LAMINAR NATURAL MIXED CONVECTION IN A TRAPEZOIDAL ENCLOSURE WITH INPUT AND OUTPUT OF AIR

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Abstract. *A numerical study is presented for mixed convection flow in trapezoidal enclosures heated from above with isothermal vertical walls, with the bottom cooled . Forced flow conditions are imposed by providing an inlet at the bottom of the isotherm left wall and a vent at the top of the right one. The interaction between the buoyancy and the forced flow is examined. Two-dimensional equations for mass, momentum and energy conservation, with the Boussinesq approximation are numerically solved using a finite volume method. The discretized equations were obtained through the Power-Law scheme and a fully implicit formulation. The adopted numerical procedure for pressure calculation is based on the SIMPLE algorithm. Governing parameters used were: Grashof number varies from* 10^3 *to* 10^7*, Prandtl number equal 0.72 and Reynolds number varying form zero to 2000. The inclination of the upper surface varying from zero to sixty degree. The streamlines, isotherms, and profiles of temperature and velocities components in x and y directions along the middle of vertical and horizontal line, and the normalized average Nusselt number at the base of the cavity, are presented in terms of some values of the Grashof and Reynolds numbers. The influence of these numbers in the heat transfer in the cavity is discussed. Nusselt number increases when Grashof number or Reynolds number increase.*

Keywords. *Mixed laminar, convection, , trapezoidal enclosure*

1. Introduction

The nature of buoyancy-induced flows or natural convection and heat transfer in enclosures has been studied and is well-understood. In addition, the enclosure geometry has been widely studied in heat transfer because of its fundamental importance and its many applications, including heat transfer in attic areas of domestic buildings and in some solar energy, electronic cooling systems and thermal environment control of dwellings. With an inlet and an air vent, forced convection conditions can be imposed inside an enclosure. The interaction between the buoyancy stemming from one heated element inside the enclosure and the imposed forced flow is the goal of the this investigation. The imposed forced flow may aid the natural convection and mixed convection or oppose it, depending on the location of the inlet and the direction of the forced flow. Conversely, the buoyancy may aid or oppose the forced flow, depending on whether the surface temperature is higher or lower than the temperature of the incoming forced fluid flow.

Torrance et al. (1972) numerically studied the fluid flow generated in a rectangular cavity by moving the upper wall, which is maintained at a temperature different from the remaining walls of the cavity. The combined effects of wall-shear and buoyancy drive the fluid motion. At higher Grashof numbers, the buoyancy effects dominate for all aspect ratios. Oberkampf and Crow (1976) numerically simulated the fluid dynamics and temperature fields in a reservoir. In the rectangular reservoir the flow was assumed to be two-dimensional in a vertical plane. Inflow was allowed at the surface in one end and outflow occurred at any depth of the opposite end. The inflow was set at a given temperature and velocity. The effect of inflow and outflow wind shear, and heat transfer on the reservoir were discussed. Sparrow and Samic (1982) studied numerically engineering applications related to mixed convection in enclosure, such as the oven of an electric stove, and natural convection recirculation in an attic. They numerically studied the fluid flow and heat transfer in a vertically oriented cylindrical enclosure with apertures for forced flow in the lower and upper horizontal circular walls. Natural convection flows are induced in the enclosure owing to the temperature difference between the entering stream and the enclosure walls. Cha and Jaluria (1984) numerically studied the effect of the buoyancy on the flow and thermal fields in shallow and rectangular reservoir with applications to solar ponds. Kumar and Yuan (1989) studied laminar, two-dimensional mixed convection flow in a rectangular enclosure with an inlet and outlet ports. Oosthuizon (1985) also studied the mixed convective heat transfer in a cavity. Papanicolaou and Jaluria (1990, 1992, 1993, 1995) studied various aspects of the mixed convection flow in adiabatic vented enclosure with isolated thermal sources flush with the inner walls. Pérez-Segarra et al. (1995) numerically studied that combines for some of these considerations. Mohamad (1995) studied the natural convection in open cavities and slots.

Hsu et al. (1997) studied numerically mixed convection in a partially divided rectangular enclosure.

Angirasa (2000) presented numerical investigation of a square enclosure with one of the vertical walls at a constant temperature. The horizontal walls and the others vertical wall are assumed to be adiabatic.The inlet for the forced flow is fixed at the bottom, and the temperature difference is considered to be either positive or negative. Khanafer et al. (2002) studied the mixed convection heat transfer in two-dimensional open-ended enclosures;

The present numerical investigation considers the mixed convection flow in trapezoidal enclosures heated from up with a uniform and constant temperature at the upper surface, with isothermal vertical walls and the bottom cooled. Forced flow conditions are imposed by providing an inlet in the isotherm left wall, H/10 long, close to the bottom, and a vent at the top in right one. The interaction between the buoyancy and the forced flow is examined. Two-dimensional equations for mass, momentum and energy conservation, with the Boussinesq approximation are numerically solved using a finite volume method. The discretized equations are obtained through the Power-Law scheme and a fully implicit formulation. The adopted numerical procedure for pressure calculation is based on the SIMPLE algorithm. Governing parameters used were: Grashof number varies from 10^3 to 10^7, Prandtl number equal 0.72 and Reynolds number varying form zero to 2000. The inclination of the upper surface can vary from zero to sixty degree.

2. Mathematical Modeling

The considered geometry is shown in Fig. (1). The gravity vector is normal to the base of the trapezoidal, L is the base length and the height of left side of the cavity; H is the right height, α is the inclination angle of the cavity top. H/10 is the length of the vents. In order to formulate the fluid convection in the cavity it was considered the following assumptions: the flow is two dimensioned, incompressible and laminar; the temperature gradients are moderate for which the Boussinesq approximation is valid; viscous dissipation and the work done by compression forces are negligible.

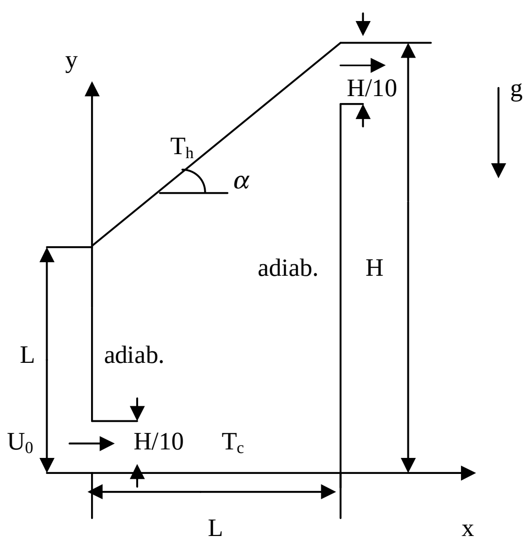

Figure 1. The vented enclosure and the coordinate system

The basic equations for the unsteady-state natural convection can be written in the dimensionless form as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{Gr}{\mathrm{Re}^2}\theta-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right) \tag{4}$$

For the construction of dimensionless quantities the following non-dimensional variables are defined, where the superscript ´ indicates the dimensional variables:

$$x=\frac{x'}{L} \qquad y=\frac{y'}{L} \qquad u=\frac{u'}{U_0} \qquad v=\frac{v'}{U_0} \qquad \theta=\frac{T-T_c}{T_h-T_c} \qquad t=\frac{U_0}{L}t' \tag{5}$$

$$\Pr=\frac{\nu}{\alpha} \qquad Gr=\frac{g\,\beta\,L^3\Delta T}{\nu^2} \qquad \mathrm{Re}=\frac{U_0 L}{\nu}$$

where Pr is the Prandtl number, Gr is the Grashof number, α and ρ_o are the fluid thermal difusivity and density respectively, K is fluid thermal conductivity.

The boundary conditions in this case are:

$$\begin{array}{llll}
\text{for } 0\le x\le 1, & y=1+\frac{H-L}{L}x, & u=v=0, & \theta=1 \\
\text{for } x=0, & 0\le y\le \frac{H}{10L}, & u=v=0, & \theta=0 \\
\text{for } x=0, & \frac{H}{10L}< y\le 1, & u=v=0, & \frac{\partial\theta}{\partial x}=0 \\
\text{for } 0\le x\le 1, & y=0, & u=v=0, & \theta=0 \\
\text{for } x=1, & 0\le y\le H/L, & u=v=0, & \frac{\partial\theta}{\partial x}=0
\end{array} \tag{6}$$

As it was not possible to determine the pressure, but only pressure differences, from equations (2) and (3), for the sake of definiteness, it will be assumed that the pressure is zero (or a reference value) at the point with the coordinates x = y = 0.

In the initial instant it was assumed that the fluid is motionless and the temperature field is uniform throughout the entire convection region

$$t=0, \qquad u=v=0, \qquad \theta=0 \tag{7}$$

The local Nusselt number at the base of the cavity is calculated using

$$Nu_L=\frac{hL}{k}=\left.\frac{\partial\theta}{\partial y}\right|_{y=0} \tag{8}$$

and the average Nusselt number can be obtained by integrating the local Nusselt number along the wall

$$\overline{Nu}=\frac{1}{L}\int_0^l Nu_L\,dl \tag{9}$$

The normalized Nusselt number is defined as

$$\mathrm{Nu}=\frac{\overline{\mathrm{Nu}}\,\big|_{\mathrm{Gr}}}{\overline{\mathrm{Nu}}\,\big|_{\mathrm{Gr}=0}} \tag{10}$$

where $\overline{\mathrm{Nu}}\,\big|_{\mathrm{Gr}=0}$ is for pure conduction for the same conditions.

3. Numerical Procedure

In order to perform the discretization of the governing equations, the power law scheme introduced by Patankar (1980) is adopted. As in all control volume methods, the discretized equations are obtained by the integration of the equation over control volumes surrounding each grid point.

The solution of the momentum equations requires a procedure to calculate the pressure fields. Since there is no specific equation for the pressure, the SIMPLE algorithm developed by Patankar and Spalding (1972), which uses the mass conservation equation to obtain an equation in terms of the pressure in the grid points, is considered. To solve the set of algebraic equation an iterative line-by-line process is used. This process is a convenient combination of a direct method TDMA for one-dimensional situations and the Gauss-Siedel method. The construction of the discretized equations was done considering a fully implicit method.

As shown by Patankar (1980), to avoid non-realistic solutions, independent grids for the variables u, v and p are required. The use of staggered grids was adopted here. This process facilitates the application of the boundary conditions. Other advantage is that the u and v velocity components coincide with the faces of the control volume for pressure and temperature. On the other hand impose the necessity of using interpolation schemes for the u and v grids.

It is very difficult to guarantee the convergence of a non-linear system of equations. To achieve the convergence, under-relaxation factors are applied in the solution procedure to avoid large corrections in one step of iteration, which may cause the divergence of the process.

The convergence must be verified in each iteration following a predetermined criterion. In this work we selected the criterion, which considers the average error for each control volume, as

$$\frac{\left\| r_p \right\|^k}{n_p} = \varepsilon \tag{11}$$

where $\left\| r_p \right\|^k$ is the Euclidian norm of the iteration k, and n_p is the number of grid points. The convergence parameter ε could be different for each variable depending on the order of magnitude of the variable in study. Here we set the convergence parameter as 10^{-5} to all variables.

4. Results and Discussion

In order to obtain grid independent results in this work, grids with 2964, 4704 and 6844 uniform volumes were tested. The differences values of quantities between the 4704 and 6844 internal volume grids were minimal.

Figures 2, 3 and 4 show developed streamlines (right side) and isotherms (left side) in the semi-open trapezoidal cavity for steady state regime, for $\alpha = 45^{\circ}$, respectively, for Gr=10^5, 10^6 and 10^7, for some valeus of Reynolds numbers.

It could be noticed for Gr=10^5, (Fig. 2) when the Reynolds number increase up to value 250 temperature decreases in the lower part of the cavity. For Re values greater than 250 temperatures increases at the center part of the cavity. The same effect can be noticed in the Fig. 5, witch shows the adimensional temperature along the middle vertical line of the cavity for some values of the Reynolds number. When Gr=10^6 (Fig 3) it can be notice that temperature get decreasing in the center of the cavity to Re value up to 500. When Re overcome this value the temperature increases at the center. When Gr=10^7 (Fig 4) temperature get decreasing in the center of the cavity to Re value up to 1500; the same can be seen in the Fig. 6. So, we conclude that as much as Gr and Re numbers increase, so much the temperature decreases in the center of the cavity.

When Re=0, case of pure natural convection, there are two circulations cells of the fluid in the cavity for Gr=10^5 and 10^6 (Fig. 2b and 3b), and there are four circulations cells for Gr=10^7 (Fig. 4b). As the increasing of Re number flows get a completely different form and new circulations cells arises in the cavity, moving to its center (Fig. 2, 3, 4 - d, f, h). As the increasing of Reynolds number the air flow in the cavity get more concentrate between the inlet and outlet of the cavity; a main circulation of the fluid arises at the center of the cavity and a small one in the lower right corner (Fig. 2, 3, 4 – f, h).

Figures 7 presents the vertical components of the velocities along the middle horizontal line of the cavity for some values of the Reynolds number, for Gr=10^6, and shows that the velocities increase at the right side of the cavity where the height is higher.

Figure 8 presents the horizontal components of the velocities along the middle vertical line for some values of the Reynolds number, for Gr=10^6, and we can notice the increasing of the velocity near the base of the cavity toward the imposed flow direction.

Figures 9 and 10 present the average normalized Nusselt number at the base of the cavity as a function of Reynolds number, respectively, for $\alpha = 30^{\circ}$ and 45°. Nusselt number increases when the Grashof number also increases, and its value is biggest nearly by Re=300, for $\alpha = 30^{\circ}$ for all values of Gr. When $\alpha = 45^{\circ}$ Nusselt number gets the biggest value for Re varying from 100 to 300 approximately. This is the best situation to the heat transfer in the cavity. After getting its biggest value Nu tends to stabilize.

Figure 11 and 12 presents also the average normalized Nusselt number as a function of Reynolds number for some different inclinations of the cavity top, respectively, for Gr=10^5 and 10^7. We can notice that Nusselt number increases rapidly for inclination zero, being this growing less accentuated when Gr=10^7. For the inclination top from zero to 30^0, we can notice that the values of Nusselt number decreases for Reynolds number greater than 300, for Gr=10^5, and the biggest value of Nusselt increases when the inclination top growths from 30^0 to 60^0 .

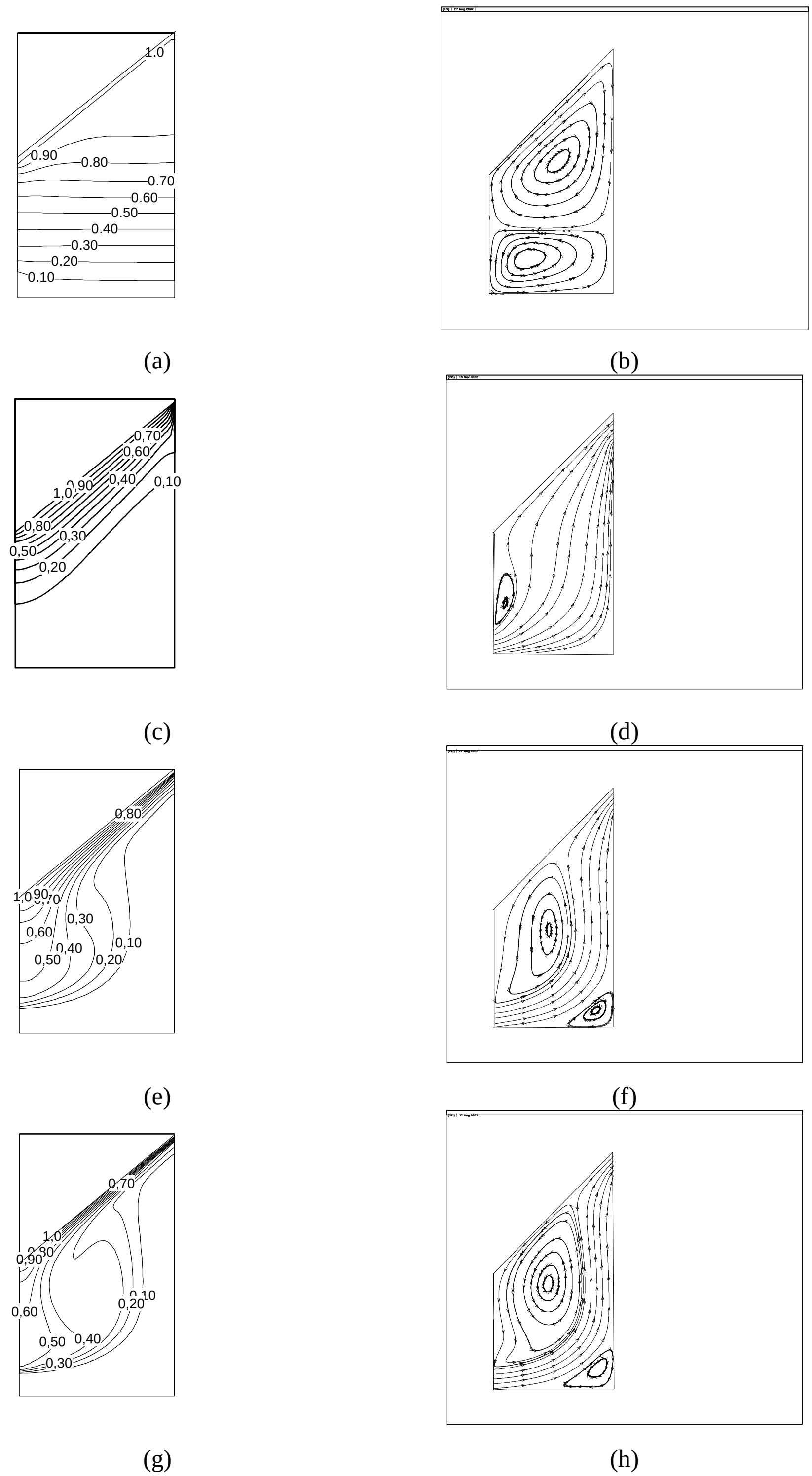

Figure 2 – Isotherms (left) and stream lines (right) for Gr=10^5 and $\alpha = 45°$ for Re=0 (a, b); Re=100 (c,d); Re=500 (e,f); Re=1500 (g,h).

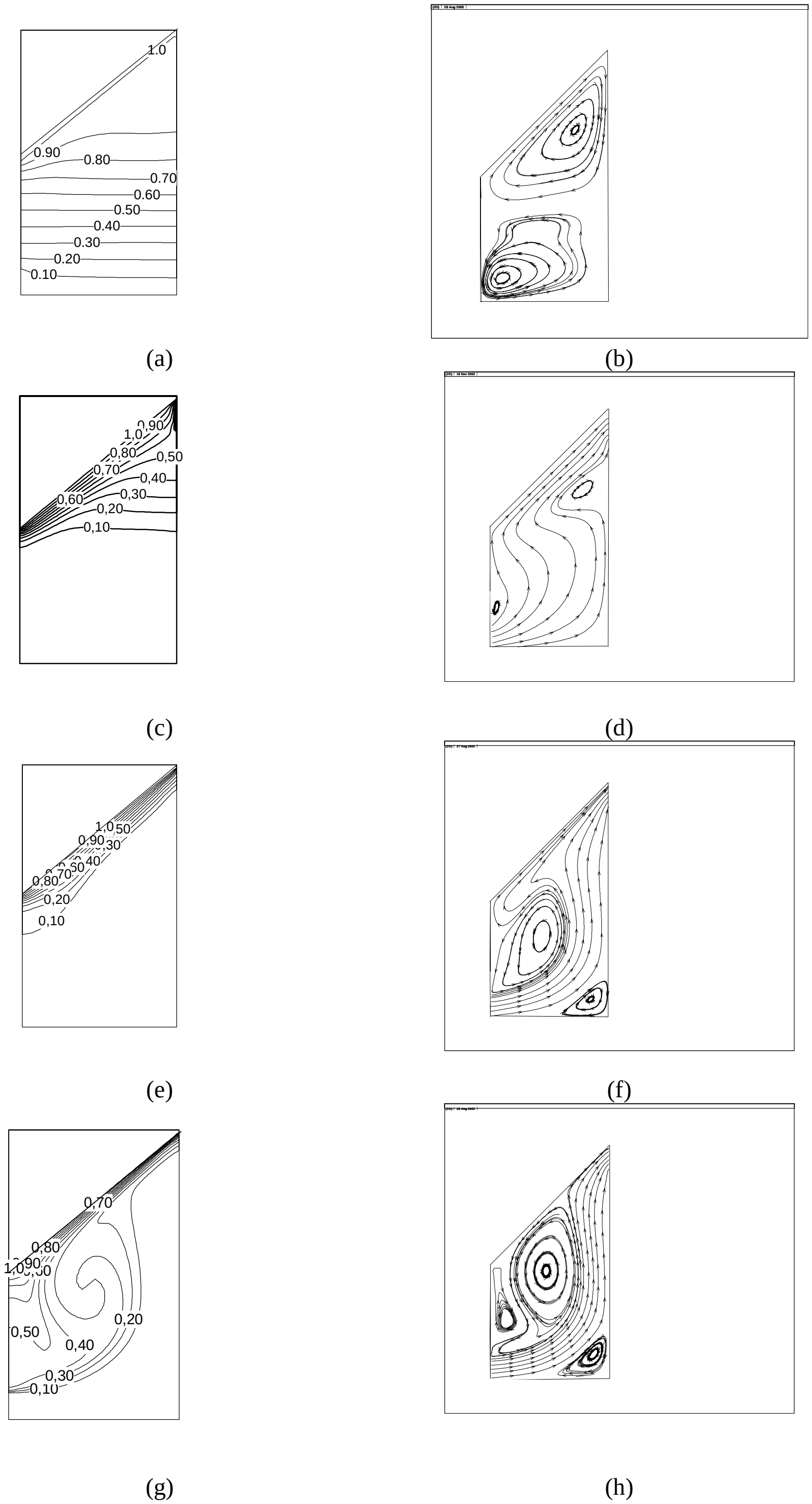

Figure 3 – Isotherms (left) and stream lines (right) for Gr=10^6 and $\alpha = 45°$ for Re=0 (a, b); Re=100 (c,d); Re=500 (e,f); Re=1500 (g,h).

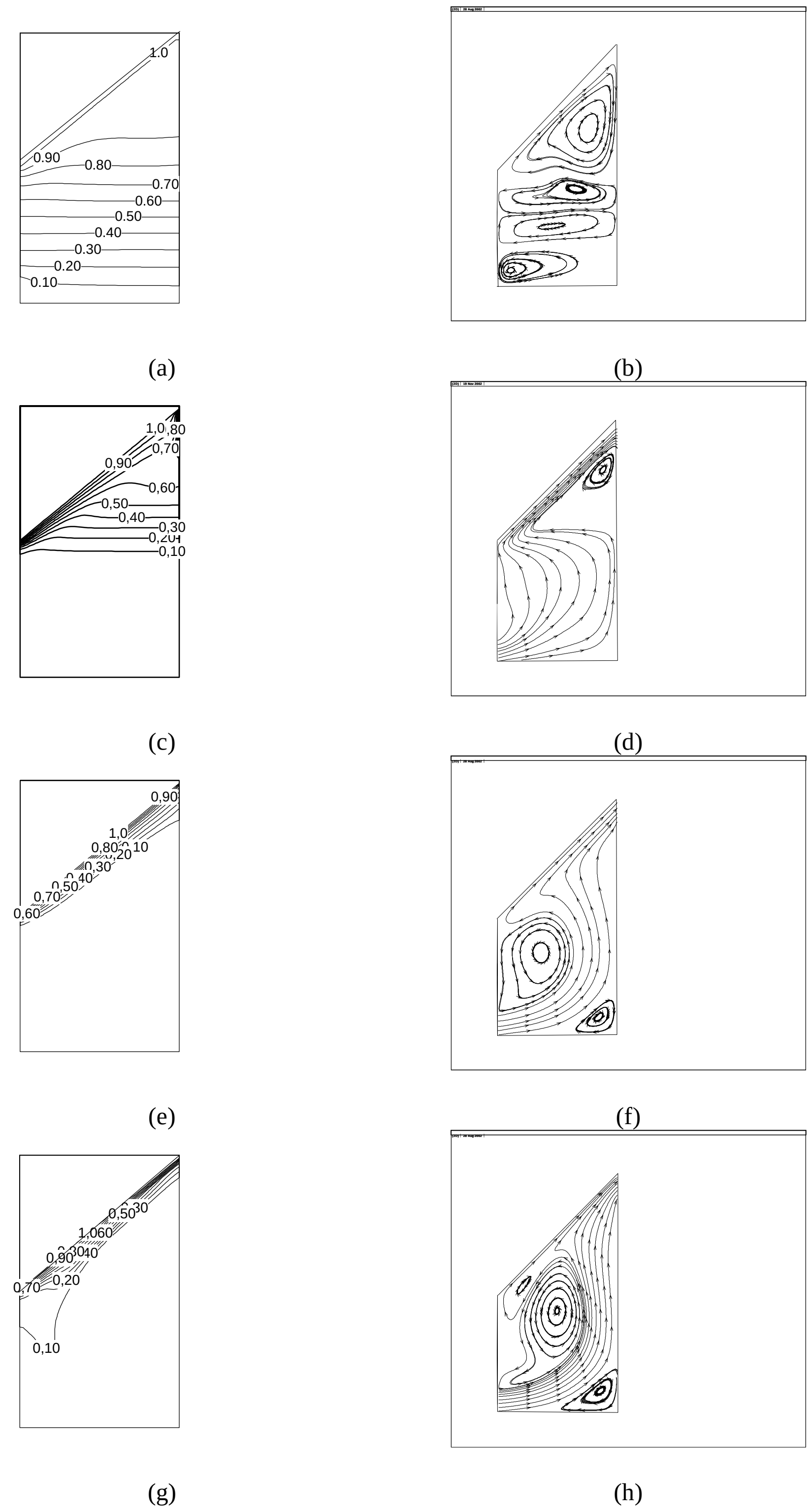

Figure 4 – Isotherms (left) and stream lines (right) for $Gr=10^7$ and $\alpha = 45°$ for Re=0 (a, b); Re=100 (c,d); Re=500 (e,f); Re=1500 (g,h).

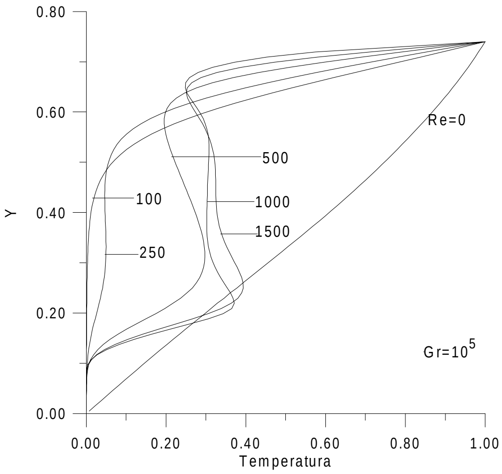

Figure 5. Adimensional temperature along the middle vertical line for some values of the Reynolds number for Gr=10^5.

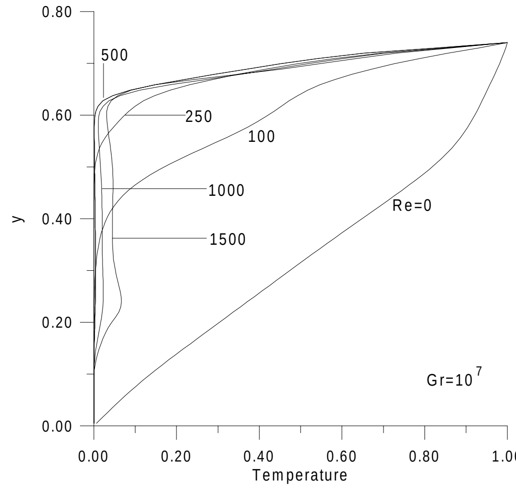

Figure 6. Adimensional temperature along the middle vertical line for some values of the Reynolds number for Gr=10^7.

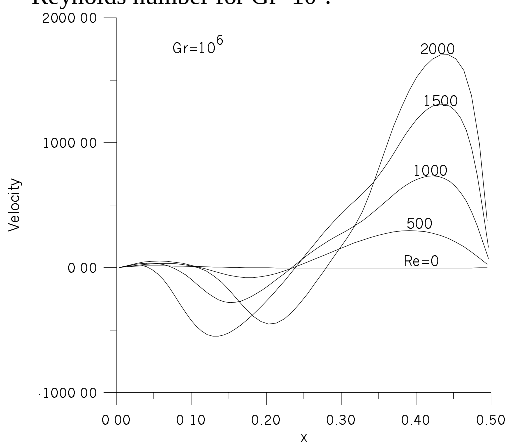

Figure 7. Vertical components of the velocity along the middle horizontal line for values of the Reynolds number.

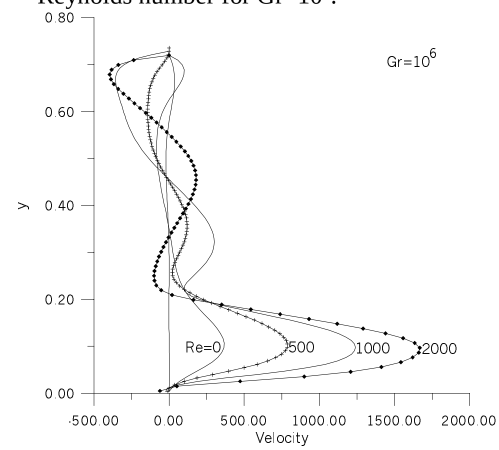

Figure 8. Horizontal components of the velocity along the middle vertical line for values of the Reynolds number.

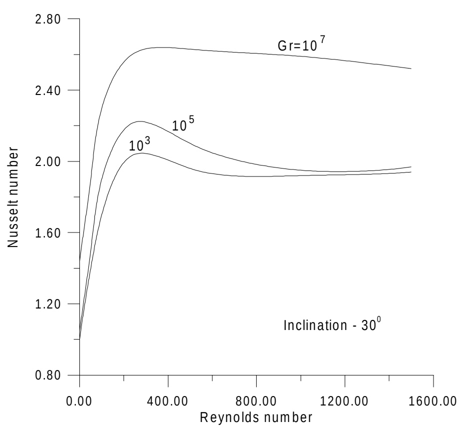

Figure 9. Average normalized Nusselt number in function of Reynolds number for some Gr numbers and $\alpha = 30^0$.

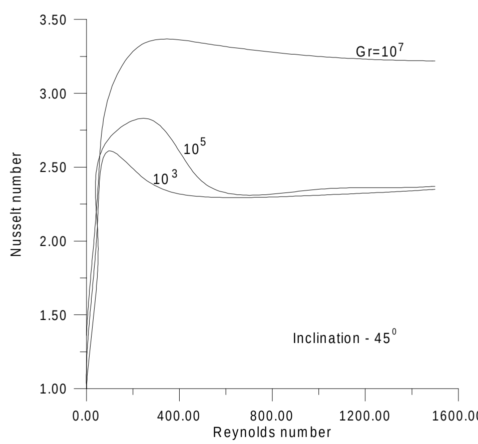

Figure 10. Average normalized Nusselt number in function of Reynolds number for some Gr numbers and $\alpha = 45^0$.

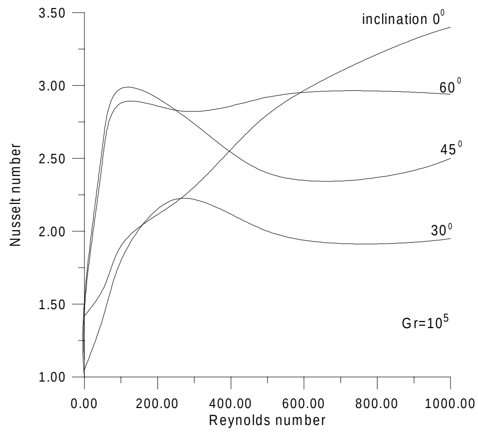

Figure 11. Average normalized Nusselt number in function of Reynolds number for various inclinations and $Gr=10^5$.

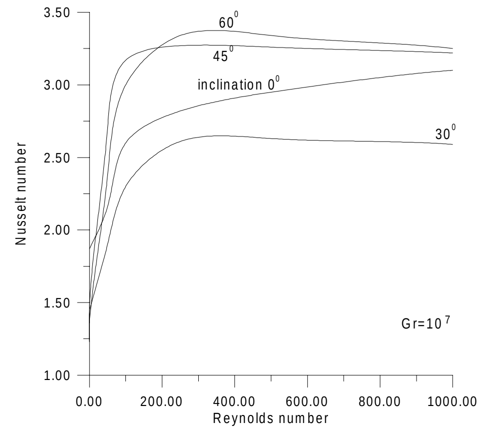

Figure 12. Average normalized Nusselt number in function of Reynolds number for various inclinations , for $Gr=10^7$.

5. Conclusion

This work presents numerical investigation of mixed convection flow in trapezoidal enclosures heated from up with a uniform and constant temperature at the upper surface, with isothermal vertical walls and the bottom cooled. Forced flow conditions are imposed by providing an inlet in the isotherm left wall, close to the bottom, and a vent at the top in right one. Governing parameters used were: Grashof number varies from 10^3 to 10^7, Prandtl number equal 0.72 and Reynolds number varying form zero to 2000. The inclination of the upper surface can vary from zero to sixty degrees.

Based on the presented results we can note that as much as Gr and Re numbers increase, so much the temperature decreases in the bottom and the center of the cavity.

Nusselt number increases when the Grashof number also increases, and its biggest value occurs for Reynolds number varying from 100 up to 300, depending on the cavity top inclination and the Gr value. In this range we have the best situation to the heat transfer in the cavity. After getting its biggest value Nu tends to stabilize.

Nusselt number increases rapidly for inclination zero, being this growing less accentuated when Gr increases. For the inclination top varying from zero to 30^0, Nusselt number decreases for Reynolds number greater than approximately 300, and the biggest value of Nusselt increases when the inclination top growths from 30^0 to 60^0 .

6. Acknowledgments

This work was developed during the pos-doctor stage of Aquino F.R.Q., in the Energy Department of the Mechanical Engineering College of the São Paulo State University “Júlio de Mesquita Filho” – UNESP - Guaratinguetá. The authors wist to thank the UNESP – Guaratinguetá.

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