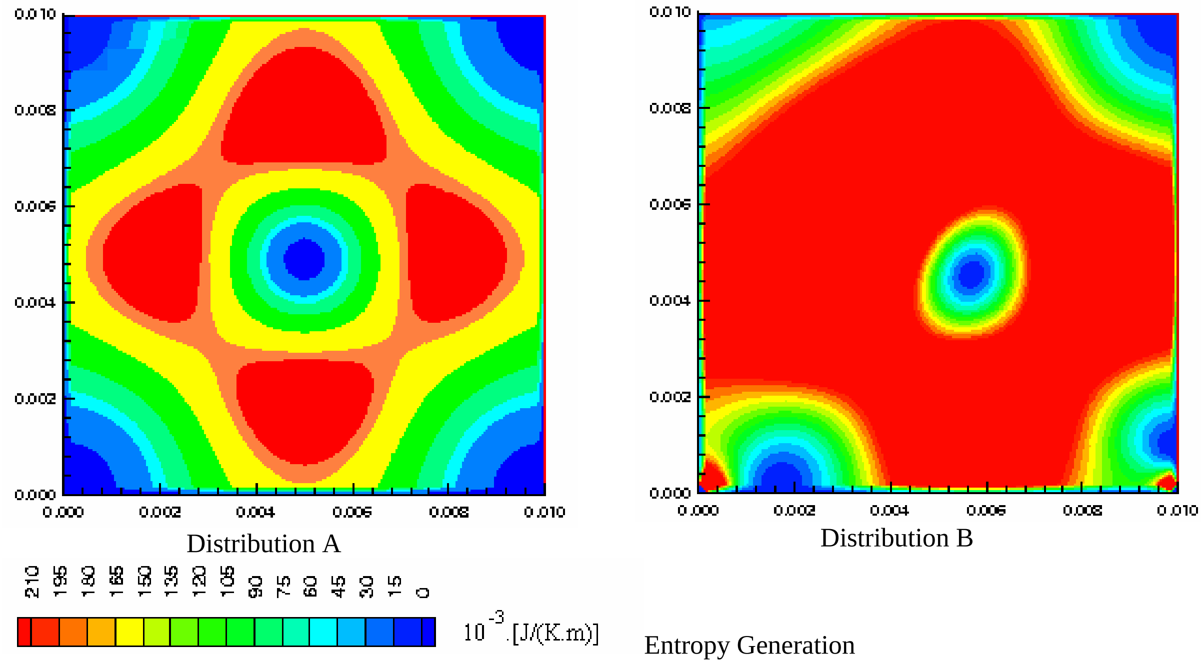

Figure 8 - Entropy generation rate per unit length due to heat transfer.

Comparing Fig. (7) and Fig. (8) it is seen that there is little contribution of viscous flow to the total entropy generation rate. This is also expected and it is almost a rule in laminar flows.

6. Conclusions

A computational method has been built for entropy generation rates in fully developed laminar flows in rectangular ducts. The method allows the imposition of any temperature distribution on duct wall. The model proved to be numerically stable and consistent and it is intended to be a tool in engineering design, using the EGMM approach, of thermal systems.

7. References

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