A NEW APPROACH TO CALCULATE THE TRANSMISSION ERROR OF HELICAL GEAR PAIRS WITH MODIFIED TOOTH SURFACE

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Abstract. *This paper describes a procedure to calculate static transmission error of helical gears under load, eliminating the assumption that the tooth contact occurs on the plane of action. Lead and profile modifications, manufacturing errors and tooth deflections under load are considered in the procedure. The method of influence coefficients is employed to calculate the tooth deflections and a method to determine the load distribution on gear meshing is developed. The procedure is implemented through a computer code and some numerical examples are verified. The results are analyzed and compared to the results of other reliable computer program called LDP (developed by The Ohio State University). The differences between the results are discussed and their causes are investigated.*

Keywords*. Transmission error, tooth contact, gear, gear noise, gearbox.*

1. Introduction

Gear noise has been a concern to gear designers for many years. Noise regulations, customer requirements and competitive pressures have increased the attention to sound quality in geared systems. The requirement of designing quiet gear pairs becomes even more crucial as other components of mechanical equipments are becoming less noisy.

The gear noise is characterized by frequencies often close to pure tone, which are often near human ear´s most sensitive region and with a high sound pressure level causing uncomfortable situations (Houser, 1991).

The transmission error is widely recognized in the gear community as being the main source of gear noise and vibration (Beacham et al, 1999; Houser, 1991; Munro, 1990 and Smith, 1987). The transmission error was defined by Welbourn (1979) as "the difference between the actual position of the output gear and the position it would occupy if the gear drive were perfectly conjugate". The transmission error occurs due to deviations from the theoretical contact of conjugate teeth. These deviations come from topological modifications of the tooth surfaces, manufacturing errors, shaft misalignments. Also, when gear pairs are under load an additional component of transmission error arises due to deflections of the gear teeth and their supporting structure (Houser, 1991).

The transmission error can be mathematically expressed by Eq. (1) and (2):

$$\delta\theta_2 = \theta_2 - \frac{N_1}{N_2}\theta_1 \quad , \tag{1}$$

$$\delta t_2 = r_{g2}(\theta_2 - \frac{N_1}{N_2}\theta_1) \quad , \tag{2}$$

where, $\delta\theta_2$ is the angular transmission error in radian, δt_2 is the longitudinal transmission error in mm, N_1 is the number of teeth of the driving gear, N_2 is the number of teeth of the driven gear, θ_1 is the angular position of the driving gear, θ_2 is the angular position of the driven gear and rg_2 is the base radius of the driven gear.

The transmission error can be considered as a continuous periodic function with number of periods per revolution equal to the number of teeth of the driven gear and length of each period equal to a base pitch of the gears. Since the transmission error is periodic at mesh frequency, some studies have presented the harmonics of the transmission error instead of linear deviations as an analysis tool (Houser et al, 1996 e Smith, 1987).

The method of influence coefficients has been extensively applied to analytical modeling of tooth deflections (Beacham et al, 1999). It generates a stiffness matrix with direct and cross compliance to points on the loaded and adjacent teeth (Conry & Seireg, 1973). The results obtained with the method of influence coefficients have shown to achieve good correlation to measured transmission error in an automotive transmissions (Beacham et al, 1999). The method of coefficients of influence provides a useful development tool because topological modifications of the teeth can be easily handled.

The conventional transmission error analysis usually considers assumptions or approximations that each tooth pair is expected to contact on the plane of action. In this case the contact deviations from the plane of action are neglected. In the case of perfect gears under no load, this assumption is correct. However, there are some studies that show the occurrence of the contact out of the plane of action for some real cases (Kurokawa et al, 1996).

During a meshing cycle, it is expected that more than one pair of teeth is in contact. An evaluation of load distribution between the tooth pairs must be taken in order to calculate tooth deflections. The formulation of the solution of the generalized elastic contact problem can be applied to the solution of load distribution in gears. In some works all elastic deflections and forces are assumed to be acting along the line of action. For the gear teeth to be in contact at any point, the sum of the elastic deflections and initial gaps is stated to be equal to the rigid body displacement, and applying an optimization algorithm the load distribution is calculated (Conry & Seireg, 1973). This approach neglects that as the deflections occur on loaded conjugate teeth, gears rotate and the contact points change from its initial position.

The present paper describes a method to calculate transmission error of helical gear pairs under load. The developed procedure does not consider the usual assumption that the tooth contact will occur on the plane of action, considering that the tooth contact analysis is performed on the whole tooth surface instead. A gradual applied load method is proposed in order to obtain a better representation of the physical contact phenomenon in gears.

2. Gear model

Kurokawa et al (1996) proposed a method to analyze the tooth contact out of the plane of action and to calculate the transmission error of spur gears. In such a method, the gear pair was analyzed as a two dimensional component. The topological modifications and the manufacturing errors were averaged to one value, and consequently, only one profile error was considered. Helical gear modeling is naturally more complex because a tree dimensional analysis is required to represent the tooth geometry.

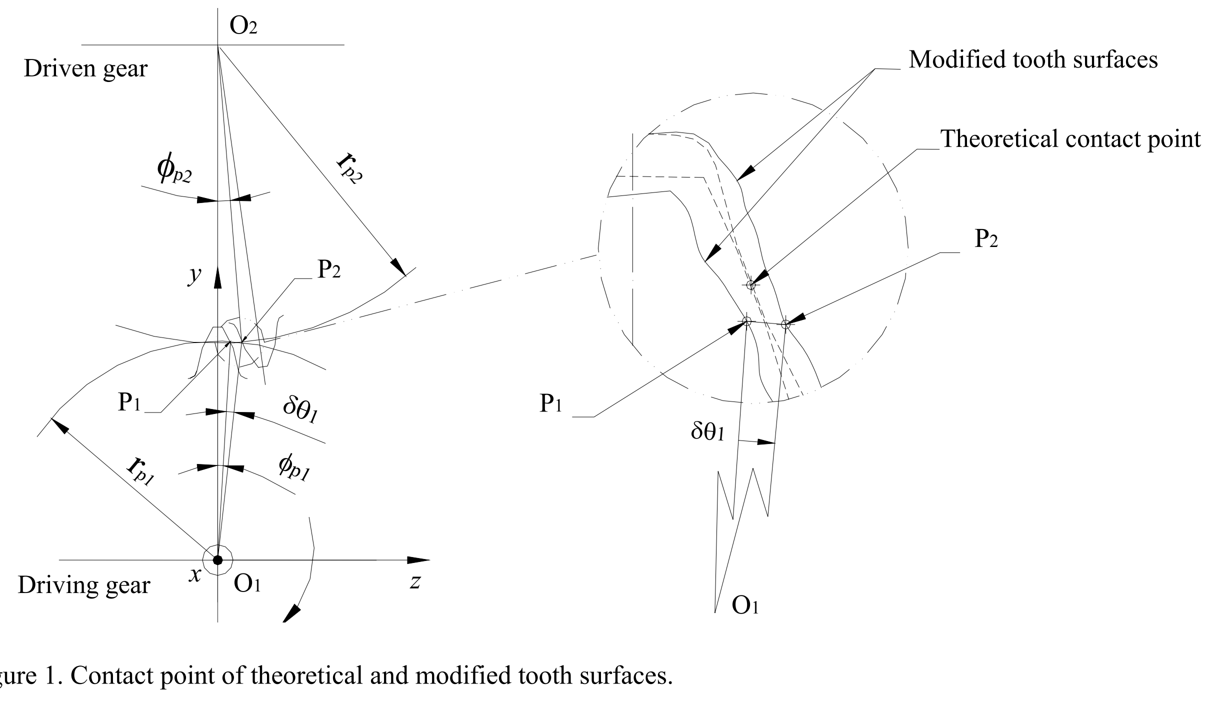

Figure 1. Contact point of theoretical and modified tooth surfaces.

In this work it is assumed that the gear pair surfaces can be represented by a number of discrete points. The driven gear tooth surface is divided into some grid points and their coordinates are appropriately calculated. In order to represent the gear meshing kinematics, it is stated that each candidate point to contact on the driving gear is located on a radius from the driving gear origin (O_1) to the discrete point on driven tooth surface, as shown in Fig. (1). The angular displacement, $\delta\theta_1$, is calculated for each grid point from driven gear to driving gear. The minimum angular displacement, $\delta\theta_{1min}$, corresponds to the angular transmission error measured on driving gear.

In gear pairs, the contact points change as a function of the gear rotational motion. It is assumed in this work that a number of discrete points along the gear meshing cycle can represent the gear pair motion. The angular displacement, $\delta\theta_{1min}$, can be found for each position of the meshing cycle.

2.1. The tooth contact analysis model

In the gear pair model shown in Fig. (1), the driven gear is located above of the driving gear. The center distance is defined over the axis y. Axis x is coincident to the rotational center of the driving gear.

In this model it is considered that a discrete point P_1 of the driving gear belongs to a radius r_{p1} that passes on point P_2 of driven gear ($\overline{O_1P_1} = \overline{O_1P_2}$). It means that if any point on tooth surface of driven gear is known, a respective candidate point to contact on tooth surface of driving gear can be calculated. The radius r_{p1} related to point P_1 can be calculated by Eq. (3):

$$r_{p1} = \sqrt{(ab - r_{p2} \cdot cos\phi_{p2})^2 + (r_{p2} \cdot sen\phi_{p2})^2} \quad , \tag{3}$$

where, ab is the mounting center distance, r_{p2} is the radius related to point P_2, and

$$\phi_{p2} = \theta_1 \cdot \frac{N_1}{N_2} \pm (\varphi_{\beta p2} + \Delta\beta_2) + \frac{\pi}{N_2} - (\varphi_{p2} + \Delta\alpha_2) \quad . \tag{4}$$

In Eq. (4), θ_1 is the angular position on the meshing cycle taken from the axis y to the tooth center line of driving gear, N_1 and N_2 are the number of teeth of driving and driven gears respectively, the positive signal (+) or negative signal (-) corresponds to left hand or right hand of helix angle of the driven gear respectively, $\Delta\beta_2$ is an angular value correspondent to the sum of lead modification and lead error at point P_2 of driven gear tooth, $\Delta\alpha_2$ is an angular value correspondent to the sum of profile modification and profile error at point P_2 of driven gear tooth and:

$$\varphi_{\beta pj} = \frac{x_{pj} \cdot tan\,\beta_{pj}}{r_{pj}} \quad , \tag{5}$$

$$\varphi_{pj} = \frac{s_{gscj}}{2 \cdot r_{gj}} - inv\alpha_{psj} \quad , \tag{6}$$

where, the index j is 1 to the driving gear and 2 to the driven gear, x_{pj} is the distance measured on axis x from the origin plane (yz) to the discrete point P_j, β_{pj} is the helix angle at radius r_{pj}, s_{gscj} is the transverse circular tooth thickness of driven gear at base radius, r_{gj} is the base radius, $inv\ \alpha_{psj}$ is the value of involute function correspondent to the transverse pressure angle at point P_j.

Similar to Eq. (4), Eq. (7) is employed to calculate the angle ϕ_{p1} related to point P_1 of driving gear which is the candidate point to be in contact with point P_2 of driven gear,

$$\phi_{p1} = \theta_1 \pm (\varphi_{\beta p1} + \Delta\beta_1) + (\varphi_{p1} + \Delta\alpha_1) \quad , \tag{7}$$

where, the positive signal (+) or negative signal (-) corresponds to right hand or left hand of helix angle of the driving gear respectively, $\Delta\beta_1$ is an angular value correspondent to the sum of lead modification and lead error at point P_1 of driving gear tooth, $\Delta\alpha_1$ is an angular value correspondent to the sum of profile modification and profile error at point P_1 of driving gear tooth, $\varphi_{\beta p1}$ and φ_{p1} are calculated by Eq. (5) and Eq. (6) respectively.

Since the point P_2 is determined, its respective candidate point to contact on driving member P_1 can be calculated by Eq. (3) and Eq. (7). The angle between P_1 and P_2 taken on driving gear can be calculated by Eq. (8):

$$\delta\theta_1 = sen^{-1}(\frac{r_{p2} \cdot sen\phi_{p2}}{r_{p1}}) - \phi_{p1} \quad . \tag{8}$$

The angle $\delta\theta_1$ is calculated for all discrete points that represent the tooth surfaces. The minimum angle found among all points analyzed, $\delta\theta_{1min}$, is considered the angular transmission error taken on driving gear and under no load. Physically, it means that the driving gear could rotate $\delta\theta_{1min}$ to contact the driven gear and no rotation would be observed on driven gear.

It is known that gear pairs present more than one pair of teeth in contact along the meshing cycle. Thus, some pairs of teeth should be considered in the analysis and also several positions on the meshing cycle should be analyzed.

In order to follow the transmission error convention previously stated, Eq. (9) is employed.

$$\delta t_2 = -\delta\theta_{1\,min} \cdot \frac{N_1}{N_2} \cdot r_{g2} \quad , \tag{9}$$

where, δt_2 is the longitudinal transmission error under no load for a particular position on the meshing cycle, $\delta\theta_{1min}$ is the minimum angular displacement of driving gear so that tooth surfaces can be in contact, N_1 / N_2 is the designed gear ratio and r_{g2} is the base radius of the driven gear.

2.2. The model for tooth contact analysis under load

The elastic deflections in geared systems are calculated in this work considering that the principal elastic deflections are the deflection of the local contact zone and the deflection due to bending of the contacting teeth, and the total elastic deflection is the sum of the principal deflections. The gear bodies, except teeth, are assumed to be rigid.

2.2.1. Deflection of the local contact zone

The local contact zone is based on Hertz's compressive stress model. The contact deflection is assumed to be present only on points where loads are applied. The contact deflection is calculated using an equation presented by Conry & Seireg (1973) and modified in order to take into account the effects of free edge according to Umezawa & Ishikawa (1973). Thus, the equation employed in this work to calculate contact deflections is:

$$w_H = \frac{25 \cdot (1-\nu^2) \cdot p_N}{\pi \cdot E} \cdot c(\hat{\rho}) \cdot c(\hat{\varepsilon}) \quad , \tag{10}$$

where, w_H is the deflection of tooth contact surface at the point where load is applied and it is taken in the normal direction to the tooth surface, p_N is the normal load applied on discrete point, E is the Young´s modulus of the material, ν is the Poisson´s ratio, $c(\hat{\rho})$ and $c(\hat{\varepsilon})$ are the correction functions for the effect of the free edge shown in Umezawa & Ishikawa (1973). $\hat{\rho}$ is the distance between the loaded point and a free edge related to gear faces. $\hat{\varepsilon}$ is the distance between the loaded point and the free edge related to the gear outside radius.

2.2.2. Bending deflection of the contacting teeth

In this work, the bending deflections of the gear teeth are calculated using the approximate formula proposed by Umezawa (1972), which considers the deflections of a rack-shaped cantilever plate with finite width due to a concentrated load according to Fig. (2).

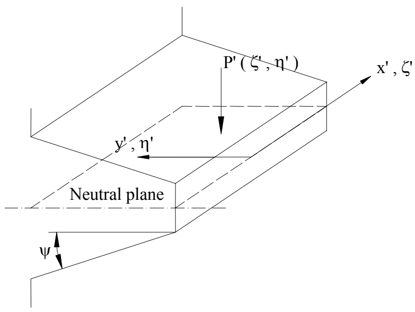

Figure 2. Rack-shaped cantilever plate model.

A concentrated normal force is applied on the neutral plane of rack and produces a deflection that is measured in the neutral plane of the rack.

Umezawa (1972) proposed some functions to the plate bending deflections based on numerical solutions obtained by finite differences calculation related to the corresponding differential equations. More recently, Park & Lee (1993) proposed new approximation functions based on results of a finite element analysis. According to these last authors the proposed functions are more realistic because they consider the tooth foundation deflections.

The equation presented by Umezawa (1972) with the new functions proposed by Park & Lee (1993) is employed in this work, i.e.,

$$w_B(x',y') = \frac{U \cdot f(\lambda \cdot \bar{x}') \cdot g(\bar{y}')}{MP} \cdot \sum_{k=1}^{N} p_k' \cdot \frac{v(\bar{r}_k)}{f\left(\lambda \cdot \left|\bar{x}' - \bar{\zeta}_k'\right|\right) \cdot g\left(\left|\bar{y}' - \bar{\eta}_k'\right|\right)} \cdot f(\lambda \cdot \bar{\zeta}_k') \cdot g(\bar{\eta}_k') \quad , \tag{11}$$

where,

$w_B(x',y')$ is the total deflection at point (x',y');

U is the absolute value of deflection of the rack-shaped cantilever plate at its origin when a concentrated load is applied there. It can be found in Park & Lee (1993) e Umezawa (1972);

$g(\bar{y}') = \sqrt{G(\bar{y}')}$ is a function that determines the deflection caused by a concentrated load applied in the heightwise direction of the plate. It is presented in Park & Lee (1993) e Umezawa (1972);

$f(\bar{x}') = \sqrt{F(\bar{x}')}$ is a function that determines the deflection caused by a concentrated load applied in the widthwise direction. It is presented in Park & Lee (1993) e Umezawa (1972);

$v(\bar{r}_k)$ is a function that defines the deflection measured at a point distant r from the concentrated load point. It is presented in Park & Lee (1993) e Umezawa (1972);

$\bar{r}_k$ is the distance from the concentrated load point $P(\zeta, \eta)$ to the point where the deflections are measured $P(x', y')$. It can be obtained using the Eq. (12);

$\bar{x}' = \frac{x'}{H}$, (variables with upper bar mean a division by plate height);

Variables with apostrophe such as x' and y' mean coordinates related to the rack-shaped cantilever plate as shown in Fig. (2).

k=1,2,...,N, where N is the number of loaded discrete points;

$$\bar{r}_k^2 = (\lambda \cdot \bar{x}' - \lambda \cdot \bar{\zeta}_k')^2 + (\bar{y}' - \bar{\eta}_k')^2 \quad , \tag{12}$$

and λ is a scale of coordinate applied on the coordinates x' e ζ' and presented in Park & Lee (1993) e Umezawa (1972).

The method of influence coefficients is employed in this work using the Eq. (11), which makes possible to calculate the bending deflection in a discrete point of the tooth caused by concentrated loads applied in another point of the tooth. Thus, a matrix relation between bending displacements and applied forces can be determined.

An appropriate function should be used to transfer the deflection of the local contact zone and the bending deflection to angular values according to the adopted coordinate system. The deflections expressed in angular form can be added to the angles ϕ_{p2} of Eq. (4) and ϕ_{p1} of Eq. (7) related to driven gear and driving gear respectively.

2.2.3. Load distribution evaluation

In order to calculate the elastic tooth deflections it is necessary to determine the intensity of the load applied on each discrete point over the tooth surfaces. The load distribution on gear teeth has been the subject of several investigations. The procedure based on a simplex-type algorithm firstly proposed by Conry & Seireg (1973) has been widely employed to solve the contact problem and to determine the load distribution in the gear teeth (Beacham et al, 1999; Norman, 1995 e Park & Lee, 1993).

In this work, a complete contact analysis is performed without restricting the contact analysis only to the theoretical plane of action. Based on this assumption, a method for load distribution is proposed by applying the load gradually and interactively. In order to obtain a better representation of the physical phenomenon, the total load is divided into discrete forces, which are applied on points over the tooth surfaces that are in contact. The force increments are applied on the contact points, the resultant deflections are calculated in all points (loaded and unloaded points) by Eq. (10) and Eq. (11), and new contact points are determined. This routine is repeated until the contact forces reach the input torque on the gear pair. The error of this method is related to the degree of discretization of the total load (number of load increments) and also to the degree of discretization of the tooth surfaces (number of contact possible points).

3. Numerical examples

The previously described methodology was implemented into a computer code and some numerical examples were used to verify the results of transmission error. The numerical results were calculated using the developed computer program, called in this work ETE, and compared to the results obtained from a reliable computer program, called LDP ("Load Distribution Program"), which was developed by the Gear Laboratory of the Faculty of Mechanical Engineering of the Ohio State University, USA (Houser, 1982).

The main differences between LDP and ETE can be pointed out as: LDP program assumes that all tooth contact is along the theoretical line of action and ETE analyzes the contact in whole tooth surface, and LDP applies a formulation for the solution of the load distribution based on the work of Conry & Seireg (1973) pertaining to elastic bodies in contact and ETE employs a method of gradual and interactive (incremental) load application described in section 2.2.3.

Two pairs of gears, a spur gear pair and a helical gear pair with different modifications on tooth surfaces were used to evaluate the proposed method. The gear design parameters are shown in Tab. (1).

Table 1. Design parameters of the gear pairs.

	Spur gear pair		**Helical gear pair**	
	Driving gear	Driven gear	Driving gear	Driven gear
Number of teeth	27	43	27	43
Center distance (mm)	70		70	
Normal module	2		1.93	
Normal pressure angle	20°		20°	
Normal helix angle	0°		15°	
Outside diameter (mm)	58	90	58	90
Root diameter (mm)	48	80	48	80
Rack cutter modification	0	0	0	0
Face width (mm)	15	15	15	15

3.1. Spur gear pair with geometry modifications (unloaded pair): first case

A case of tooth geometry modification by roll motion was applied for both members of the spur gear pair shown in Tab. (1). Parabolic modification starting at roll angle of the operating pitch point and tooth tip modification magnitude of 0.050mm was applied for both gears.

The longitudinal transmission error was calculated in ten positions of meshing cycle for the spur gear pair with the parabolic modification on the tooth profiles. The discretization parameters employed were 500 discrete points on the heightwise direction and one discrete point on the widthwise direction. The results obtained, without applied loads from ETE and from LDP, are presented in Fig. (3).

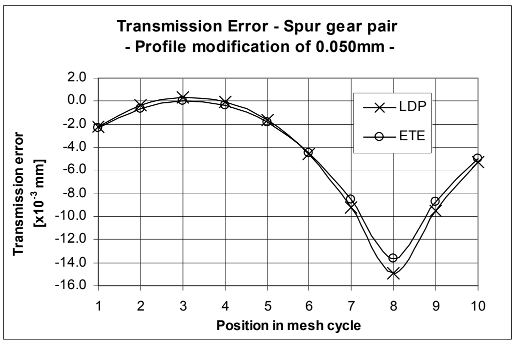

Figure 3. Comparison of the transmission error for the spur gear pair with modification on the tooth surfaces - Case 1.

The results shown in Fig. (3) present some agreement between LDP and ETE. The maximum difference between the results is observed at position 8 and corresponds to $1.221 x 10^{-3}$mm or 8.2% to the LDP result. It was expected to find no relevant difference between results, since it was considered unloaded gears. Only geometry and gear kinematics are involved in this analysis. In order to investigate the cause of that difference, a geometric model using a CAD (computer aided design) system was employed. The spur gear pair was modeled in two-dimensions considering the profile modifications. The gear pair model was rotated to the position corresponding to the position 8 of the meshing cycle and the difference between the theoretical position (perfect gears with no modification) and the actual position imposed by the profile modifications was determined. The angular value was multiplied by the base radius of the driven gear to calculate the correspondent longitudinal transmission error. Figure (4) illustrates the actual contact point at position 8 of the meshing cycle. It can be observed that the actual contact is out of the line of action. The graphic procedure presented maximum difference of 1.1% in comparison to the results numerically obtained by using LDP and ETE.

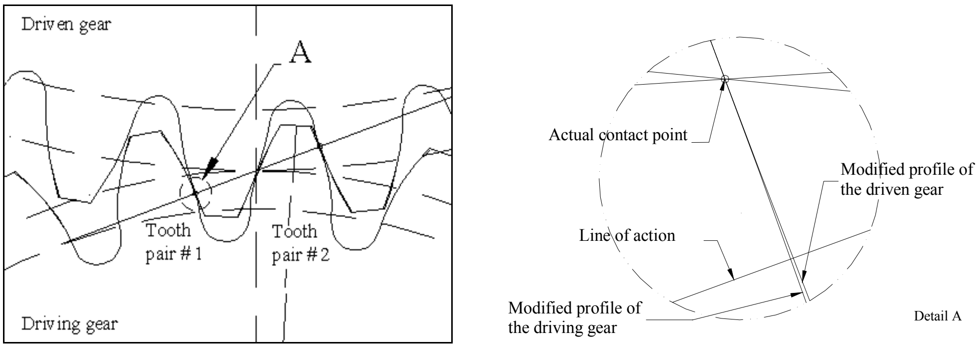

Figure 4. Actual contact point at position 8.

It can be concluded that the approach that considers the contact only on the line of action leads to some imprecision in the contact analysis and on the transmission error prediction even for the unloaded condition.

3.2. Spur gear pair with geometry modifications (loaded pair): second case

In order to compare results of transmission error under load using LDP and ETE, it was assumed a small tooth surface modification so that the initial error of LDP analysis for the spur gear pair under no load was reduced. For this second case a parabolic modification with 0.010mm of magnitude on the tooth tip was introduced on both members of the spur gear pair. The parabolic modification started at the roll angle of operating pitch point.

Seven levels of torque from zero to 30Nm were applied on this second case of the spur gear pair and the respective transmission errors were calculated using the LDP and the ETE. Material properties used were: Young's modulus of 2.1×10^4kgf/mm^2, and Poisson's ratio of 0.3. The discretization parameters used for calculations on ETE were 100 discrete loads per torque unit, 150 discrete points on heightwise and on widthwise directions.

The maximum and the minimum longitudinal transmission error obtained for ten positions of meshing cycle with LDP and ETE were compared for each torque level. A good agreement between the values calculated by LDP and ETE was observed. The maximum difference between the results of LDP and ETE was found for a torque of 15Nm and represents 6.2% in comparison to LDP.

Figure (5) shows the longitudinal transmission error for ten positions in the mesh cycle for torque of 15Nm. Positions 8 and 9 presented the largest differences in relation to the results of LDP.

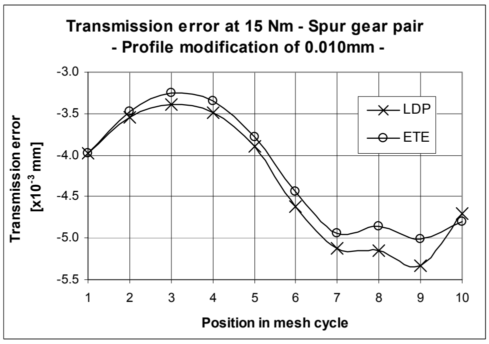

Figure 5. Comparison of transmission error for the spur gear pair with modification on tooth surfaces - Case 2, under 15Nm of load.

Two pairs of teeth would be simultaneously conjugated at positions 8 and 9, if the gear pair was perfect (without modifications). As profile modifications were considered in this study case, for no load and for certain levels of load, the tooth deflections are too small so that the two pair of teeth could be in contact. In those cases only one pair of teeth can be actually carrying load. Modifications of tooth surfaces lead to lesser contact ratios affecting significantly the load distribution specially for light loads.

The initial gaps determined by the two programs for the two pairs of teeth in contact called in this work tooth pair # 1 and tooth pair # 2 were investigated. It was found differences in the initial gaps calculated by ETE and LDP due to the assumption of LDP in considering the contact only on the line of action. Figure (6) illustrates that condition at position 9, as an example.

In fact, the good agreement between results of LDP and ETE found without applied loads is not enough to guarantee that all other point that represent the tooth surface present the same behavior. Although the first pair of teeth in contact presented near values of transmission error regarding LDP and ETE programs, the second pair of teeth candidate to be in contact presented larger deviation between LDP and ETE. This can be explained since the procedure proposed in this paper, ETE, considers the contact in whole tooth surface and not only on the plane of action.

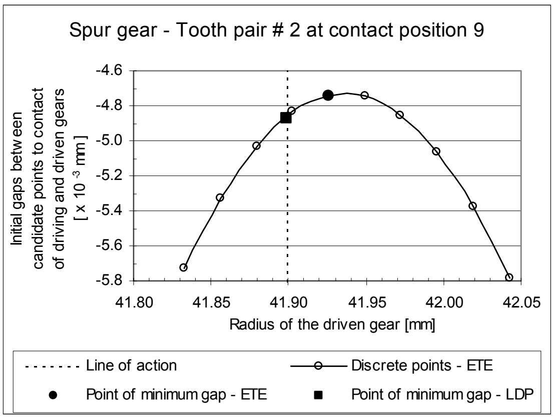

Figure 6. Initial gaps at contact position 9.

For higher torques, the influence of differences on initial gaps is reduced due to the effect of load distribution. When the contact of the second pair of teeth begins, it carries a small part of the total load. But increasing the applied torque, the load distribution tends to be equally divided between the two pairs of teeth in contact.

3.3. Helical gear pair with geometry modifications

A tooth geometry modification by roll motion was considered for both members of the helical gear pair with design parameter shown in Tab. (1). A parabolic modification starting at roll angle of the operating pitch point was employed. A tip modification magnitude of 0.010mm was applied on tooth profiles, and also a circular lead crown of 0.005mm was considered in both gears.

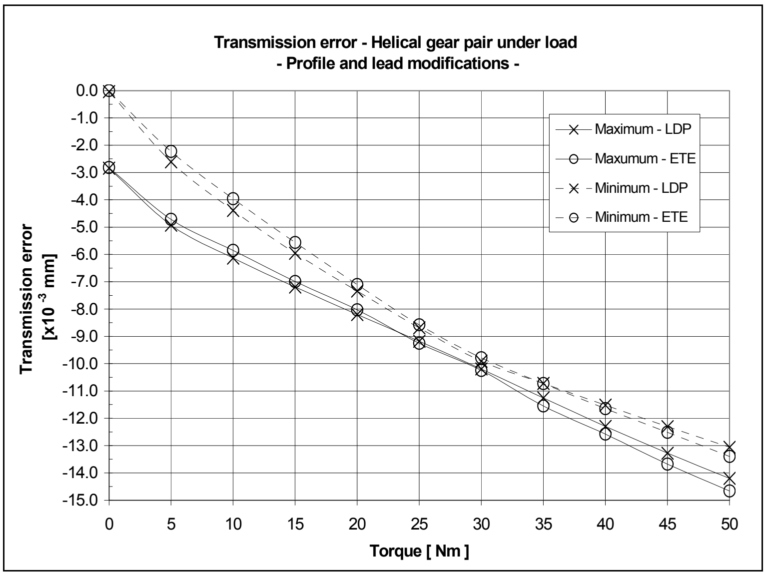

Figure 7. Comparison of transmission error under load obtained using LDP and ETE for the helical gear pair with modified tooth surfaces.

Eleven levels of torque from zero to 50Nm were applied on the helical gear pair with profile and lead modifications, and the respective transmission errors were calculated using the LDP and the ETE. Young's modulus of $2.1x10^4 kgf/mm^2$ and Poisson's ratio of 0.3 were used in the calculations. The discretization parameters used for calculations on ETE were 100 discrete loads per torque unit, 80 discrete points on both heightwise and widthwise directions. The maximum and the minimum longitudinal transmission error obtained on ten positions of meshing cycle with LDP and ETE are shown in Fig. (7).

The transmission errors as shown in Fig. (7), for example, do not provide much information about the performance of the gear pair in terms of noise because it is a single value analysis. A good correlation can be obtained between gear noise and the two first harmonics of transmission error (Smith, 1987). Transmission error harmonics is an useful tool in analyzing the transmission error behavior along the meshing cycle because it allows to take into account the shape of the transmission error function and not only the analysis based on single value.

In this paper, the two first harmonics of the transmission error were calculated using the fast Fourier transform algorithm. The ten points of the transmission error calculated along the meshing cycle were considered as discrete points of a periodic signal taken at a certain frequency of sample.

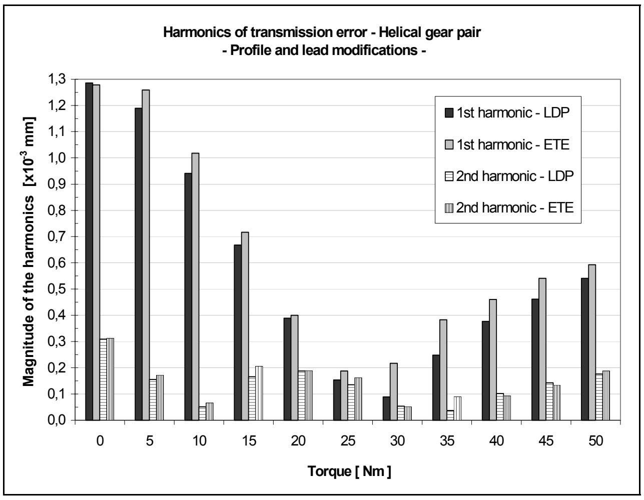

Figure 8. Comparison of 1st and 2nd harmonics of transmission error under loads using LDP and ETE for the helical gear pair with modified tooth surfaces.

Figure (8) shows the magnitudes of the first and second harmonic of the transmission errors calculated for each torque level. A comparison of LDP and ETE can be established. The first harmonic presents higher magnitudes at low torque and tends to the minimum around 25 to 30Nm of torque. In this case, it can be expected that the gear noise is higher at unloaded condition and in very light torque levels.

The harmonics calculated by LDP and ETE present the same behaviour and similar results. However, at a range of torque from 30 to 35Nm, a relevant difference can be found on the magnitude of the first harmonic. This difference can be explained based on Fig. (9), where a comparison between the transmission error in ten positions of the meshing cycle calculated by LDP and ETE for 30Nm of torque is presented. The results obtained by the two programs show close values of transmission error. The maximum difference is found at position 9 and corresponds to $1.580x10^{-4}$mm or 1.6 % related to LDP result. It can be observed in Fig. (9) that the shape of the curve formed by the ten discrete points of LDP and ETE are slightly different. This difference causes the harmonics of the transmission error to be different.

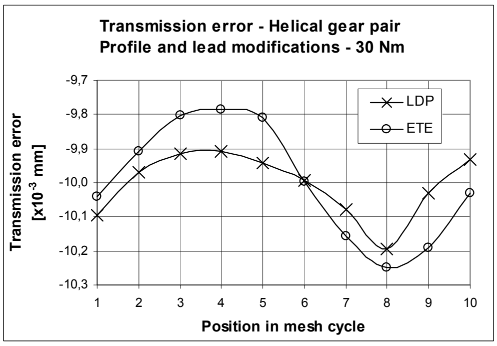

Figure 9. Comparison of transmission error for the helical gear pair under 30 Nm of torque.

The differences observed on transmission error results of LDP and ETE in Fig. (9) can be explained by:

1. The initial gaps determined by the two programs are slightly different due to the different assumptions for the contact analysis between LDP and ETE. Consequently load distribution and tooth deflection are affected;

2. LDP and ETE present different approaches for the load distribution: LDP applies loads on discrete points over theoretical lines of action, while ETE applies loads on discrete points over "an area" of contact. In ETE, the discrete point in contact (with applied load) are spread approximately over an elliptical area, which seems to be a better representation of the contact of curved surfaces like profile of gears;

3. For the gear teeth to be in contact at any point, the sum of the elastic deflections and initial gaps should be equal to the rigid body displacement. Comparing the sums of the elastic deflections and initial gaps of each point in contact, it was observed that there are some differences between values calculated by ETE (the maximum difference found was 9.2×10^{-5}mm for 30Nm of torque applied through of 3000 discrete loads). These differences are related to the degree of discretization of the load applied. It means that higher degrees of discretization can reduce these differences.

4. Conclusion

This paper proposed a model to calculate the transmission error of helical gear pairs under load. The proposed model provides more realistic simulation results on tooth contact analysis and transmission errors than the existent models that made the assumption of contact only on the plane of action. A computer program where developed and tested for a pair of spur gears and for a pair of helical gears with modifications on tooth surfaces. The numerical examples tested show that modifications on tooth surfaces can lead to contact deviations from the plane of action. It was also shown that correct calculation of the initial gaps is fundamental for transmission error calculation. Based on these results, the proposed procedure can be more precise in calculating transmission error of spur and helical gear pairs with modified tooth surfaces since a complete tooth contact analysis employed.

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