ON DYNAMIC BEHAVIOR OF A SIMPLE PORTAL STRUCTURE: THEORETICAL ASPECTS

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Abstract. *In this paper, we analyze a simple portal structure with two degrees of freedom which mathematical model has quadratic nonlinearities. The non-linear planar vibrations of this structure are of great interest in Engineering. By using Center Manifolds, we investigate the stability of the equilibrium point in the origin, as well as the existence of the phenomenon of bifurcation. In view of experimental studies done before, by a number of authors, the actual analysis shows that the vertical damping plays the most important role in the dissipative part of this system. When there are forced oscillations, and in the presence of the horizontal and vertical damping, we have the existence of asymptotically stable periodic orbits.*

Keywords. *Nonlinear Oscillations, Center Manifolds, Equilibrium Points, Asymptotic Stability, Bifurcation.*

1. Introduction

An analytical study of nonlinear vibrations of a simple machine portal frame foundation is presented in this paper. The portal frame studied here is similar to another one previously analyzed for support excitations see: Barr and Macwannell (1971). The goal of this paper is to discuss some theoretical and rigorous aspects of previous works that were obtained for a single motor foundation see Brasil and Mazilli (1990). Some extensions of this kind of problem were done by a number of authors Brasil (1996), Brasil and Mazilli, (1993) and Brasil and Mazilli (1995)). Brasil and Mazilli (1993). These authors analyzed this model, by using the Andros program that was implemented by these authors. We notice that: these results were, in good accordance, with the results obtained before, by using perturbation methods, studied before by Nayfeh and Mook (1979). We also remarked that this theory was used in several other nonlinear analysis, see Brazil and Mazilli (1995).

Here, the portal frame of Figure 1 is considered in the analysis. We take into account a structural system defined by two elastic columns of same h height, clamped in their bases. On each free extremity from these columns, a mass m is leaned on. These columns have constant cross section of I_c moment of inertia. A horizontal elastic beam of L length is pinned to these masses. This beam has constant cross section of I_b moment of inertia. This structural system consists of a linear elastic material which Young modulus is E. Geometric nonlinearities are introduced by considering the shortening due to bending of the columns and of the beam, that is quadratic type, Brasil and Mazilli (1990). Besides, a body of mass M is supported in the middle point of the horizontal bar. . Of course, the non-linear planar vibrations of this structure are of great interest in Engineering. Under the earlier assumptions, the motion equations of the mass M are deduced, as we see in Brasil and Mazilli (1990), Balthazar et all. (1997), Brasil and Balthazar (2002). . We mention that the coordinate q_1 is proportional to horizontal displacement and q_2 is proportional to vertical displacement, see Fig. (1).

Next, taking into account that the potential energy and kinetic energy were expanded to cubic terms, the Lagrangian equations of motion, are the following ones (see, for an example: Brasil and Mazilli, (1995)):

$$\begin{cases} \ddot{q}_1 + k_1 q_1 + \mu_1 \dot{q}_1 + \alpha_1 q_1 q_2 = 0, \\ \ddot{q}_2 + \omega_2^2 q_q + \mu_2 q_2 + \alpha_2 q_1^2 + \dfrac{g}{L} = 0,. \end{cases} \tag{1}$$

where

$$k_1 = \frac{2(k_c - mgC)}{2m+M}, \quad w_2^2 = \frac{k_b}{M}, \quad m_1 = \frac{C_1}{(2m+M)h},$$
$$m_2 = \frac{C_2}{ML}, \quad a_1 = \frac{k_b CL}{2m+M}, \quad a_2 = \frac{k_b Ch^2}{2mL}, \tag{2}$$
$$k_c = \frac{3EI_c}{h^3}, \quad k_b = \frac{48EI_b}{L^3}, \quad C = \frac{6}{5h},$$

g is the acceleration of gravity and w_2 is a natural frequency of the system.

Here, in view of (2), we assume that the parameter k_1 can be taken as positive or negative. Later, we will show that, the changes of the signal of the coefficient of q_1 will play an important role in proof of the existence of bifurcations. C_1, C_2 are the coefficients of viscous damping. We also mention that Palacios et. alL., and (2002) used nonlinear saturation phenomenon and internal resonance in order to obtain a control of the vibrations of the portal frame model that was defined by Fig. (1). They showed that the steady state vibration could be controlled and they proved that this kind of control technique was efficient and robust.

We remarked that the goal of this paper is to investigate some questions about stability and bifurcation of (1). It is clear that all of them depend on the coefficients m_1, m_2. Therefore, we examine four distinct cases:

1. $m_1 > 0, m_2 > 0$,
2. $m_1 = 0, m_2 > 0$,
3. $m_1 > 0, m_2 = 0$,
4. $m_1 = 0, m_2 = 0$,

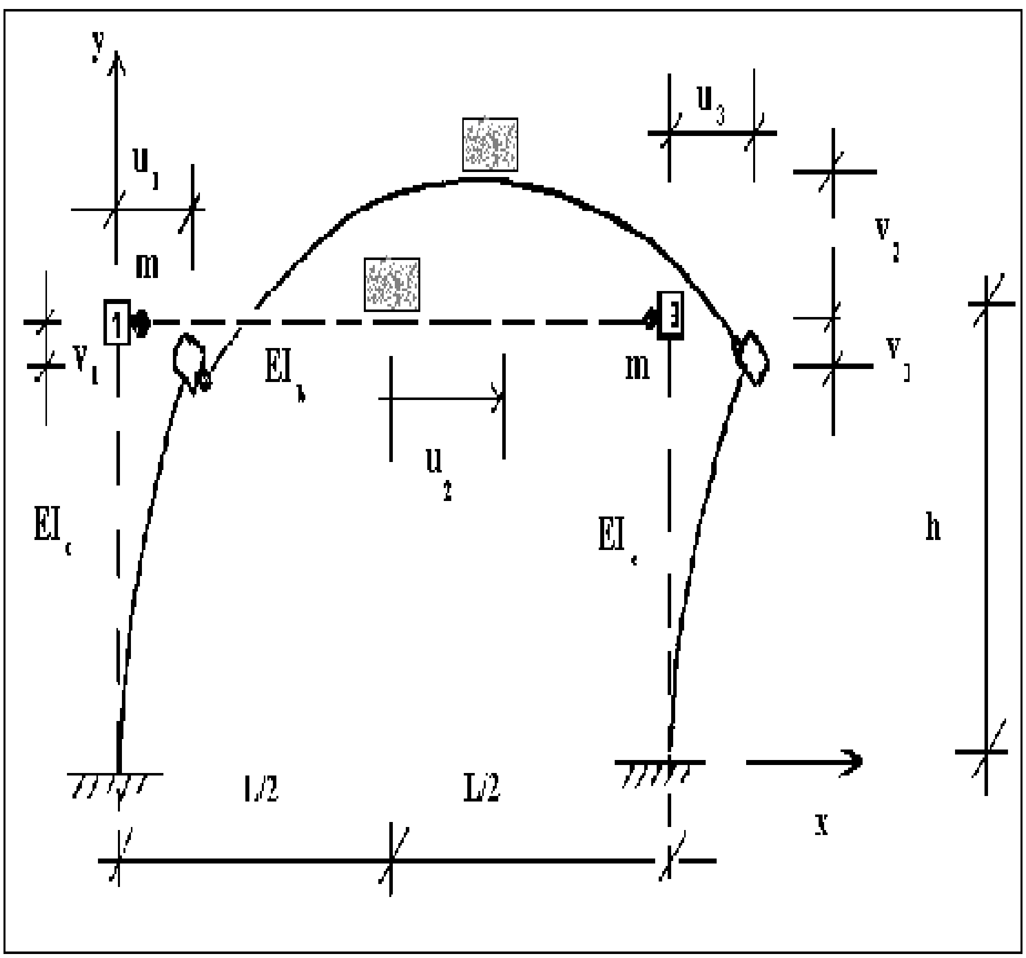

Figure 1. A Portal Structure

Unexpectedly, from the qualitative viewpoint, cases 1 and 2 are both similar. However, case 2 has a decaying rate of some quantities that suggests that there is a better agreement with experiences than case 1. In case 3 we prove that there are stable periodic solutions of (1). From experiments see Brasil et all (2002), there is no periodic behavior, so we will discard such possibility. In case 4, a simple change of variables shows that (1) is a Hamiltonian system

So, away from resonance, there is a handful of periodic orbits close to the equilibrium point. According to the experimental evidence, that was done by Brasil et all (2002), we may conclude that this dynamic behavior is not acceptable. We also proved that in all cases there is bifurcation, which happens due to the action of the gravitational field.

If we taking into account the portal structure subject to forced oscillations, the equation of motion is given by

$$\begin{cases} \ddot{q}_1 + k_1 q_1 + \mu_1 \dot{q}_1 + \alpha_1 q_1 q_2 = A\cos\Omega t, \\ \ddot{q}_2 + \omega_2^2 q_q + \mu_2 q_2 + \alpha_2 q_1^2 + \dfrac{g}{L} = B\sin\Omega t,. \end{cases} \tag{3}$$

where A, B are constants. Here, we will prove that if condition 1 holds, then (3) has periodic solutions and in this case, there is a bifurcation (where these solutions change its stability). We remarked that this problem was investigated before, in aspects different from those analyzed here.

2. Preliminaries

Making $q_2 = q_3 - \dfrac{g}{\omega_2^2 L}$, (1) may be written as

$$\begin{cases} \ddot{q}_1 + kq_1 + \mu_1 \dot{q}_1 + \alpha_1 q_1 q_3 = 0, \\ \ddot{q}_3 + \omega_2^2 q_3 + \mu_2 q_2 + \alpha_2 q_3^2 = 0, \end{cases} \tag{4}$$

where $k = k_1 - \dfrac{g\alpha_1}{\omega_2^2 L}$. Taking new variables, defined by $x = q_1, y = \dot{q}_1, u = q_3, v = \dot{q}_3$, we will obtain from (4) that

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -k & -\mu_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_2^2 & -\mu_2 \end{pmatrix} \begin{pmatrix} x \\ y \\ u \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ -\alpha_1 xu \\ 0 \\ -\alpha_2 x^2 \end{pmatrix}. \tag{5}$$

Note that, the equilibrium points (x_0, y_0, u_0, v_0) of (5) are exactly $(0,0,0,0)$ and $\left(\pm\sqrt{\dfrac{k\omega_2^2}{\alpha_1\alpha_2}}, 0, -\dfrac{k}{\alpha_1}, 0\right)$ if $k > 0$. In addition, $(0,0,0,0)$ is the unique equilibrium point if $k < 0$. By using the Hurwitz Criterion, see Meirovitch (1970), it is easy to obtain the following result.

Proposition 1. *a) If $k < 0$ then $(0,0,0,0)$ is an unstable equilibrium point of (5).*

b) If $k > 0$ then $\left(\pm\sqrt{\dfrac{k\omega_2^2}{\alpha_1\alpha_2}}, 0, -\dfrac{k}{\alpha_1}, 0\right)$ are unstable equilibrium points of (5).

Next, we will show that the dynamical behavior of the system, in the equilibrium point $(0,0,0,0)$, changes according to the signal of k.

3. Case $\mu_1 > 0, \mu_2 > 0$

If $k > 0$, all of the eigenvalues of (5) have negative real parts, then $(0,0,0,0)$ is an asymptotically stable equilibrium point. If $k = 0$, the expressions of the eigenvalues of (5) are given by

$$0, -\mu_1, -\frac{1}{2}\mu_2 + \frac{1}{2}\sqrt{\mu_2^2 - 4\omega_2^2}, -\frac{1}{2}\mu_2 - \frac{1}{2}\sqrt{\mu_2^2 - 4\omega_2^2}.$$

Note that, Since 0 is an eigenvalue, we will use the center manifold approach as it was given in Carr and Al-Amood (1980).

Making the following change of variables $x=\frac{z-w}{\mu_1}$, $y=w$ in (5), we will obtain that

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{u}\\ \dot{v}\end{pmatrix}=\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & -\mu_1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -\omega_2^2 & -\mu_2\end{pmatrix}\begin{pmatrix}x\\ y\\ u\\ v\end{pmatrix}+\begin{pmatrix}-\frac{\alpha_1}{\mu_1}(z-w)u\\ -\frac{\alpha_1}{\mu_1}(z-w)u\\ 0\\ -\frac{\alpha_2}{\mu_1^2}(z-w)^2\end{pmatrix}. \tag{6}$$

We obtain that the center manifold of (6) is the graph of a mapping $z\to(p(z),q(z),r(z))$ near 0, where $p(0)=p'(0)=q(0)=q'(0)=r(0)=r'(0)=0$ and $w=p(z),u=q(z),v=r(z)$. Hence, the center manifold, (6) may be written as

$$\dot{z}=-\frac{\alpha_1}{\mu_1}(z-p(z))q(z). \tag{7}$$

Now, we need to find an approximation of p,q and r. For this, we will use the Theorem 3 of Carr and Al-Amood (1980). Letting

$$M(\varphi)(z)=-\frac{\alpha_1}{\mu_1}(z-p(z))q(z)\begin{pmatrix}p'(z)\\ q'(z)\\ r'(z)\end{pmatrix}-\begin{pmatrix}-\mu_1 & 0 & 0\\ 0 & 0 & 1\\ 0 & -\omega_2^2 & -\mu_2\end{pmatrix}\begin{pmatrix}p(z)\\ q(z)\\ r(z)\end{pmatrix}-\begin{pmatrix}-\frac{\alpha_1}{\mu_1}(z-p(z))q(z)\\ 0\\ -\frac{\alpha_2}{\mu_1^2}(z-p(z))^2\end{pmatrix}, \tag{8}$$

where $\varphi=(p,q,r)$. In addition, taking $p(z)=Az^2, q(z)=Bz^2, r(z)=Cz^2$ in the earlier equation, we will obtain that

$$M(\varphi)(z)=\begin{pmatrix}\mu_1 Az^2\\ -Cz^2\\ \left(\frac{\alpha_2}{\mu_1^2}+\omega_2^2 B+\mu_2 C\right)z^2\end{pmatrix}+O(3). \tag{9}$$

So, if $A=C=0, B=-\frac{\alpha_2}{\mu_1^2\omega_2^2}$ we get $M(\varphi)(z)=O(3)$. By using the Theorem 3 of Carr and Al-Amood (1980), (7) may be rewritten as:

$$\dot{z}=\frac{\alpha_1\alpha_2}{\mu_1^3\omega_2^2}z^3+O(4). \tag{10}$$

Hence, 0 is an unstable equilibrium point of (10). By using Theorem 2 of Carr and Al-Amood (1980) we will obtain that $(0,0,0,0)$ is an unstable equilibrium point of (6).

If $k<0$, it follows from Proposition 1 that $(0,0,0,0)$ is an unstable equilibrium point.

In summary, we remarked, that by one hand if $\alpha_1<\frac{k_1\omega_2^2 L}{g}$, then $(0,0,0,0)$ is an asymptotically stable equilibrium point and by another hand, if $\alpha_1\geq\frac{k_1\omega_2^2 L}{g}$ this point is an unstable equilibrium point.

4. Case $m_1 = 0, m_2 > 0$

If $k > 0$, the eigenvalues of (5) are the following ones:

$$-\frac{1}{2}m_2 + \frac{1}{2}\sqrt{m_2^2 - 4\mathbf{w}_2^2}, -\frac{1}{2}m_2 - \frac{1}{2}\sqrt{m_2^2 - 4\mathbf{w}_2^2}, \pm i\sqrt{k}\ .$$

From this and by using (5), we may use the results of Carr and Al-Amood (1980). In this particular case, a center manifold is given by $u = f(x,y), v = g(x,y)$ where $f(0,0) = g(0,0) = 0$ and $f'(0,0) = g'(0,0) = 0$. Note that the mapping $M(\cdot)$ defined in Carr and Al-Amood (1980) is given by

$$M(\boldsymbol{j}) = \begin{pmatrix} \frac{\partial f(x,y)}{\partial x} & \frac{\partial f(x,y)}{\partial y} \\ \frac{\partial g(x,y)}{\partial x} & \frac{\partial g(x,y)}{\partial y} \end{pmatrix}\left[\begin{pmatrix} 0 & 1 \\ -k & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ -\boldsymbol{a}_1 x f(x,y) \end{pmatrix}\right] - \begin{pmatrix} 0 & 1 \\ -\mathbf{w}_2^2 & m_2 \end{pmatrix}\begin{pmatrix} f(x,y) \\ g(x,y) \end{pmatrix} - \begin{pmatrix} 0 \\ -\boldsymbol{a}_2 x^2 \end{pmatrix},$$

where $\boldsymbol{j} = (f, g)$. Now, taking $f_0(x,y) = Ax^2 + Bxy + Cy^2, g_0(x,y) = Dx^2 + Exy + Fy^2$ where

$$A = -\frac{\left(2m_2^2 k + (4k - \mathbf{w}_2^2)(2k - \mathbf{w}_2^2)\right)\boldsymbol{a}_2}{\mathbf{w}_2^2\left(4m_2^2 k + (4k - \mathbf{w}_2^2)^2\right)},$$
$$B = \frac{2m_2\boldsymbol{a}_2}{4m_2^2 k + (4k - \mathbf{w}_2^2)^2},$$
$$C = \frac{-2(m_2^2 + 4k - \mathbf{w}_2^2)\boldsymbol{a}_2}{\mathbf{w}_2^2\left(4m_2^2 k + (4k - \mathbf{w}_2^2)^2\right)},$$
$$D = \frac{-2km_2\boldsymbol{a}_2}{4m_2^2 k + (4k - \mathbf{w}_2^2)^2},$$
$$E = \frac{2(4k - \mathbf{w}_2^2)\boldsymbol{a}_2}{4m_2^2 k + (4k - \mathbf{w}_2^2)^2},$$
$$F = \frac{2m_2\boldsymbol{a}_2}{4m_2^2 k + (4k - \mathbf{w}_2^2)^2},$$

after a long, but straightforward computation, we will obtain $M(\boldsymbol{j}_0) = O(3)$, where $\boldsymbol{j}_0 = (f_0, g_0)$. By using the Theorem 3 of Carr and Al-Amood (1980) we get $\boldsymbol{j} - \boldsymbol{j}_0 = O(3)$. In this way the graph of the center manifold, the equation (5) becomes

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -k & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ -\boldsymbol{a}_1 x f_0(x,y) \end{pmatrix} + O(4). \quad (11)$$

Making the following change of variable $y \to \frac{y}{\sqrt{k}}$ in (11), we will obtain

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{k} \\ -\sqrt{k} & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ -\boldsymbol{a}_1 \frac{x f_0(x, \sqrt{k}y)}{\sqrt{k}} \end{pmatrix} + O(4). \quad (12)$$

The normal form of (12) is given by

$$\begin{aligned} \dot{u} &= a(u^2 + v^2)u - (-\mathbf{w}_1 + b(u^2 + v^2))v + O(4), \\ \dot{v} &= (-\mathbf{w}_1 + b(u^2 + v^2))u + a(u^2 + v^2)v + O(4). \end{aligned} \quad (13)$$

By using Guckenheimer and Holmes (1983), pg. 151-152), the constant a is given by

$$a = \frac{1}{16}\left(-2B\boldsymbol{a}_1\right) = -\frac{1}{4}\frac{m_2\boldsymbol{a}_1\boldsymbol{a}_2}{4m_2^2 k + \left(4k - \boldsymbol{w}_2^2\right)^2} < 0.$$

The equation (13) is expressed by using polar coordinates, as

$$\begin{cases} \dot{r} = -\frac{1}{4}\frac{m_2\boldsymbol{a}_1\boldsymbol{a}_2}{4m_2^2 k + \left(4k - \boldsymbol{w}_2^2\right)^2} r^3 + O(4), \\ \dot{\boldsymbol{q}} = \left(-\sqrt{k} + br^2\right) + O(4), \end{cases} \tag{14}$$

From (14) we obtain that $lim_{t\to\infty} r(t) = 0$. Hence, $(0,0)$ is an asymptotically stable equilibrium point of (13). From Theorem 2 of Carr and Al-Amood (1980) we get that $(0,0,0,0)$ is an asymptotically stable equilibrium point of (6). By ignoring the $O(4)$ terms in $(14)_1$ we will obtain

$$r(t) = \sqrt{\frac{2\left(4m_2^2 k + \left(4k - \boldsymbol{w}_2^2\right)^2\right)}{m_2\boldsymbol{a}_1\boldsymbol{a}_2}}\sqrt{\frac{1}{t}} + O\left(\left(\sqrt{\frac{1}{t}}\right)^3\right).$$

So, when $2\sqrt{k} = \boldsymbol{w}_2$ (resonant case) the solution $r(t)$ has the least rate of decay.

Anyway, we obtain the same conclusions of the earlier section about stability of the equilibrium point $(0,0,0,0)$.

5. Case $m_1 > 0, m_2 = 0$

If $k > 0$,the eigenvalues of the linear part of (5) are a pair of negative numbers and a pair of pure imaginary. Moreover, the mapping $h(u,v) = (0,0)$, $(x = 0, y = 0)$ is a center manifold of (5). On the graph of h, (5) becomes

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\boldsymbol{w}_2^2 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}. \tag{15}$$

Since $(0,0)$ is a stable equilibrium point of (15), it follows from Theorem 2 of Carr and Al-Amood (1980) that $(0,0,0,0)$ is a stable equilibrium point of (5). Let $(x(t), y(t), u(t), v(t))$ be a solution of (5). Then, from the Theorem 2 of Carr and Al-Amood (1980), there is a solution $w(t)$ of (15) such that $(x(t), y(t)) = O\left(e^{-gt}\right)$ and $(u(t), v(t)) = w(t) + O\left(e^{-gt}\right)$ where $g > 0$ is a constant.

As $h(u,v) = (0,0)$ is a center manifold, we obtain from Remark 2.16, pg. 322 of Chow and Hale (1982), that (5) is topologically equivalent to the linear system

$$\begin{cases} \dot{w} = \begin{pmatrix} 0 & 1 \\ -\boldsymbol{w}_2^2 & 0 \end{pmatrix} w, \\ \dot{z} = \begin{pmatrix} 0 & 1 \\ -k & 0 \end{pmatrix} z. \end{cases} \tag{16}$$

Hence the periodic orbits $(x, y, u, v) = (0, 0, u_0(t), \dot{u}_0(t))$, where $\ddot{u}_0 + w_2^2 u_0 = 0$, of (5) are *stable orbits*.

Again, in this case, $\boldsymbol{a}_1 = \frac{k_1\boldsymbol{w}_2^2 L}{g}$ is a bifurcation point.

6. Case $m_1 = 0, m_2 = 0$

Making $u \to \sqrt{\frac{2\boldsymbol{a}_2}{\boldsymbol{a}_1}} u$, $v \to \sqrt{\frac{2\boldsymbol{a}_2}{\boldsymbol{a}_1}} v$ in (5), we obtain the following Hamiltonian system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} \frac{\partial H}{\partial y} \\ -\frac{\partial H}{\partial x} \\ \frac{\partial H}{\partial v} \\ -\frac{\partial H}{\partial u} \end{pmatrix}, \tag{17}$$

where, $H(x,y,u,v) = \frac{kx^2 + y^2}{2} + \frac{\omega_2^2 u^2 + v^2}{2} + \alpha x^2 u$, and $\alpha = \sqrt{\frac{\alpha_1 \alpha_2}{2}}$. As in the foregoing section $x = y = 0$ is an invariant manifold of (17). If $k > 0$, and since

$$D^2 H(0,0,0,0) = \begin{pmatrix} k & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \omega_2^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

is a positive matrix and by using Theorem 3.4.17, pg. 207 of Abraham and Marsden (1978), we obtain that $(0,0,0,0)$ is a stable equilibrium point of (17). Moreover, if k and ω_2 satisfy the non-resonance condition, *i.e.*, k/ω_2 and $\omega_2/k \notin IN$, then by Liapunov Subcenter Stability Theorem, see Theorem 8.4.2, pg. 580 of Abraham and Marsden (1978), there are two two-dimensional invariant manifold of (17). Each invariant manifold is a union of closed orbits. Clearly $x = y = 0$ is one of these manifolds.

As earlier, there is bifurcation at $\alpha_1 = \frac{k_1 \omega_2^2 L}{g}$.

We would like to notice that (17) is equivalent to the equation (1.3), pg. 75 of Starzhinsky (1980), which models coupling of radial and vertical oscillations of particles in cycling accelerators.

7. Forced Oscillations

By using, the Poincaré mapping as it was made in Theorem 4.1.1 of Guckenheimer and Holmes (1983), we will obtain, by a simple argument, the following result.

Proposition 2. Consider the following system:

$$\dot{x} = g_0(x) + \varepsilon g_1(t); \tag{18}$$

where $g : \Delta \subset IR^n \to IR^n$ is a C^∞ mapping, Δ an open subset, 0 is a hyperbolic fixed point of g and g_1 is a C^∞ T-periodic mapping. Then, for all ε adequately small, (18) has a periodic orbit with period T of the same stability type as 0 .

By using this result, we will obtain that if $\mu_1, \mu_2 > 0, k \neq 0$ and A, B are adequately small, then (3) has periodic orbits. Moreover, if $\alpha_1 < \frac{k_1 \omega_2^2 L}{g}$ these orbits are asymptotically stable and if $\alpha_1 > \frac{k_1 \omega_2^2 L}{g}$, we have unstable orbits.

8. Concluding Remarks

In this paper, we obtain results about the existence of bifurcation points of (1). We also analyzed the local behavior of this system where some of the damping coefficients are equal to zero. Our results suggest strongly that it is enough to consider only the vertical damping. For equation (3), where there exists ``harmonic forcing'', we obtain by use of a simple argument, a bifurcation of periodic orbits. We used the techniques of the Center Manifold to obtain rigorous results about the dynamic behavior of (1) and (3). These results are new, according author's knowledgements.

9. Acknowledgements

The second author acknowledges financial support by FAPESP - Fundação de Amparo à Pesquisa do Estado de São Paulo and CNPq - Conselho Nacional de Desenvolvimento Científico, both Brazilian research funding agencies.

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