WING STRUCTURE MODAL PARAMETER ESTIMATION AND QUALIFICATION USING ERA

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Abstract. The determination of the dynamic characteristics of aircraft structures has become an extremely important issue in the aerospace industry, primarily due to the continuous demand for lighter and consequently more flexible structures. In this context, most aerospace structural system must be subjected to some form of modal verification prior to flight in order to ensure that the aircraft is free from any dangerous aeroelastic instability phenomena. The verification procedure often includes the experimental identification of structural characteristics such as the natural frequency and normal modes using modal testing. This paper presents a ground vibration testing (GVT) of a metallic wing of the Neiva Regente aircraft in order to assess the frequency response functions. Identification algorithm (ERA) is used to determine mode shapes from experimental data. The experimental results, nevertheless, may present computational modes that must be removed from the model. The Consistent-Mode Indicator (CMI) is used for assessing the consistency of structural modal parameter identified with the (ERA). The first two natural frequencies and modes are presented. Because of nonlinearities and numerous local modes, modal identification of aircraft wing has proved to be surprisingly difficult.

Keywords: Aeroelasticity, GVT, ERA, Structural Dynamics, Identification.

1. INTRODUCTION

Aircraft have been in constant development in order to become lighter, strengthener and more reliable. As consequence of such demand, aircraft structures have also become more flexible increasing the susceptibility to vibrations and aeroelastic instabilities. The study of aeroelastic phenomena, such as flutter, has been of great attention in aeronautical science. Flutter is a dynamic aeroelastic instability, which involves the interaction, or coupling, of aerodynamic, elastic and inertial forces, yielding to undesired divergent oscillatory behaviour that can lead to the structure failure or fatigue problems (Bisplinghoff *et al.*, 1955). Particularly for fixed- and rotary-wing aircraft, unwanted aeroelastic phenomena can raise from inappropriate structural dynamic features. Aircraft structures may also suffer from excessive vibration because of their structural flexibility, causing discomfort to the crew and fatigue problems. To avoid structural dynamic problems, aeronautical engineers must be able to attain the main dynamic characteristics, such as: natural/resonant frequencies and mode shapes. With those information it is possible to verify and re-design both structure and aerodynamics avoiding excessive vibration problems and suppressing unwanted aeroelastic instabilities.

Nonetheless, for complex structures, typical in aerospace industry, the attainment of dynamic characteristics is not an easy task. The finite element methods (FEM) have been the most successful tool for obtaining the dynamic features during preliminary design. Although the FEM has been consolidated in the aerospace industry, experimental verification of aircraft design is still necessary. Advances in acquisition system and in the field of sensors and/or actuators devices are also providing new possibilities to the aerospace structural dynamics. Kehoe and Freudinger (1993) and Abel (1997) present overviews on the most recent development in the main NASA research centres on aerospace structural dynamics. They have shown the main researches on this area, the modern techniques in use and highlight the vital role of modal parameters assessment, in particular, the so-called ground vibration testing, in assuring the prevention of aeroelastic instabilities.

The need for experiments to modal identification, i.e., the identification methods of modal parameters, is basically due to the complexity of typical aerospace structures. Accurate predictions of the structural modes are desired to understand the dynamic behaviour of the aircraft. According to Juang (1994), experimental approaches for modal identification usually work with data in the form of free-decay vibration measurement, frequency response functions (FRFs), impulse response functions (IRFs), etc. In this work, an Experimental Modal Analysis, or Ground Vibration Test, associated with an Identification Algorithm have been performed on an aeronautical structure to identify its structural modes and their associated natural frequencies and damping. This type of testing is an important part of the final aircraft design flight activity. The reasons for its important role in flight test include correlating and verifying the test modal data with dynamic finite-element models used to predict potential structural instabilities (such as flutter), assessing the significance of modifications to the aircraft structure by comparing the modal data before and after the modification, and helping to resolve in-flight anomalies (Sczibor, 2002).

Numerical algorithms have been also developed to calculate the modal parameters from the aforementioned experimental data. The most powerful modal identification algorithm in use today is the Eigensystem Realization Algorithm (ERA), which is a time domain algorithm that can identify many modes simultaneously (Tsunaki, 1999; Juang, 1994). Moreover, the mathematical formulation of ERA also allows the direct application of reliability coefficients to distinguish between computational and physical modes (Juang and Pappa, 1985). Recently, Pappa (1997), Cooper (1997) and Tasker *et al.*(1998) have presented variations and extensions to the ERA.

This paper presents a modal identification process based on ERA. Initially, a brief mathematical description of the ERA and CMI are developed. After this, the semi-span wing of the Neiva Regente aircraft has been subjected to a ground vibration test to get FRFs. Then, the ERA is used to identify the mode shapes from experimental data. Finally, the confidence factor CMI has been used to qualify the results, like in Pappa *et al.* (1993). The modes presenting higher CMI values have been considered leading to the identified model. Difficulties and assumptions adopted during the wing experimental analysis are also presented and discussed.

2. EIGENSYSTEM REALIZATION ALGORITHM APPROACH

A finite dimensional, discrete-time, linear, time-invariant dynamical system has the state-variable equations

$$x(k+1) = Ax(k) + Bu(k)$$
 (1)

$$y(k+1) = Cx(k) + Du(k)$$
 (2)

where x is a *n*-dimensional state vector, u a *m*-dimensional control input, and y a *p*-dimensional output or measurement vector. The integer k is the sample indicator. The transition matrix A characterizes the dynamics of the system. For flexible structures, it is a representation of mass, stiffness, and damping properties.

For the system Eqs. (1) and (2) with free pulse response, the time domain description is given by the function know, as the Markov parameter

$$Y(k) = C A^{k-1} B$$
⁽³⁾

or in the case of initial state response

$$Y(k) = C A^{k} [x_{1}(0), x_{2}(0), ..., x_{m}(0)]$$
⁽⁴⁾

where $x_i(0)$ represents the *i*th set of initial conditions and *k* is an integer. Note that *B* is an $n \times m$ matrix and *C* is a $p \times n$ matrix. The problem of system realization is: Given the functions Y(k), construct constant matrices [A, B, C] in terms of Y(k) such that identities of Eq. (3) hold and the order of *A* is minimum.

The ERA approach, Juang and Pappa (1985), begins by forming the $r \times s$ block matrix (generalized Hankel matrix), that is:

$$H_{rs}(k-1) = \begin{bmatrix} Y(k) & Y(k+t_1) & \cdots & Y(k+t_{s-1}) \\ Y(j_1+k) & Y(j_1+k+t_1) & \cdots & Y(j_1+k+t_{s-1}) \\ \vdots & \vdots & \ddots & \vdots \\ Y(j_{r-1}+k) & Y(j_{r-1}+k+t_1) & \cdots & Y(j_{r-1}+k+t_{s-1}) \end{bmatrix}$$
(5)

where $j_i (i = 1, ..., r - 1)$ and $t_i (i = 1, ..., s - 1)$ are arbitrary integers. For the system with initial state response measurements, simply replace $H_{rs}(k-1)$ by $H_{rs}(k)$. Now observe that

$$H_{rs}\left(k\right) = V_{r} A^{K} W_{s}$$
⁽⁶⁾

in which

$$V_{r} = \begin{bmatrix} C \\ CA^{j_{1}} \\ \vdots \\ CA^{j_{r-1}} \end{bmatrix} \qquad W_{s} = \begin{bmatrix} B & A^{t_{1}}B & \cdots & A^{t_{s-1}}B \end{bmatrix}$$
(7)

where V_r and W_s are the observability and controllability matrices, respectively. Note that V_r and W_s are rectangular matrices with dimensions $rp \times n$ and $n \times ms$, respectively. Assume that there exists a matrix H^+ satisfying the relation

$$W_s H^+ V_r = I_n \tag{8}$$

where I_n is an identity matrix of order *n*. It will be shown that the matrix H^+ plays a major role in deriving the ERA. Observe that, from Eqs. (7) and (8),

$$H_{rs}(0)H^{+}H_{rs}(0) = V_{r}W_{s}H^{+}V_{r}W_{s} = V_{r}W_{s} = H_{rs}(0)$$
⁽⁹⁾

The matrix H^+ is thus the pseudoinverse of the matrix $H_{rs}(0)$ in a general sense.

The ERA process starts with the factorisation of the block data matrix Eq. (5), for k=1, using singular value decomposition:

$$H_{rs}(0) = R \Sigma S^{T}$$
⁽¹⁰⁾

where the columns of matrices R and S are orthonormal and Σ is a rectangular matrix, that is

$$\Sigma = \begin{bmatrix} \Sigma_n & 0\\ 0 & 0 \end{bmatrix}$$

with $\Sigma_n = diag[\sigma_1, \sigma_2, \dots, \sigma_i, \sigma_{i+1}, \dots, \sigma_n]$ and monotonically non-increasing σ_i (*i*=1, 2, ..., *n*), $\sigma_1 \ge \sigma_2 \ge \dots \ge \sigma_i \ge \sigma_{i+1} \ge \dots = \sigma_n \ge 0$.

Next, let R_n and S_n be the matrices formed by the first *n* columns of *R* and *S*, respectively. Hence, the matrix H(0) and its pseudoinverse become

$$R_{2n} \sum_{2n}^{\frac{1}{2}} = V_r$$

$$\sum_{2n}^{\frac{1}{2}} S_{2n}^T = W_s$$
(11)

where

and

$$R_n^T R_n = I_n = S_n^T S_n$$

$$H^+ = S_n \Sigma_n^{-1} R_n^T$$
(12)

Equation (12) can be readily proved by observing Eq. (9).

Defining 0_p as a null matrix of order p, I_p as an identity matrix of order p, $E_p^T = [I_p, 0_p, \dots, 0_p]$, (where p is the number of outputs), and $E_m^T = [I_m, 0_m, \dots, 0_m]$, (where m is the number of inputs). Using Eqs. (3), (6), (8), (11), and (12), a minimum order realization can be obtained as follows:

$$Y(k+1) = E_{p}^{T} H_{rs}(k) E_{m} = E_{p}^{T} V_{r} A^{k} W_{s} E_{m}$$

$$= E_{p}^{T} V_{r} \left[W_{s} H^{+} V_{r} \right] A^{k} \left[W_{s} H^{+} V_{r} \right] W_{s} E_{m}$$

$$= E_{p}^{T} H_{rs}(0) \left[S_{n} \Sigma_{n}^{-1} R_{n}^{T} \right] V_{r} A^{k} W_{s} \left[S_{n} \Sigma_{n}^{-1} R_{n}^{T} \right] H_{rs}(0) E_{m}$$

$$= E_{p}^{T} H_{rs}(0) S_{n} \Sigma_{n}^{-1/2} \left[\Sigma_{n}^{-1/2} R_{n}^{T} H(1) S_{n} \Sigma_{n}^{-1/2} \right]^{k} \Sigma_{n}^{-1/2} R_{n}^{T} H(0) E_{m}$$

$$= E_{p}^{T} R_{n} \Sigma_{n}^{1/2} \left[\Sigma_{n}^{-1/2} R_{n}^{T} H_{rs}(1) S_{n} \Sigma_{n}^{-1/2} \right]^{k} \Sigma_{n}^{1/2} S_{n}^{T} E_{m}$$
(13)

This is the basic formulation of realization for the ERA. The triplet below is a minimum realization

$$A = \sum_{n}^{-1/2} R_{n}^{T} H(1) S_{n} \sum_{n}^{-1/2} B = \sum_{n}^{1/2} S_{n}^{T} E_{m}$$

$$C = E_{p}^{T} R_{n} \sum_{n}^{1/2} C_{n}^{T} E_{n}$$
(14)

The realized discrete-time model represented by matrices A, B, C and D can be transformed to modal coordinates using the eigenvalues Λ and eigenvector matrix Ψ of A

$$A' = \Psi^{-1} A \Psi = \Lambda \quad (diagonal)$$

$$B' = \Psi^{-1} B$$

$$C' = C \Psi$$
(15)

The modal damping rates σ_i and damped natural frequencies ω_i are the real and imaginary parts of the eigenvalues after transformation back to the continuous domain

$$s_i = \sigma_i \pm j\omega_i = \ln(\Lambda_i)/\Delta t \tag{16}$$

Modal participation factors and mode shapes are the corresponding rows of B' and columns of C', respectively. In practice, some modal parameters obtained using this approach are inaccurate due to high modal density, nonlinearity, etc. CMI is used to assess the relative accuracy of the various results (Pappa *et al.*, 1993).

2.1 Consistent-Mode Indicator (CMI)

This section shows the fundamental ideas of the Consistent-Mode Indicator (CMI) as proposed in Pappa *et al.* (1993) and Tsunaki (1999) for assessing the consistency of structural modal parameter identified with the (ERA).

CMI is computed for mode *i* as the product of two other parameter, EMAC and modal phase collinearity (MPC)

$$CMI_i = EMAC * MPC_i \tag{17}$$

EMAC quantifies the temporal consistency of the identification results. MPC quantifies the spatial consistency of the identification results. Practical experience has shown that both conditions must be satisfied simultaneously to ensure accurate results. These two parameters are described separately in the following sections.

2.1.1 Extended Modal Amplitude Coherence

EMAC is computed using the identified modal parameters. Mode shape and modal participation component for data at *t*=0 are compared with corresponding components at *t*=*T*₀ (for outputs) and *t*=*T*₁ (for inputs) located in final block row and final block column, respectively, of the modal observability matrix $G' = R_n \sum_{n=1}^{1/2} \Psi$ and modal controllability matrix $F' = \Psi^{-1} \sum_{n=1}^{1/2} S_n^T$. Data in the corresponding final block row and final block column of the Hankel matrices

are shifted by 10 times samples (by default) from the previous block row and column, providing an extension of the primary data analysis window. An EMAC value is computed for each of the p inputs (initial conditions) and q outputs (response measurements), for every mode.

Let $(\phi_{ij})_0$ be the identified mode shape component for mode *i* and response measurement *j* at *t*=0 and $(\phi_{ij})_{T_0}$ be the corresponding identified component at *t*=*T*₀. The identified eigenvalue for mode *i* is *s*_i. Compute a predicted value of $(\phi_{ij})_{T_0}$ as follows:

$$\left(\widetilde{\phi}_{ij}\right)_{T_0} = \left(\phi_{ij}\right)_0 e^{siT_0}$$
(18)

Temporal consistency is quantified by comparing $(\phi_{ij})_{T_0}$ and $(\tilde{\phi}_{ij})_{T_0}$. The actual and predicted magnitudes are compared using the ratio of the magnitudes

$$R_{ij} = \frac{\left| \left(\phi_{ij} \right)_{T_0} \right|}{\left| \left(\tilde{\phi}_{ij} \right)_{T_0} \right|} \quad \text{for} \quad \left| \left(\phi_{ij} \right)_{T_0} \right| \leq \left| \left(\tilde{\phi}_{ij} \right)_{T_0} \right|$$

$$= \left| \left(\tilde{\phi}_{ij} \right)_{T_0} \right| / \left| \left(\phi_{ij} \right)_{T_0} \right| \quad \text{otherwise.}$$

$$(19)$$

The actual and predicted phase angles are also compared. Letting $P_{ij} = ARG\left[\left(\phi_{ij}\right)_{T_0} / \left(\tilde{\phi}_{ij}\right)_{T_0}\right], \quad \pi \leq P_{ij} \leq \pi$, a weighting factor is determined as follows:

$$W_{ij} = 1.0 - \left(\frac{|P_{ij}|}{(\pi/4)}\right) \quad \text{for} \quad |P_{ij}| \le \pi/4$$

= 0 otherwise. (20)

An output EMAC for mode i and response measurement j is then computed as

$$EMAC_{ij}^{0} = R_{ij} W_{ij}$$
⁽²¹⁾

An input EMAC for mode *i* and initial condition *k*, $EMAC_{ik}^{I}$, is similarly computed using the identified modal participation factors.

Using these results, an EMAC value is associated with every $j - k^{th}$ input-output pair as follows:

$$EMAC_{ijk} = EMAC_{ij}^{0} EMAC_{ik}^{I}$$
⁽²²⁾

Finally, to condense all EMAC results for mode i into a single value, a weighted average of the individual results is computed

$$EMAC_{i} = \frac{\sum_{j=1}^{q} \sum_{k=1}^{p} EMAC_{ijk} |\phi_{ij}|^{2} |\phi_{ik}|^{2}}{\sum_{j=1}^{q} \sum_{k=1}^{p} |\phi_{ij}|^{2} |\phi_{ik}|^{2}} = \frac{\left(\sum_{j=1}^{q} EMAC_{ij}^{0} |\phi_{ij}|^{2}\right) \left(\sum_{k=1}^{p} EMAC_{ik}^{I} |\phi_{ik}|^{2}\right)}{\sum_{j=1}^{q} |\phi_{ij}|^{2} \sum_{k=1}^{p} |\phi_{ik}|^{2}}$$
(23)

where ϕ_{ij} and ϕ_{ik} are mode shape and modal participation components, respectively. The numeric values that EMAC can assume vary between 0 and 1. In general, is considered that a value above 0.9 means good temporal consistency in the mode shape identified and a value below 0.1 means low consistency.

2.1.2 Modal Phase Collinearity

MPC quantifies the spatial consistency of the identification results. For classical normal modes, all locations on the structure vibrate exactly in-phase or out-of-phase with one another, i.e., the corresponding mode shape is a real or "monophase" vector.

With monophase behaviour, the variance-covariance matrix of the real and imaginary parts of the mode shape vectors has only one nonzero eigenvalue. If the identified mode shape phase angles are uncorrelated, on the other hand, the two eigenvalues of this matrix will be approximately equal. MPC quantifies the degree of monophase behaviour by comparing the relative size of the eigenvalues of the variance-covariance matrix.

Let Φ'_i and Φ''_i be the real and imaginary parts, respectively, of the identified mode shape for mode *i*. Calculate the variance and covariance of the real and imaginary parts

$$S_{xx} = \Phi_i^T \Phi_i'$$

$$S_{yy} = \Phi_i^T \Phi_i''$$

$$S_{xy} = \Phi_i^T \Phi_i''$$
(24)

letting

$$\eta = \frac{S_{yy} - S_{xx}}{2S_{xy}} \tag{25}$$

the eigenvalues of the variance-covariance matrix are

$$\lambda_{1,2} = \frac{S_{xx} + S_{yy}}{2} \pm S_{xy} \sqrt{\eta^2 + 1}$$
(26)

MPC for mode *i* is then defined as follows:

$$MPC_{i} = \left(\frac{\lambda_{1} - \lambda_{2}}{\lambda_{1} + \lambda_{2}}\right)^{2}$$
(27)

 MPC_i values range from 0 for a mode with completely uncorrelated phase angles to 1 for a monophase result.

3. IDENTIFICATION APPROACH APPLIED TO AN AERONAUTICAL STRUCTURE

In this section are presented the necessary steps in a GVT, structure under testing, equipment setup, data acquisition, and frequency response analysis.

3.1. Structure Under Testing

Figure (1) shows the Neiva Regente aircraft. The airplane was manufactured in the 60's by the Brazilian Aeronautical Industry Neiva Ltda., Botucatu, SP. The construction is almost totally metallic comprising a semi-monocoque fuselage and semi-wings with simple mounts. Only a semi-wing (of conventional construction) has been taken to the experiments. The wing has been suspended through cables and springs in order to achieve a free-in-space boundary condition. The system to be identified and testing set-up, is shown in Fig. (2).



Figure 1. Neiva Regente aircraft (dimensions in meters).



Figure 2. Test environment and apparatus.

3.2. Experimental Set-up and Analysis

The experimental set-up used to the wing modal characterization is presented in this section. The GVT has been based on measured FRFs assuming a single excitation point and one acceleration measurement point (single-input/single-output approach).

Figure (3) shows the excitation and measure devices used in the modal testing. The structure under test has been driven by an electrodynamic shaker attached to the suspended wing by a flexible stinger. Although have been used other excitation and windows forms, the FRFs measured have been obtained using chirp excitation signal, Hanning windows in the 0-500 Hz frequency range. The FRFs have been calculated between the applied forces and the accelerometer. For each excitation position a FRF has been measured for each point scattered along the wing spars external surface. Figure (4) illustrates all the 30 measurement points used. The two excitations points used are: front and rear spar mountings. As shown in the Fig. (4), the experimental tests used to the wing modal characterization do not present driving point. The input and output signals have been gathered by standard piezoelectric sensors. The force and acceleration measurements are in the perpendicular direction to the wing surface (y), which emphasizes the primary out-of-plane

bending and torsional modes of the structure (Pappa *et al.*, 1997). The electrodynamic exciter used has been a B&K 4812 associated with a B&K 2707 power amplifier. The signals have been measured by a four channel 2630 Tektronix Spectrum Analyser. Input and output signals have been measured respectively by a Kistler 912 (13,3 *pC/N*) force transducer and a B&K 4383 (30,5 *pC/g*) accelerometer. The Fig. (5) shows all data used simultaneously in the analysis, i.e., the Average Power Spectrum (APS) of all 60 measured FRFs. It provides an overview of the natural frequencies of the structure and their relative strengths in the measurements. It indicates that there are two predominant modes of the structure from 0 to 100 Hz.



(a) Excitation point (shaker)

- Figure 3. Excitation and measurement devices.
- (b) Measure point (accelerometer)



Figure 4. Test set-up wing excitation/response locations.



Figure 5. Average Power Spectrum (APS) of 60 measured FRFs.

4. RESULTS AND DISCUSSION

This section presents the results of the identification process of the modal parameters obtained through the application of the Eigensystem Realization Algorithm (ERA). The supplied results of the ERA has been submitted the a qualification process through of CMI and it has been presented the identified modes whose CMI has been superior to 60%. In modal-survey tests, identification difficulties arise primarily from high modal density, nonlinearity, weakly excited modes, local modes, nonstationarities, rattling, etc., not from instrumentation noise. The simultaneous effects of these conditions are in general impossible to include explicitly in reliable calculations. It has been observed a large amount of noise in the measured FRFs. This feature has been assigned by the influence of wing structure construction method (riveted plates and stringers). This problem is common in aeronautical structures and some time must be expended in searching for unreliably fixations. Moreover, the excitation position has facilitated the identification of particular modes, specially the one between 0 and 10 Hz, probably a rigid body mode. Lower frequency values related to rigid body have been totally neglected.

Natural frequencies, damping factors and accuracy indicator are documented in Tab. (1) and modes shapes are presented in Figs. (6) and (7).

ERA			
Mode	$oldsymbol{\omega}_{nd}$ [Hz]	ξ _n %	CMI %
$\frac{1}{2}$	45.6 58.2	0.72	72.0

Table 1. Modal Parameters and Accuracy Indicator



Figure 6. First bending mode identified by the ERA – 45.6 Hz.



Figure 7. First torsional mode identified by the ERA – 58.2 Hz.

5. CONCLUSIONS

The main scope of this work has been the modal parameter identification in a typical aeronautical structure by the Eigensystem Realization Algorithm (ERA). Firstly, a ground vibration testing (GVT) of a metallic wing of the Neiva Regente aircraft has been realized in order to assess the frequency response functions. The wing has been driven by an electrodynamic vibration exciter using chirp excitation signals and one accelerometer to collect the structural response at 30 points on the wing external surface. The FRFs have been measured on the wing span using standard piezoelectric sensors. Secondly, the algorithm identification (ERA) has been used to determine mode shapes from experimental data. Finally, the Consistent-Mode Indicator has been used for assessing the consistency of structural modal parameter identified with the ERA. A total of two mode shapes have been identified based on ERA. According to the results previously shown, the first bending and first torsional mode shapes have been identified.

The modal identification of an aircraft wing has proved to be surprisingly difficult. Due to nonlinearities and numerous local modes achieved in the ground vibration test, the results of identification using the ERA are not completely satisfactory.

For the future, it seems clear that most sophisticated methodologies must be used in order to overcome noisy measurements the undesired influence of localized modes to the whole structure dynamic characteristics. In this way, the use of new ground vibration test techniques and ERA variations, seems to be an adequate approach for further investigations to the wing structure used in this work.

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7. REFERENCES

- Abel, I., 1997, "Research and Applications in Aeroelasticity and Structural Dynamics at the NASA Langley Research Center", NASA TM-112852.
- Bisplinghoff, R.L., Ashley, H. and Halfman, R.L., 1955, "Aeroelasticity", Addison-Wesley, Massachusetts, USA.
- Cooper, J.E., 1997, "On-line version of the Eigensystem Realization Algorithm using Data Correlations", Journal of Guidance, Control, and Dynamics, Vol.20, No.1, January-February, pp. 137-142.
- Juang, J-N., 1994, "Applied System Identification", Prentice Hall PTR, New Jersey, USA.
- Juang, J-N., Pappa, R.S., 1985, "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction", Journal of Guidance, Control, and Dynamics, Vol. 8, No.5, pp. 620-627.
- Kehoe, M.W., Freudinger, L.C., 1993, "Aircraft Ground Vibration Testing at the NASA Dryden Flight Research Facility", NASA TM-104275.
- Pappa, R.S., Elliott, K.B., Schenk, A., 1993, "Consistent-Mode Indicator for the Eigensystem Realization Algorithm", Journal of Guidance, Control, and Dynamics, V.16, No.5, pp. 852-858.
- Pappa, R.S., James, G.H.III, Zimmerman, D.C., 1997, "Autonomous Modal Identification of the Space Shuttle Tail Rudder", NASA TM-112866.
- Sczibor, V., 2002, "Identificação Modal de uma estrutura aeronáutica via Algoritmo de Realização de Sistemas", Dissertação de Mestrado, School of Engineering of São Carlos, Universidade de São Paulo, Brazil, 147p.
- Tasker, F., Bosse, A., and Fisher, S., 1998, "Real-Time Modal Parameter Estimation using Subspace Methods: Theory", Mechanical System and Signal Processing, Vol.12, No.6, pp. 797-808.
- Tsunaki, R.H., 1999, "Identificação Automatizada de Modelos Dinâmicos no Espaço de Estado", Doctorate Thesis, School of Engineering of São Carlos, Universidade de São Paulo, Brazil, 143p.