EXERGOECONOMIC OPTIMIZATION OF REFRIGERATION SYSTEMS

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ABSTRACT. Thermal systems constitute a field of great interest amongst the subjects of research in energy for its ample application of the electric energy generated for them, for the available thermal energy, or still for the use of refrigeration systems for the environment acclimatization of work or determined industrial areas that thus demand it. For all these cases, the evaluation of the project operational conditions of each application is important, as well as the study of optimization by means of which improvements are given in terms of efficiency increase and reduction of costs. This work aims at the study of a vapor compression refrigeration plant, which operates with the working fluid R-134a, which will be analyzed on the basis of the Exergetic Costs Theory. For the process development of exergoeconomic optimization it is used the Engineering Equation Solver (EES) software that has as structure standard of its database the thermodynamic properties of the working fluid.

Keywords. *refrigeration, R-134a, exergy, exergetic cost theory.*

1. Introduction

Optimization analysis in industrial systems are widely used in the most diverse types of studies, as is showed in the analysis by Allen (1971), about definition of the optimum final products of a processing refinery of raw oil, Chinneck and Chandrashekar (1984) and their energetic system in wide scale and Ferreira and Silva (2001) regarding the optimal condition of the steam flow in steam networks.

Nowadays, refrigeration cycles have been used due to the increasing necessities of work and industrial areas environment acclimatization that demands this requirement. The present work aims at to the analysis of a refrigeration plant for vapor compression, operating with the cooling fluid R-134a, whose related analysis was based on the Theory of the Exergetic Cost (TCE). The refrigeration cycle considered in this analysis was the one proposed by d'Accadia and Rossi (1998) with the necessary modifications adopted by Ferreira (2003).

The work considered by Ferreira (2003) not only analyzes the refrigeration plant studied but also the implementation of the cooling fluid R-134a thermodynamic properties in an optimization software capable of solving analysis of linear and nonlinear programming problems.

2. Configuration and Formularization

The plant studied was initially proposed by d'Accadia and Rossi (1998) and adjusted to the present analysis conditions. The plant of refrigeration for vapor compression is composed, as indicated in Fig. 1, by a refrigeration cycle that includes the cooling tower (TR) and the condenser (CD), a throttling valve (VE), an evaporator (EV) and an electric engine (EM1) that drives the compressor (CP). Figure (1) indicates, still, the referring points to the thermodynamic cycles, and the mass flow rate of refrigerant (m_{ref}), of circulation water (m_W), air (m_{air}) and supplying water ($m_{W,SU}$).

For the refrigeration plant, Exergetic Cost Theory (TCE) was applied and the incidence matrix (A) was composed and modified to obtain the incidence ($\underline{A}$) and inverse modified incidence ($\underline{A}^{-1}$) matrices, to the calculus of the unitary exergetic costs of each flow in the system.

In this way, exergetic and exergoeconomic costs vector (Y) determination will depend on the assessment vector (E), which one could take on exergetic (E') or exergoeconomic (E'') values, respectively, according to d'Accadia e Rossi (1998) and mathematically represented by Eq. (1).

$$Y = \underline{A}^{-1}.E \qquad (1)$$

To apply TCE, the incidence matrix will need to contain the mass (AM), energy (AE) and exergy (AB) balance vectors, with theirs respective vectors of associated costs. The energy and mass balance must be equal to zero, while the exergetic equation is equal to a non-zero vector (named [1]), as demonstrated by Eq. (2), (3) and (4).

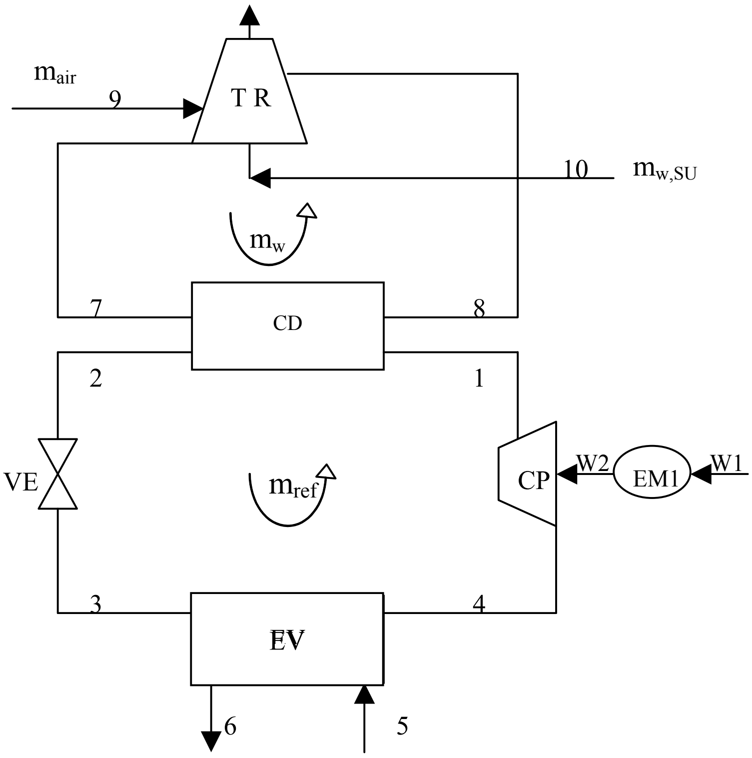


Figure 1. Refrigeration cycle by compression.

$$AM = 0 \tag{2}$$

$$AE = 0 \tag{3}$$

$$AB = 1 \tag{4}$$

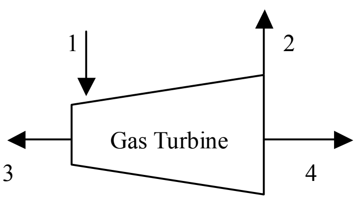
The energy and mass balances can easily be gotten by simple transport relations, being the input flow equal to output flow. For the exergy balance, the use of Valero and Lozano (1993) proposals becomes necessary, as described by Ferreira (2003) and Balestieri (2001):

- Proposal 1: exergetic cost of a flow (B*), product (P*) or fuel (F*) is the amount of needed exergy to produce; therefore, it's a conservative property. This proposal allows that as many equations of exergetic cost balance are formulated as many components to compose the installation. In a matrix way:

$$AB^* = 0 \tag{5}$$

- Proposal 2: the exergetic cost of input flows of the installation (fuels, air, water, etc.) is equal to its exergy.

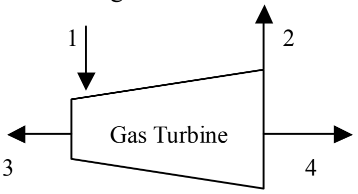
- Proposal 3: if an output stream of a component is part of fuel (F), it must be considered that exergy must not be considered and, therefore, its unitary exergetic cost (B*/B) is identical to the unitary exergetic cost of the input flow that precedes it. In Fig. 2 it's showed an example to a gas turbine and its exergetic balance.



$$F = B_1 - B_2$$
$$\frac{B_1^*}{B_1} = \frac{B_2^*}{B_2}$$

Figure 2. Unitary exergetic costs to fuel flows in a gas turbine.

- Proposal 4: if a component has a product (P) formed by some flows, it must be associated with these flows the same unitary exergetic cost. This explains the fact that if two or more products could be identified in one same equipment, their process of formation are indistinct in the considered level of aggregation and, therefore, a proportional exergetic cost must be associated with the exergy that contains. Figure 3 illustrates the proposal application to a gas turbine.



$$P = B_3 - B_4$$

$$\frac{B_3^*}{B_3} = \frac{B_4^*}{B_4}$$

Figure 3. Unitary exergetic costs to fuel flows in a gas turbine.

- Proposal 5: in the external values absence to the losses flows, as heat yielded to the environment, emission of chimney gases, or any other flow, null exergetic cost is attributed, since they do not present posterior utility.

According to these proposals, matrix system can be written as showed bellow:

flow \ equipment	1	2	$\cdots$	n
1	$bm_{1,1}$	$bm_{1,2}$	$\cdots$	$bm_{1,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
m	$bm_{m,1}$	$bm_{m,2}$	$\cdots$	$bm_{m,n}$
1	$be_{1,1}$	$be_{1,2}$	$\cdots$	$be_{1,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
m	$be_{m,1}$	$be_{m,2}$	$\cdots$	$be_{m,n}$
1	$bb_{1,1}$	$bb_{1,2}$	$\cdots$	$bb_{1,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
m	$bb_{m,1}$	$bb_{m,2}$	$\cdots$	$bb_{m,n}$

Figure 4. Schematic equations of mass, energy and exergy balance.

and its respective costs vector.

$$\begin{array}{c} \underline{vector\, Y^*} \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ B_1^* \\ \vdots \\ B_m^* \end{array}$$

Figure 5. Schematic cost vector.

with m mass, e energy, b exergy and B^* the exergetic cost of the analyzed flow.

The unitary exergetic cost is a term of the flow efficiency, showed in the relation:

$$\frac{1}{\eta} = k^* = \frac{B^*}{B}$$

with k* as the unitary exergetic cost and η the flow efficiency.

With the refrigeration plant defined, it is necessary to apply the TCE to the scheme. The plant presented in Fig. (1) is modified for a better visualization and understanding of a productive plant according to Fig. (6), with 3 fictitious divisions, related to the exergy formers components: the pressure, temperature and negentropy, which means exergetic loss of a determined flow, components.

The corresponding equations to the thermal (BT), pressure (BP) and irreversibility (S) components are given by the equations bellow, described in Kotas (1985), and, also it is, determined all equations of flow represented in Fig. (6). All of them are given in kW units.

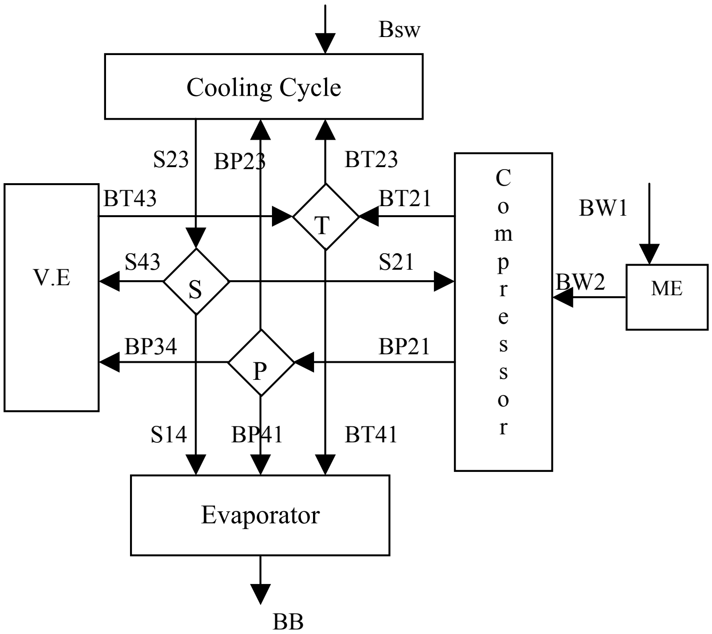


Figure 6. Productive structure of the refrigeration plant.

$$BT = m_{ref}\left(\Delta h\left(1-\frac{T_0}{T_R}\right)\right) \quad (6)$$

$$S = m_{ref}\Delta s T_0 \quad (7)$$

$$BW1 = m_{ref}\left(h_2 - h_1\right) \quad (8)$$

$$BW2 = BW1.\eta_{EM1} \quad (9)$$

$$BP = m_{ref}\left(\Delta h - T_0\Delta s\right) \quad (10)$$

with:

Δh – enthalpy flow difference, in kJ/kg,
T_0 – environment temperature, in K,
T_R – temperature which heat transfer occurs, in K
Δs – entropy flow difference, in kJ/kg.K,
h2, h1 – enthalpy on points 2 and 1, respectively, in kJ/kg,
η_{EM1} – electric engine efficiency, dimensionless.

According to the above equations, the incidence matrix can be presented relatively to the considered refrigeration plant, as well as the exergetic and exergoeconomic costs vectors, as described below. Note that the entire network matrix will be composed of pressure, thermal and irreversibility components, which results in a seventeen flow set of equations, shown in Fig. (7).

	BW1	BW2	BP21	BP23	BP34	BP41	BW3	BSW	BT21	BT23	BT43	BT41	S23	S21	S43	S14	BB
1	1	-1															
2		1	-1						-1					1			
3				1			1	1		1			-1				
4					1						-1				1		
5						1						1				1	-1
6			1	-1	-1	-1											
7									1	-1	1	-1					
8													1	-1	-1	-1	
9			BP21						BT21								
10				BP23		BP41											
11				BP23	BP34												
12										BT23		BT41					
13														1/S21	-1/S43		
14														1/S21		-1/S14	
15		1															
16							1										
17								1									

Figure 7. Equation system of exergetic and exergoeconomic costs

E'	E"
0	-ξZ1
0	-ξZ2
0	-ξZ3
0	-ξZ4
0	-ξZ5
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
BW1	c0BW1
BW3	c0BW3
BSW	Πsw

Figure 8. Vector of exergetic and exergoeconomic external assessment

with:

Z_1 – Electric engine investment cost (Z_{EM1})
Z_2 – Compressor investment cost (Z_{CP})
Z_3 – Refrigeration cycle investment cost ($Z_{CD} + Z_{TR}$)
Z_4 – Throttling valve investment cost (Z_{VE})
Z_5 – Evaporator investment cost (Z_{EV})
ξ – amortization factor ($5.54.10^{-9}$ s^{-1})
c0 – electric energy cost (19.44 US$/GJ)
Πsw – Economic cost of the flow (10^{-5} US$/s)

Once determined the incidence matrix, the next step is to create the objective function, which is constrained to the operation and equipment costs conditions in the refrigeration plant. The equipment costs equations for the refrigeration plant components are showed in Eqs. (11) to (16).

$$Z_{EM1} = Z_{0,EM1}\left(\frac{P_{EM1}}{P_{0,EM1}}\right)^m \frac{\eta_{EM1}}{(1-\eta_{EM1})} \tag{11}$$

with P_{EM1} mechanical work supplied by the engine, in kW, and η the electromechanic efficiency.

In the studied case:

$Z_{0,EM1}$ = US$150 $P_{0,EM1}$ = 10 kW m_{EM1} = 0.87 η_{EM1} = 0.9

$$Z_{CP} = Z_{0,CP}\left(\frac{P_{CP}}{P_{0,CP}}\right)^{m}\left(\frac{\eta_{CP}}{0.9-\eta_{CP}}\right)^{n_{CP}} \tag{12}$$

with $P_{CP} = BP21 + BT21$ kW $\eta_{CP} = 0.8$
$Z_{0,CP} = US\$12000$ $P_{0,CP} = 100$ kW
$m_{CP} = 1.0$ $n_{CP} = 0.5$

$$Z_{VA} = Z_{0,VA}P_{VA} \tag{13}$$

$P_{VA} = BT43$ kW $Z_{0,VA} = 37$ US$/kW

$$\left.\begin{aligned}
Z_{TR} &= a_0 + a_1\Psi_{fill} + a_2 m_{W,SU}\\
ND &= \frac{(TC-0.08)}{a}\left(\frac{m_W}{m_{air}}\right)^{n}\\
H &= 0.3ND\\
Af &= \frac{m_W}{5.4}\\
\Psi_{fill} &= Af.H
\end{aligned}\right\} \tag{14}$$

$a_0 = US\$\ 350$ $a_1 = 350\ US\$/m^3$ $a_2 = 4000\ US\$/(kg/s)$
$n = 0.62$ $a = 0.07$

Economic expressions for evaporator and condenser costs referred by Böehm (1987) is used because a dimensional problem was identified in d'Accadia and Rossi (1998) equations, which it is supposed to be probably by transcription mistakes. So:

$$Z_{EV} = Z_{0,EV}(BB/10)^{5.359} \tag{15}$$

$$Z_{CO} = Z_{0,CO}(S23/10)^{0.626} \tag{16}$$

and:

$Z_{0,CO} = Z_{0,EV} = US\$\ 3000$

with BB and S23 the energetic and entropy flow relating to the losses, respectively, of the evaporator and condenser.

In this way, the costs sum is given by the Eq. (17).

$$Z = \sum_{i=1}^{5} Z_i \tag{17}$$

The system operation data are referred to the temperatures and pressures that satisfy the request thermal charge and are shown in Tab. (1). Only the flows relative to the compression system are stated because the model calculates the other ones.

Table 1. Operation conditions to the cycle of refrigeration.

Fluid	R134a			
State	1	2	3	4
T (K)	295.8	350.6	290	280.1
P (kPa)	320	900	855	374.6

Control conditions for the objective function are initially stated, aiming at minimizing the physical exergy increase relative to the water to be refrigerated, BB flow, given by the Eq. (18), assuming as decision variables the refrigerant mass flow rate and any other parameter that will be, by any chance, of process interest. In the studied case, the refrigerant mass flow rate is within a range of 0.15 to 50 kg/s, according to Eq. (19).

$$\text{Min BB} \tag{18}$$

$$\text{S.T.:} \quad 0.15 \leq m_{ref} \leq 50 \text{ kg/s} \tag{19}$$

Applying the proposed equations, an analysis of the system behavior was done. In other words, this analysis shows a “picture” of the system before any parameter change of design or non-design variables. This picture is named base case. The base case analysis, whose results are in Tab. 2, illustrates k* value for each flow, before sensibility analysis.

Table 2. Base case exergetic costs.

Flow	Exergetic flow B (kW)	Exergetic cost B^* (kW)	Unitary exergetic cost (B^*/B)
BB	9.20	43.01	4.68
BP21	26.21	40.26	1.54
BP23	10.07	15.47	1.54
BP34	3.84	5.90	1.54
BP41	12.30	18.90	1.54
Bsw	0.01	0.01	1.00
BT21	0.71	1.09	1.54
BT23	3.93	3.64	0.93
BT41	9.32	8.64	0.93
BT43	3.21	11.19	3.48
BW1	43.00	43.00	1.00
BW2	34.40	43.00	1.25
S14	157.30	15.47	0.10
S21	16.79	1.65	0.10
S23	227.90	19.11	0.08
S43	53.84	5.30	0.10

3. Results

The sensibility analysis results are presented in Tab. (3), emphasizing that it wasn’t intended to get the mathematic optimum, as it is demonstrated in the references about mathematic programming, especially because the software that supported the present analysis is not based on this concept. A similar analysis was realized with the exergoeconomic analysis result, and the results are also showed in Tab. (3).

Table 3. Exergetic costs to the advanced condition by the sensibility analysis.

Flow	Exergetic flow B (kW)	Exergetic cost B^* (kW)	Unitary exergetic cost (B^*/B)	Exergoeconomic cost (Π - US\$/10^{-6}s)	Unitary exergoeconomic cost (Π/B - US\$/GJ)
BB	9.1960	6.4550	0.7019	231.1000	25.1305
BP21	26.2100	6.4990	0.2480	149.9000	5.7192
BP23	10.0700	2.4970	0.2480	57.6000	5.7200
BP34	3.8380	0.9517	0.2480	21.9500	5.7191
BP41	12.3000	3.0500	0.2480	70.3600	5.7203
Bsw	0.0052	0.0052	1.0000	10.0000	1923.0769
BT21	0.7096	0.1759	0.2479	4.0590	5.7201
BT23	3.9270	0.5478	0.1395	18.6500	4.7492
BT41	9.3210	1.3000	0.1395	44.2700	4.7495
BT43	3.2140	1.6720	0.5202	58.8600	18.3136
BW1	6.4500	6.4500	1.0000	125.4000	19.4419
BW2	5.8050	6.4500	1.1111	125.9000	39.9176
S14	23.5900	2.1050	0.0892	105.9000	4.4892
S21	2.5190	0.2247	0.0892	11.3100	4.4899
S23	34.1900	3.0500	0.0892	153.5000	4.4896
S43	8.0760	0.7204	0.0892	36.2500	4.4886

Table (4) shows values that are referred to the exergoeconomic base case and the advanced condition by the sensibility analysis, in which it is observed a progress in the order of 53% in the total cost of equipment investment.

Table 4. Investment costs

Economic costs (US$)	Base Case	Optimum
Z_{CD}	21238.00	6476.00
Z_{CP}	6461.00	3022.00
Z_{TR}	4802.00	98.01
Z_{EM1}	118.90	118.90
Z_{EV}	404.60	5655.00
Z_{VA}	118.90	118.90
ΣZ's	**33143.40**	**15488.81**

4. Comments

Difficulties reported by d'Accadia and Rossi (1998) in solving the mathematic optimization problem, given the strong mathematical modeling non-linearities, was also observed in this work. The approach adopted by the authors was based on the step-by-step optimization, obtained by individual analysis of each one equipment with local decision parameters generation that are transferred in a cascaded form to the rest.

In this paper, solution was obtained by proceedings and resources available in EES software, as R134a thermodynamic properties, among other fluids, and sensitivity mathematical analysis, with magnitude order appropriate to the results obtained by d'Accadia e Rossi (1998), taking into account that the other plant was considered for R-22 refrigerant fluid. Values showed in Tab. 4 were obtained directly by the proposed formulation.

From the exergoeconomic cost minimization of the flow BB, which is responsible for the process refrigeration, it can be deduced from the results that a significant reduction in the exergetic and exergoeconomic costs of all flows can be gotten considering the basic conditions to the operation system.

From the obtained results, the following aspects that has a merit to be studied in future works are:

a- to analyze thermodynamic backgrounds of the R134a refrigerant state equation, as well the reference state to this equation;

b- to estimate studies about programming that make possible the adequate implementation, in fuller optimization software, as Lingo, of routines for thermodynamics properties in dynamic process that involve loops to become actual values of these properties;

c- to estimate alternative configurations of refrigeration cycles for vapor compression, specially that one that shows multiples stages of thermal changing to perform thermodynamic efficiency;

d- to estimate economic and environmental impacts originated by the use of others refrigerants available for the proposed configuration as well as for some others structures.

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