



## MULTIBLADED ROTOR HELICOPTER GROUND RESONANCE INCLUDING EIGENVECTOR ANALYSIS

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**Abstract.** *The influence of different helicopter parameters on the self-exciting, catastrophic instability known as ground resonance is studied herein. The analytical model considers a multibladed rotor helicopter-rotors with three or more blades-and neglects aerodynamics effects. It takes into account two equations of motion sets: one considers the lead-lag degrees of freedom of each rigid blade and the longitudinal and lateral degrees of freedom of the rotor support-hub; the other set takes into account the lead-lag and flapping degrees of freedom of each rigid blade and the pitch and roll degrees of freedom of the helicopter as a whole. Results include diagrams for modal frequency and modal damping as function of rotor nondimensional rotational speed for different rotor configurations. Modal frequency and damping diagrams are presented for both rotating and nonrotating frames of reference analyses. Eigenvector phasors are plotted for different regimes of rotor rotational speed. Their behavior inside and outside the instability region is analyzed. Interpretation of the phasors behavior points towards a link to the very existence and nature of the phenomenon itself.*

**Key words:** *ground resonance, dynamical stability, helicopter*

### 1. INTRODUCTION

Ground resonance is an autoexcited instability that occurs when the frequency of the blade lead-lag mode coalesces with the frequency of the body mode. As a consequence, the combined center of mass turns violently outward producing a rotating force at the rotor hub as it can be seen

in Fig. 1. Under the circumstances for the “resonance” to happen, the rotational energy of the blades is converted into oscillatory motion of the body, that, itself, feeds back energy into the rotor. The instability generally happens when the aircraft is not airborne yet, being close to the ground and having its landing gear extended. A wind gust, a sudden motion of the control or a hard landing can motivate the displacement of the combined blades center of mass. Since the phenomenon is potentially very destructive, avoiding it is an important consideration in helicopter design.

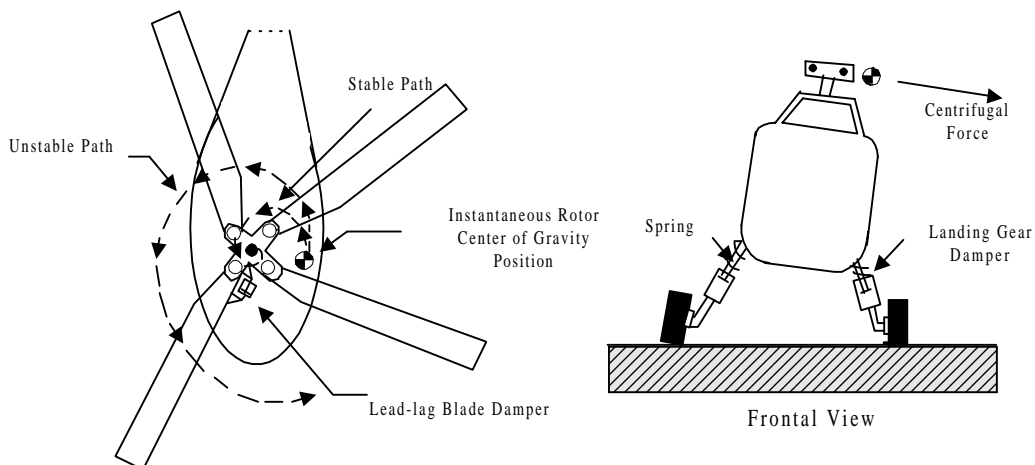


Figure 1 - Illustration of the helicopter ground resonance, from Prouty(1985).

The first paper on this subject was published by Coleman and Feingold(1958). It's still the most complete work published on the subject. Peters and Hohenemser(1971) presented a method for determining the stability of linear ordinary differential equations, turning the Floquet Theory ready for applying in rotorcraft problems as this one. The influence of the aerodynamics forces when hingeless rotors are present is presented by Johnson(1982); he correlates results from his dynamic inflow model. Bousman(1981)'s experimental data showed the real need to have aerodynamics effects included in the real problem. Arcos and de Andrade(1997a) studied the stability of both hingeless and articulated two-bladed helicopter rotors using Floquet analysis. Detailed study involving the behavior of associated eigenvectors in and out the instability region for that configuration were also presented by Arcos and de Andrade(1997b).

## 2. ANALYTICAL MODEL

The equations of motion are derived from a Lagrangean viewpoint. The systems involved as well as the involved degrees of freedom are depicted in Fig. 2. The most important assumptions taken are: (1) the rotor model involves three or more rigid blades attached to the hub, including root offsets; (2) aerodynamics effects are neglected; (3) springs are set at the blade roots to model blade stiffnesses in both flap and lead-lag; (4) external dampers are placed upon the blades for both flap and lead-lag motions; and (5) blades rotate with rotational speed  $\Omega$ .

Two set of equations are employed: the first considers two rotational degrees of freedom for the body (roll- $\phi$  and pitch- $\theta$ ) and two degrees of freedom for each blade (lead-lag- $\zeta$  and flap- $\beta$ ); the second set is the classical model, considering two translational body degrees of freedom and two degrees of freedom, flap and lead-lag, for each blade.

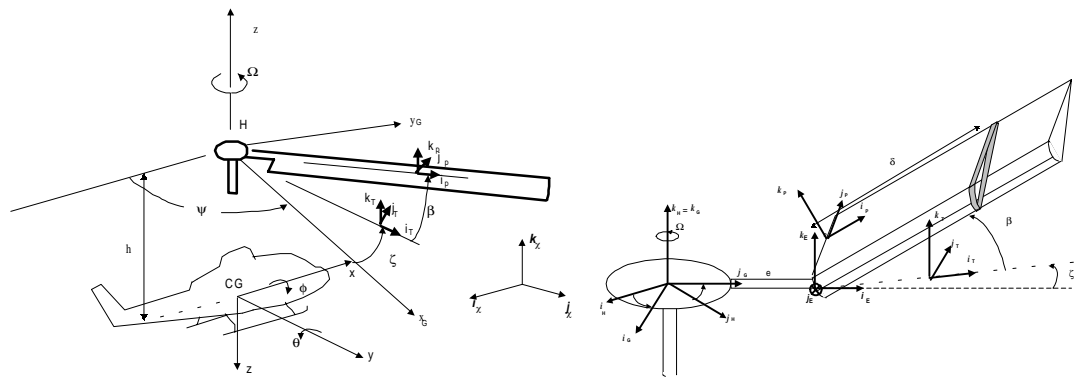


Figure 2 - Drawings that represent rotor / body and rotor / blade model.

### 3. SOLUTION APPROACH

The solution approach involves the solution of a set of ordinary linear differential equations obtained with the aid of symbolic computation as described in Pegado(1998). Linearization takes into account that both flap and lead-lag angles are small and neglects higher order terms. In general, this kind of the second-order linear differential equations shows periodic coefficients, which involves a more detailed numerical analysis due to is inherent frequency aliasing(Peters & Hohenemser(1971)). Depending on the nature of the set of rotor blades (called henceforth “rotor”)-rotor hub (called henceforth “hub”) periodic coefficients can be avoided. In this work, an isotropic rotor it that whose blades have the same properties as mass, damping, and stiffness. An isotropic hub here is the one which has the same properties in both  $x$  and  $y$  directions. When both rotor and hub are isotropic, periodic coefficients can be eliminated through the use of multiblade coordinate transformation when the blade equations are set into the nonrotating reference frame. Once the equations are established in a state-variable form, their eigenvalues can be calculated. Plots of their imaginary parts (called modal frequencies) in terms of the rotor rotational speed, called Coleman diagrams, can be obtained as well plots of their real parts, which are called modal damping. Instability ranges can be observed directly from both plots.

When an isotropic hub is in the presence of an anisotropic rotor, transforming the equations of motion of the former into the rotating reference frame yields a standard, nonperiodic coefficient set of equations to be solved. Equations of motion have periodic coefficients when an isotropic rotor is in the presence of an anisotropic hub. In this case the equations are transformed into a nonrotating reference frame through multiblade coordinate transformation, finite-state equations are set and the stability information is obtained through Floquet analysis(Peters & Hohenemser(1971)). The so-called Floquet transition matrix is calculated integrating numerically the state transition matrix over one period,  $2\pi$ . The Floquet transition matrix eigenvalues are plotted in form of damping and frequency diagrams and the stability range can be obtained.

### 4. RESULTS AND DISCUSSION

Before presenting results, it is interesting to comment on the physics involved within the phenomenon and its analysis.

The rotor blades seem to be lead-lagging independently for an observer in the rotating frame, i.e., for somebody sitting on the rotor hub and watching the blade motion. In this situation, all the

blades have the same oscillation frequency, but their phases are different. On the other hand, the blade motions are coupled to each other to form distinct patterns as seen by an observer in the nonrotating frame. These patterns are called rotor mode shapes. If a strobe light is set to the frequency of a particular mode, then the components of the mode shape can be seen by an observer in the nonrotating frame. For example, for a four-bladed rotor, the four components in the nonrotating frame are  $\zeta_0$ ,  $\zeta_2$ ,  $\zeta_{1c}$ , and  $\zeta_{1s}$ , called, respectively, collective, differential, cosine and sine modes. The last two are called cyclic modes. Under a physical standpoint,  $\zeta_{1c}$  and  $\zeta_{1s}$  are, respectively, a lateral and a longitudinal shifts of the rotor center of mass.

For isotropic rotors in hover, the collective and differential lag motions are completely uncoupled from cyclic rotor and the body modes. It isn't difficult to visualize that any collective lag motion can only introduce a torque to the main shaft. The differential mode is present in rotor having an even number of blades. Concerning the lead-lag mode, it is linked to the scissor-like movement of the blade set.

The coupled sine and cosine (cyclic) lead-lag modes can be referred to as either high frequency or low frequency mode. The high frequency mode represents a forward turning of the rotor center of mass; that means that the global rotor center of mass is swirling ahead and in the same direction as the rotor itself, as observed by an observer in the nonrotating frame. In the low frequency mode, the rotor center of mass can be turning either in the same direction of the rotor itself (progressive) or in the opposite direction (regressive). In the first case, its associated frequency,  $\nu_\zeta$ , is lower than 1/rev; in the second situation, it will be greater than 1/rev. The rotating lead-lag frequency is defined as  $\nu_\zeta^2 = K_1/\Omega^2 + K_2$ , where  $K_1$ , and  $K_2$  are the Southwell coefficients, and they represent the structural stiffness and the centrifugal force acting upon the blade, respectively. The resonance occurs when the body modes coalesce with the low frequency progressive blade mode.

In this work, simulation of anisotropic rotors is done by considering that one external blade damper is innoperative, that means, one blade damping coefficient is zero.

Figures 3, 4 and 5 show the influence of the type of rotor on the stability range. The behavior of articulated, stiff inplane and soft inplane hingeless rotors are investigated. Results herein use dimensionless rotor rotational speed of 155 RPM as a reference, same used by Coleman and Feingold(1958). Articulated rotors have  $K_1=0$  and  $\nu_\zeta < 1/\text{rev}$  as standard values. Figure 3 shows results for a typical articulated four-bladed helicopter rotor without damping and with anisotropic hub. Two instability regions can be observed: one between 0.85 and 1.2 and another ranging from 1.3 to 2.0. The modal frequency diagram shows the coalescence between the body modes and the low frequency lead-lag mode.

Like articulated rotors, soft inplane hingeless rotors have  $\nu_\zeta < 1/\text{rev}$ , but for them  $K_1$  is nonzero. Figure 4 shows its behavior on a configuration including three equal blades with isotropic hub. In this figure the corresponding eigenvalues are plotted, showing one instability range between 1.2 and 2.4, where there is a coalescence of the low frequency blade lead-lag mode with the body modes, as observed on both modal frequency and damping diagrams.

The stiff inplane hingeless rotor center of mass cannot perform the progressive low frequency rotation because its lead-lag rotating frequency is less then 1/rev. For these rotor configurations, the ground resonance does not take place, as observed in Fig. 5. Here the body modes do not coalesce with the low frequency lead-lag mode, being free from the instability.

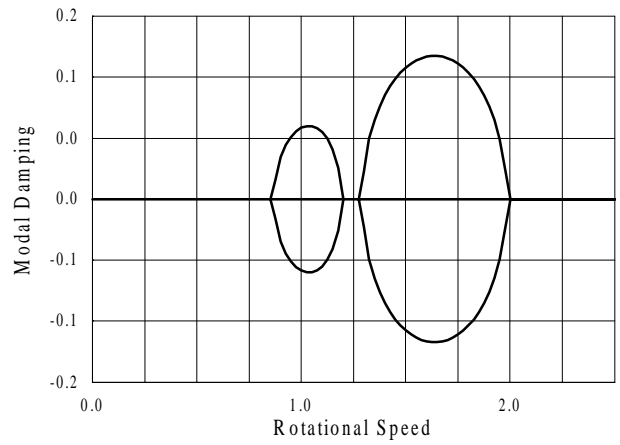
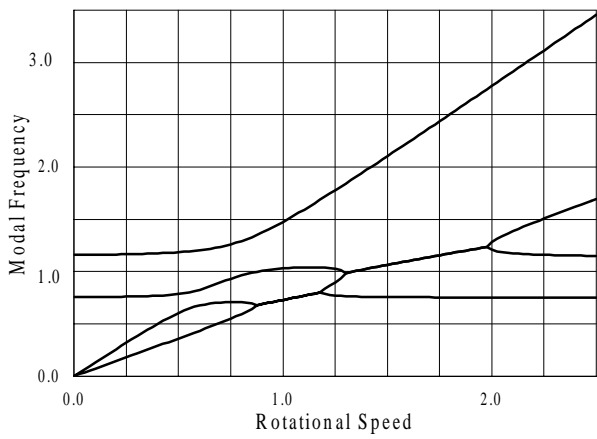


Figure 3 – Articulated isotropic four-bladed rotor, without damping and with anisotropic hub in the nonrotating frame.

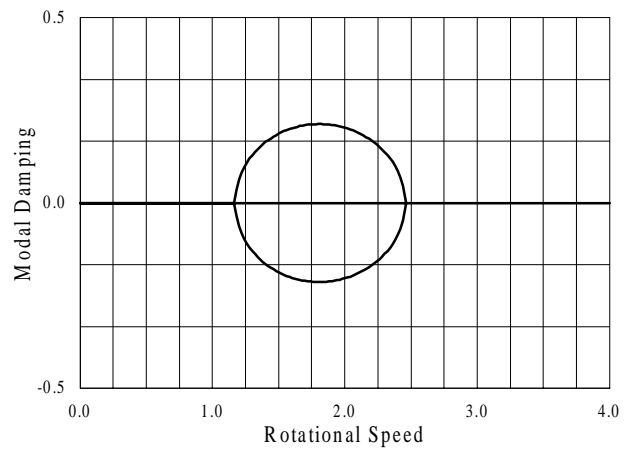
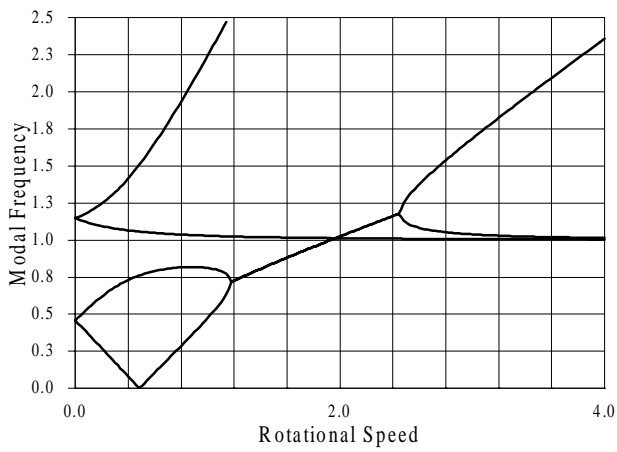


Figure 4 - Soft inplane hingeless three-bladed isotropic rotor, without damping and with isotropic hub in the nonrotating frame.

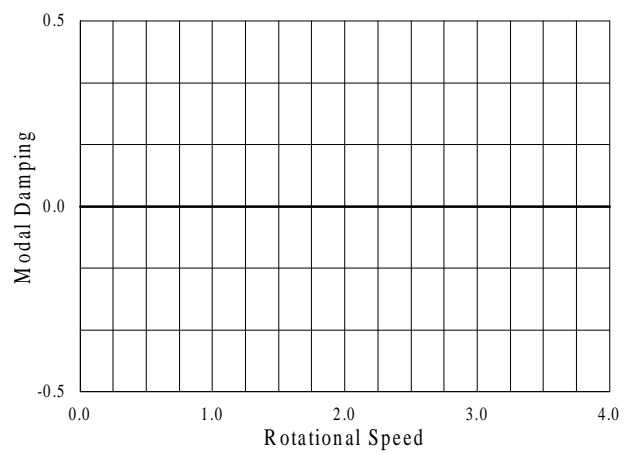
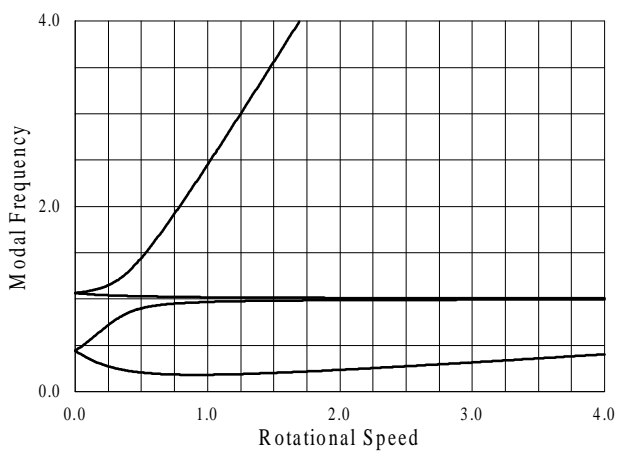


Figure 5 - Stiff inplane hingeless three-bladed isotropic rotor, without damping and with isotropic hub in the nonrotating frame.

Following, the influence of the external damping is analyzed. As mentioned before, in Fig. 3, for a four-bladed articulated rotor without external dampers, two instability regions are present. Fig. 6 shows results for a simulation involving the same basic configuration but now including installed external lead-lag dampers on each blade. Main point of comparison between the two figures is that the inclusion of external dampers prevent the occurrence of instability.

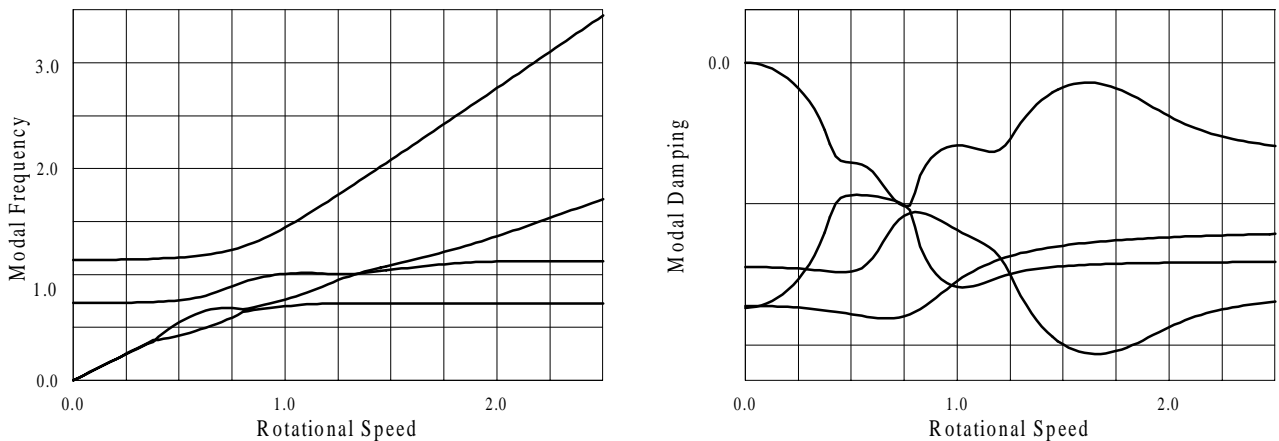


Figure 6 - Articulated four-bladed isotropic rotor, with damping and with isotropic hub in the nonrotating frame.

Results for a three-bladed isotropic hingeless rotor with damping are represented in Fig. 7. As observed in Fig. 6 here also one observe that an adequate choice for the level of damping in preliminary design can free the helicopter from ground resonance. The external damping role is to keep blade lead-lag and body frequency modes apart, decreasing or eventually making the coalescence to not happen at all. As observed, the modal damping migrates to the fourth quadrant of the corresponding plots. The value of damping required for ground resonance stability can be calculated, for instance, using Deutsch criterion as described in Johnson(1980).

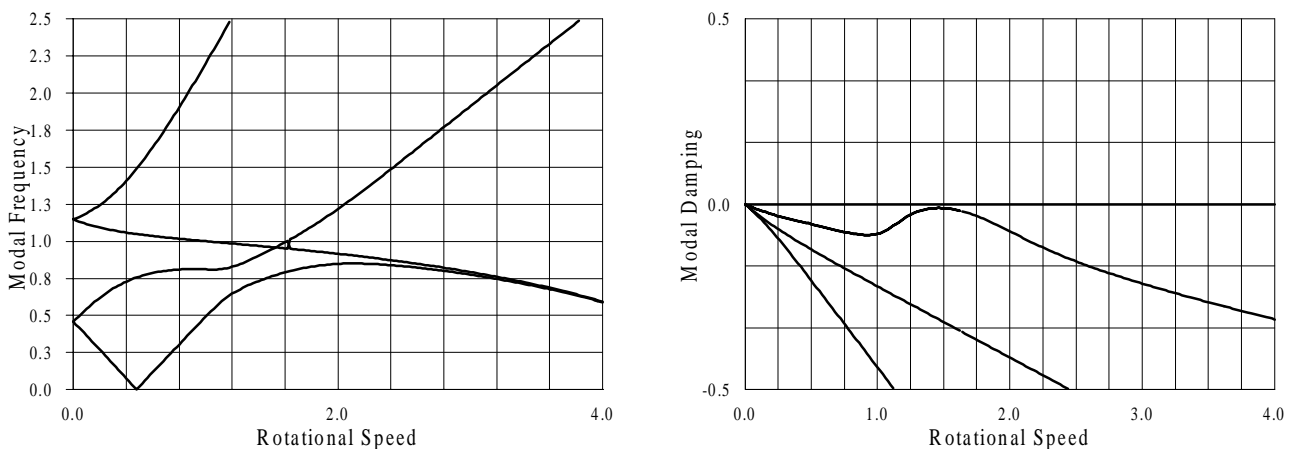


Figure 7 - Soft inplane three-bladed isotropic rotor, with damping and with isotropic hub in the nonrotating frame.

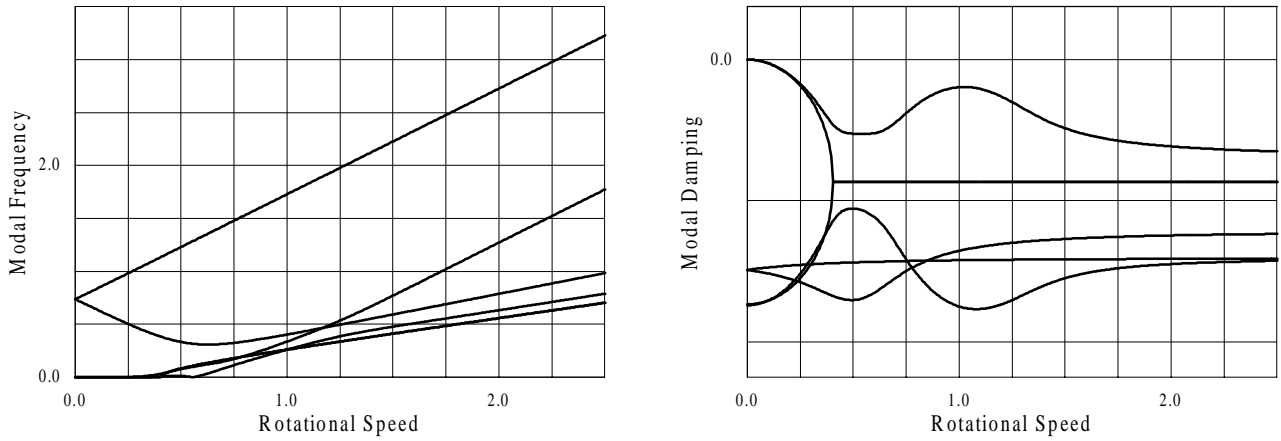


Figure 8 - Articulated four-bladed anisotropic rotor, with one innoperative damper in the presence of isotropic hub in rotating frame.

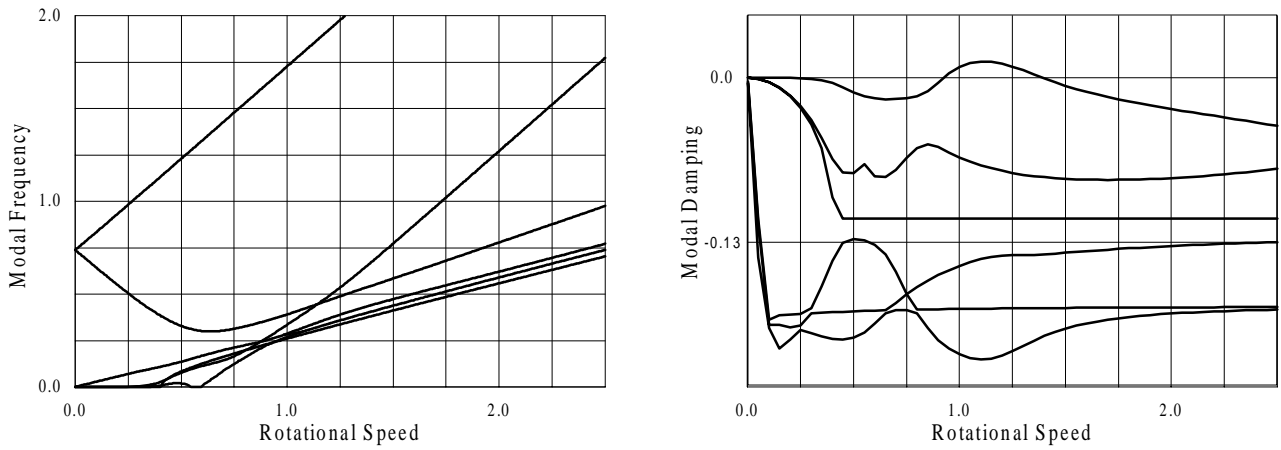


Figure 9 - Articulated four-bladed anisotropic rotor, with one innoperative damper in the presence of isotropic hub in rotating frame, Floquet's analysis.

For an articulated four-bladed rotor including one innoperative damper, there is one region of instability between 0.9 and 1.4 as shown on both Fig. 8 and 9. Figure 9 is obtained using Floquet analysis. Floquet analysis is also employed for solving the problem when both rotor and hub are anisotropic, as observed on Fig. 10.

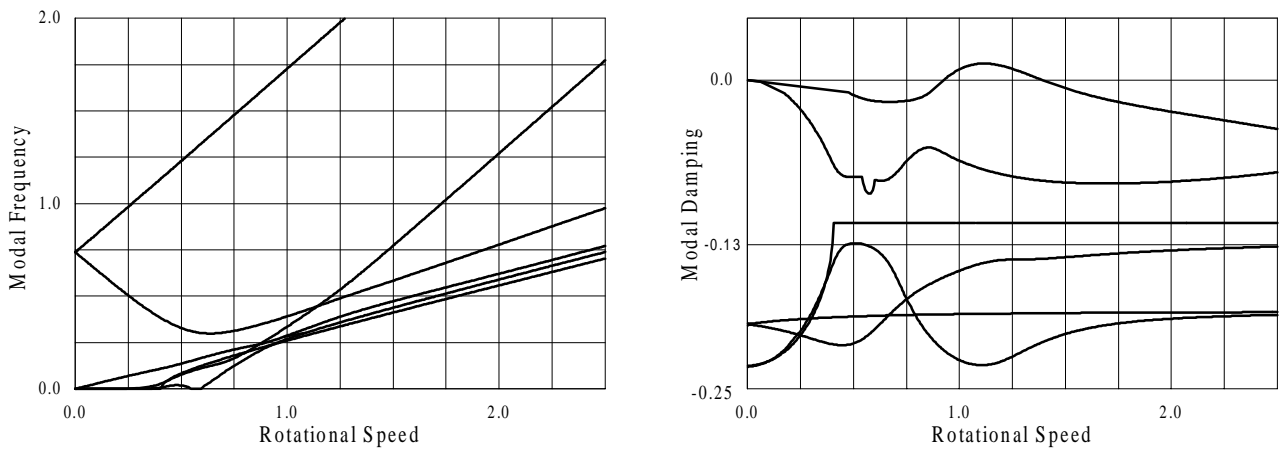


Figure 10 - Articulated four-bladed anisotropic rotor, with one innoperative damper in the presence of anisotropic hub in nonrotating frame.

Eigenvector analysis is conducted for different ranges of rotational speed and results for the nonrotating frames are shown in Figures 11-13. Figure 11 shows results for a three-bladed soft inplane hingeless rotor along with  $x$  and  $y$  body modes. Figure 12 shows similar results but including pitch and roll body modes. Figure 13 shows phasor diagrams for an articulated four-bladed isotropic rotor in the presence of an anisotropic hub.

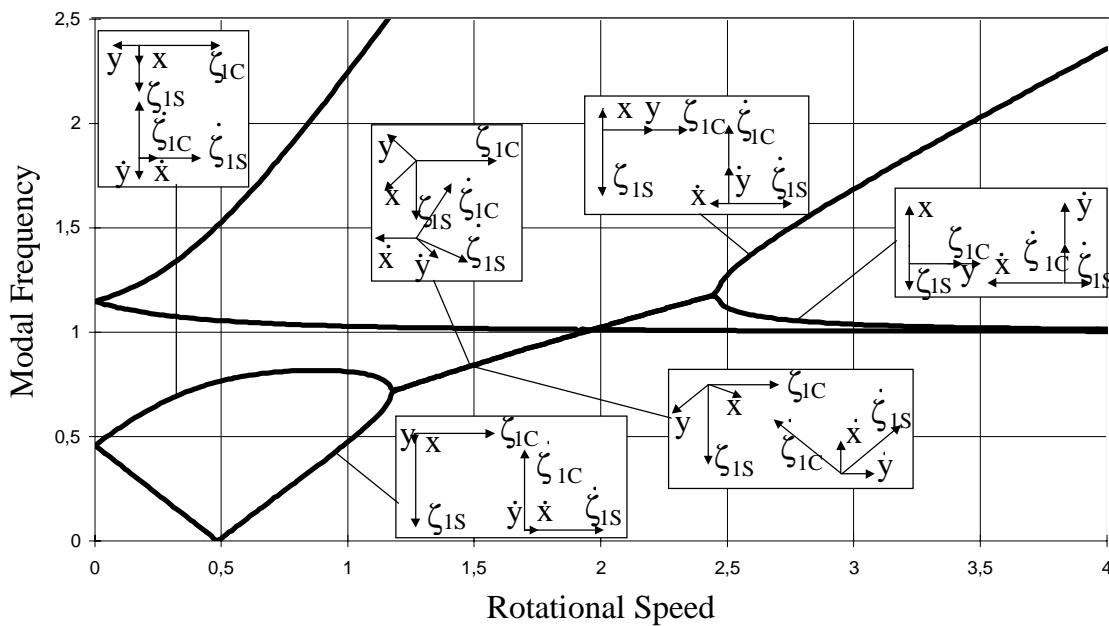


Figure 11 - Phasor diagrams for a three-bladed soft inplane hingeless rotor in the presence of an isotropic hub in rotating frame,  $x$  and  $y$  body modes.



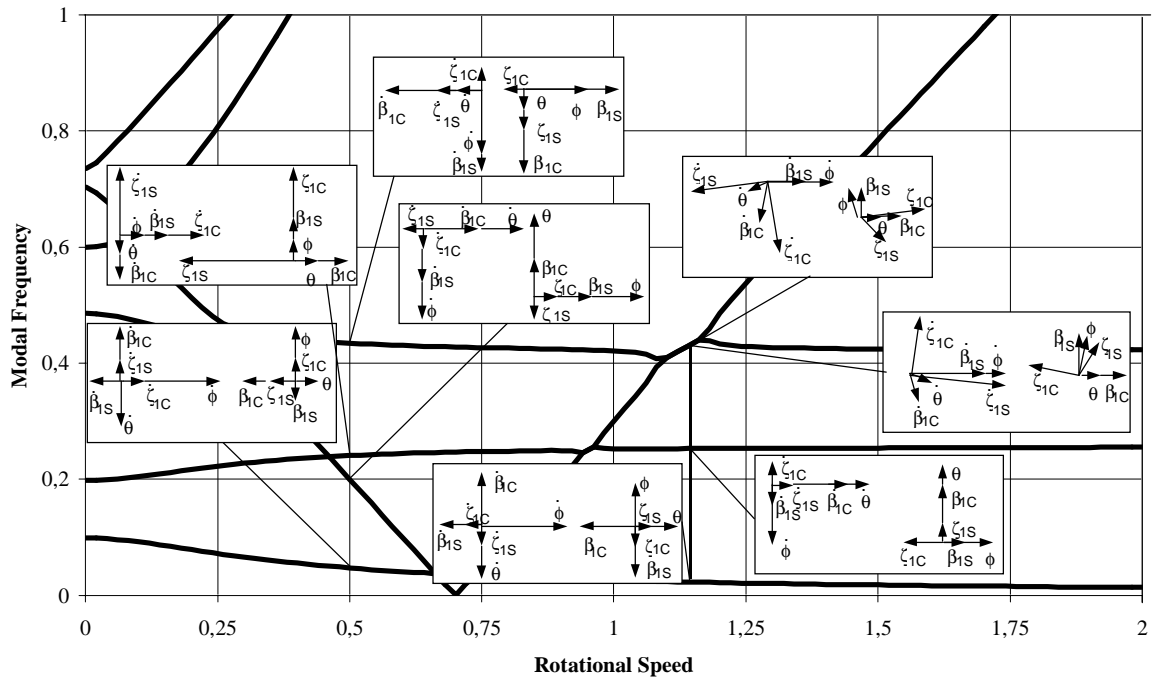


Figure 12 - Phasor diagrams for a three-bladed soft inplane hingeless rotor in rotating frame, pitch and roll body modes.

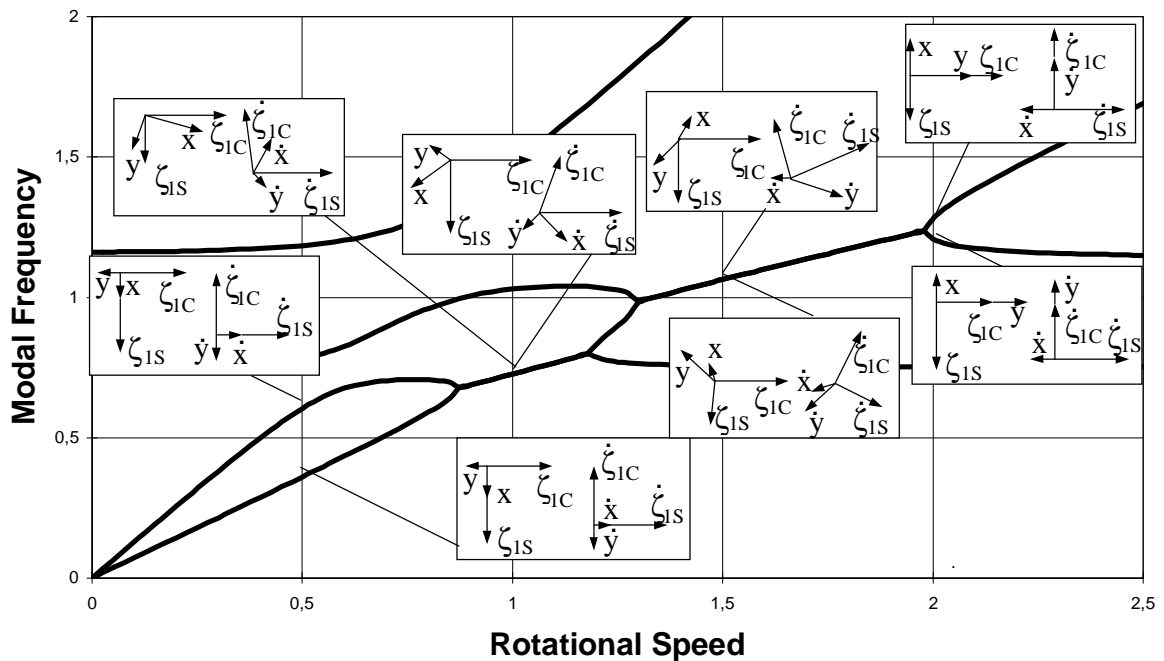


Figure 13 - Phasor diagrams for a four-bladed articulated rotor in the presence of an anisotropic hub in rotating frame.

One observes that for all stability ranges phasors for degrees of freedom from both blade and hub are 90 degrees out-of-phase with respect to their time rate counterparts. To the instability

regions, there are always inphase or out-of-phase components in this context, making clear that instability is present, with an increasing exchange of energy from blade to body motion and vice-versa.

## 5. CONCLUSIONS

This research examines the influence on the helicopter ground resonance of different types of rotor, different placement of damping, and rotor and hub anisotropy. The resonance can occur in both articulated and soft inplane hingeless rotors. In the latter, the progressive low frequency lead-lag mode is involved in the coalescence with body modes. Stiff inplane hingeless rotors are free from this instability. It is shown that an adequate level of external damping placement can prevent resonance. Floquet analysis shows itself as a very handy tool for solving configurations involving both rotor and hub anisotropy. Eigenvector analysis shows phasor diagrams with 90 degrees out-of-phase between degrees of freedom and their time rate counterparts for stability ranges of rotor rotational speed. Corresponding inphase and out-of-phase components are observed for instability regions. This eigenvector behavior can enlighten the physics of the problem since the inphase and out-of-phase phasors for degrees of freedom and their time rate counterparts present in the instability regions can be directly related to the increase of oscillations and feedbacking instability mechanism.

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