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MULTI-REFLECTION LASER INTERFEROMETERS

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Abstract. *For the absolute measurement of the vibration amplitude of a shaker table with the interferometric method, normally two-wave laser interferometers are used. To compensate for table tilting and to increase the frequency limit of the classical minimum-point method, configurations with multiple reflections on the vibrating table are used.*

These configurations can also be employed in other fields, for instance to check stability and thermal drift of structures, if an optical flat surface can be directly prepared or fixed on the structure.

This paper shows the characteristics of a four-reflection two-wave interferometer, which can be applied to one or both sides of the shaker table. In the first case the measuring beam is reflected four times while in the second case also the reference beam is similarly reflected to obtain an 8-fold multiplication of the vibration amplitude.

In particular interferometer intrinsic compensation of table rotations, to avoid shift and tilting of exit beam (important for maintaining the superposition with the reference beam) is illustrated. Tolerances and description of systematic errors are reported.

Keywords: *Laser interferometry, multi-beam interferometer, accelerometers calibration*

1. INTRODUCTION

The calibration of linear acceleration standards is usually performed by means of a suitable vibration exciter (metrological shaker), which should provide a nearly ideal sinusoidal and rectilinear motion to the transducer along its main axis (z) at the amplitudes and frequencies requested.

All imperfections of motion (transversal and rotational motions, harmonic distortions, noise and amplitude instability) will affect the output of any transducer.

Interferometric determination of the axial displacement of the accelerometer reference surface will instead be directly affected only by two rotational motions (around x and y axis), while other motions can cause uncertainties, depending on the method of analysis employed.

The needed displacement is accurately measured by means of a Michelson interferometer only if the measuring beam will impinge this surface (moving mirror) in correspondence of the transducer axis.

If this is not practically convenient, as in the case of single-ended transducers, the systematic Abbe error can be compensated by means of two possible solutions:

- to average subsequent measurements by positioning the beam in a few symmetrical points around the transducer axis (Clark. 1983); this solution is time consuming and the averaged value is valid only if rotational motions are checked and found to be constant,
- to employ a two-wave multi-pass interferometer (Siddal and Baldwin, 1984) , which presents the following advantages and characteristics:

- *s*-fold increase of displacement sensitivity, by means of *s* reflections ($s \geq 2$) of the measurement beam on the vibrating surface
- intrinsic compensation of table rotational motions, if a geometrical symmetry of spots is realised

2. DETERMINATION OF THE VIBRATION AMPLITUDE

The interference optical signal, modulated by the vibration of the reflecting mirror, can be analysed in different ways. Two methods are mostly used by the Primary Calibration Laboratories:

Fringe counting in the time domain. Its classic version is used in the frequency range up to 800 Hz (ISO Standard - 1993) and consists in the determination of the average (over a certain time) number of fringes per vibration cycle N_C ; the peak value of table vibration displacement d is given by:

$$d = N_C \bullet \lambda / 8\, s$$

where λ is the laser wavelength and s the number of reflections.

The main source of uncertainty of this value is represented by noise and odd harmonics content, which can be accounted for only by means of a frequency analysis (modulus and phase) of the transducer output and considering that acceleration amplitude distortion is n^2 times (n = harmonic order) that of displacement.

Minima search of a signal component in the frequency domain. The Fourier transform of the interferometric signal contains:
- the d.c. component
- all the harmonics of the vibration frequency, whose amplitudes may be expressed in terms of $J_n(x)$ (Bessel functions of the first kind and n-th order).

The smallest vibration amplitude d can be obtained by analysing the d.c. component (Robinson and alii, 1987**)** which is expressed by:

$$E_{dc} = A + B \bullet \cos\theta \bullet J_0\,(x)$$

where A, B (with A>B) are constants, $\theta = a \bullet 2\pi / \lambda$ (with a = optical paths difference), argument $x = s \bullet d \bullet 4\pi / \lambda$ (with s and d as defined before).

If the reference mirror is set in low frequency motion so that both values of cos $\theta = \pm 1$ are reached, this component is modulated and will oscillate in the range:

$$E_{dc} = A \pm B \bullet J_0\,(x)$$

By increasing the vibration amplitude from zero the first condition of minimum oscillation of the signal E_{dc} can be reached; it corresponds to the first zero of J_0 (x) when x ≅ 2,4048; higher order zeros, useful for higher amplitudes are tabulated.

In an even simpler way it is also possible to search for the amplitude minima values of any other component: e.g. the method described in ISO 5347/1 standard regards the component J_1 (x) at the vibration frequency for which the first zero is reached: this happens when x ≅ 3,8317, that is about 60% larger than that of J_0(x).

For each component the maximum reachable frequency corresponds to the 1st zero of J_n(x), (that is for $x = x_{1,Jn}$) . It will be determined by:

$$f_{max}=(1/2\pi)[a_{max} \bullet s /d_{1,Jn}]^{1/2}$$

where "a_{max}" is the maximum acceleration reachable by the shaker in the calibration condition and

where $d_{1,Jn}= (x_{1,Jn} /s) \bullet (\lambda /2\pi)$ which in the specific case of $\lambda \cong 633$ nm (red light) gives $d_{1,J0} \cong 121,1$ nm and $d_{1,J1} \cong 193,0$ nm, so that:

$$(f_{max})_{J0} \cong 457 \bullet [a_{max} \bullet s]^{1/2} \quad \text{and} \quad (f_{max})_{J1} \cong 362 \bullet [a_{max} \bullet s]^{1/2}$$

If a shorter wavelength (e.g. green light. $\lambda \cong 543$ nm) is used, the maximum frequency will increase (about 8 %).

3. TWO WAVE MULTI- REFLECTION POLARISED INTERFEROMETERS

In this project a polarised interferometer is used.

It consists of a combination of polarised optics , of simple prisms (corner cube, penta prism or 90° prisms) used to get multiple reflections, with the return beam shifted and parallel to the incoming one. Back reflection to the laser with consequent losses of intensity, frequency disturbances and intensity instability of the laser source are also avoided.

The limited table dimensions (Payne and Booth, 1995) and the presence of the accelerometer itself make it impossible to apply to the table any optical component; the best solution is to use a flat table surface, coated with a high reflectivity silver or aluminium film.

This solution requires that parasitic tilting of the moving element, at the starting and during the calibration, being "sufficiently" small. The initial misalignment of the mirror can be corrected , while the dynamic tilt must be intrinsically compensated.

The characteristic and the structure of the double shaker determine:

- beam size along the path
- superposition of reference and measurement beams at the exit of the interferometer
- requirement that the interferometer be insensitive to the small normal operational tilting of the moving table.

3.1 The four-reflections optical interferometer

In case of a symmetrical double shaker (Payne and Booth 1995) the moving element can be considered as a cylinder. On the face where the transducer is applied and on the opposite face, where masses necessary for the reciprocity method are applied, the laser beam can only impinge on four 4,5 mm diameter spots, located at the corners of a 15 mm side square.

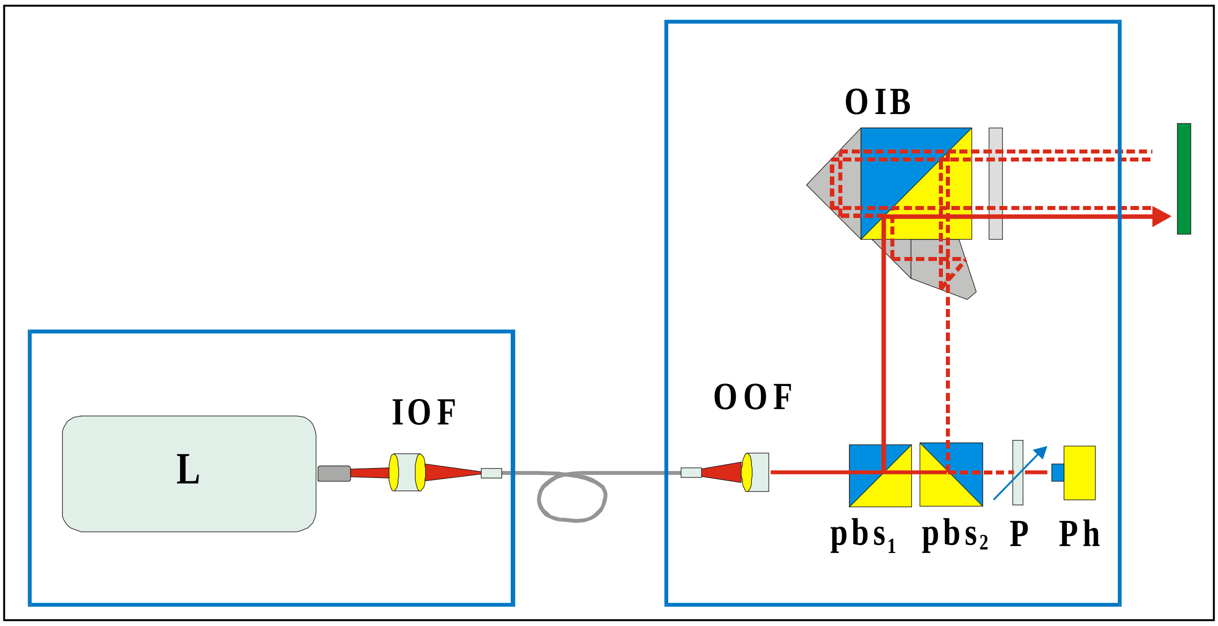

Fig.1- Overall scheme of the interferometer.

The interferometer must produce four very stable parallel beams: i.e. it must behave like an integral illumination system, together with the light source, to be aligned with respect to the external moving mirror. The complete interferometer then consists of two parts (Fig 1):
Base 1, supporting the laser source (L) and the entrance head of an optical fibre, including the matching lenses (IOF)
Base 2, supporting:
- exit head of the optical fibre, with the collimating optics (OOF)
- beam splitter (pbs1) to separate the measuring beam from the reference beam
- interferometer body (OIB)
- beam splitter (pbs2) and polariser (P) to recombine measuring and reference beams on the photodiode (Ph).

Once the alignment of the beam from the optical fibre and the beam splitter, with respect to the body, is done, all these pieces are fixed to the Base 2, in order to obtain the highest stability.

Optical scheme. In the following the interferometric body is described (Fig. 2). It is composed of commercial pieces:
- a large (40mm) polarising beam splitter cube (CPBS), with the PBS splitter surface perpendicular to the base
- a 30mm diameter CC3 corner cube.
- a special back reflecting prism (PPM) composed of a penta-prism , a spacer and a 45° reflecting prism
- a 30 mm diameter Quarter Wave Plate (QWP).

At the exit of the optical fibre, the cube pbs1 separates the beam in two components. The σ component goes through, π enters the PBS cube perpendicularly, is reflected by the partially reflecting surface PBS, then passes through the Quarter Wave Plate (QWP) , and through a Passage 1 in the shaker magnet iron to the shaker table surface. The beam reflected at the table surface (beam 1), passes again through the Passage 1 and the QWP.

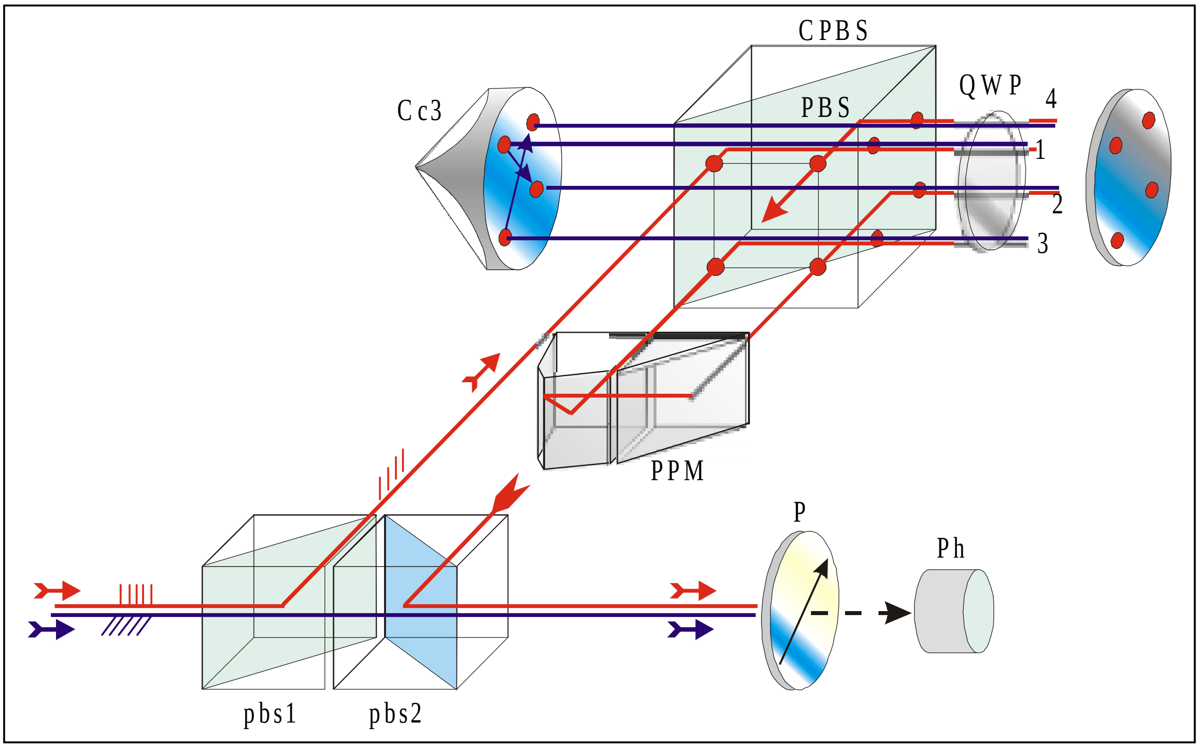

Fig. 2 Optical scheme of the four beam polarised interferometer.

The double pass through the QWP rotates the polarisation of the beam, so it passes through PBS surface to the large centred cube CC3*. The axis of CC3 is on the centreline of the shaker table and the accelerometer. The reflected beam is symmetric with CC3 centreline and is parallel to the incoming beam. It passes through PBS, QWP and Passage 2 in the shaker magnet iron, to arrive at the shaker table surface, diagonally across the first reflection.

The beam reflected at the table surface (beam 2), passes again through Passage 2 and the QWP to PBS. The double pass through the QWP rotates the polarisation of the beam, so it is reflected by PBS and enters the Prism-Penta Prism Retro-reflector PPM. PPM is fabricated so the exit beam is parallel to the entrance beam at the spacing of the passages in the shaker iron.

The polarisation is not changed and so the beam is again reflected at PBS to pass through the QWP and Passage 3 to the shaker table surface.

The beam (beam 3) is reflected at the table surface and again passes through Passage 3, the QWP and through PBS to enter the CC3: the exit beam of CC3 is parallel to the entrance beam and passes through the PBS, the QWP and Passage 4 to the shaker table.

The beam is reflected (beam 4) at the table surface and again passes through Passage 4 and the QWP. This time it is reflected at the PBS and exits the PBS Cube over the Penta-prism/prism retroreflector.

Since the four reflection points on the table surface form a square with the accelerometer at the centre, the accelerometer reads the mean of the four paths.

The main characteristics of this configuration are path stability of exit beam position at the centre of the fourth mirror (corner of the square) and insensitivity to table rotations.

** **Note**. A small rotation of the polarisation vector, introduced by the corner cube CC3, originates a small spurious component (π), which is deviated by PBS and does not interact with the measuring beam.*

Mixing the reference and measuring beams. At the end of the optical path, the beam is deviated by the small cube pbs2, which is adjusted to superpose both measuring and reference beams. Before impinging on the Photodiode, the beams cross a linear polariser P to get interference. In the case of the 4 reflection interferometer, the reference path is only 15 mm long, while the measurement path is about 2500 mm: adjustment for this large difference must be provided. A certain compensation could be obtained by using a lens between pbs1 and pbs2; this compensation is not necessary in the case of the differential interferometer, 4+4 reflections (Fig. 4).

According to the optical scheme, once CC3, PPM and the QWP are correctly positioned these items can be cemented to the Cube PBS, to get a higher mechanical stability.

Sensitivity to tilts of the moving mirror. The ideal representation of the measuring beam path, described above, does not show the actual behaviour of the interferometer, when tilting of the moving mirror is introduced.

The tilting can be decomposed in two components around the vertical (y) and horizontal (x) axes. Observed from the back of the corner cube CC3, the behaviour and trajectory of the measuring beam are shown in Fig. 3a for a rotation around the horizontal axis and Fig. 3b around the vertical one.

The cross symbol (+) represents the ideal position of the reflections on the mirrored shaker table surface for the ideal condition of zero tilt. For the condition of table tilt, the Star symbol (☆) corresponds to the positions of the reflections on the mirror and the dot symbol (●) corresponds to the position of the beam on the face of CC3. The dashed arrows show the deviation introduced by CC3, while the dot-dash arrow shows the deviation by PPM.

<u>Rotation of the mirror around the horizontal axis (Fig. 3a)</u> :

In position 1, the star (☆) on the (+) represents the arriving beam impinging at the ideal location on the mirror. Note that this beam arrives parallel to the (z) shaker axis. The dot (●) is the image on CC3 of this beam shifted by the mirror rotation.

In position 2 the dot (●) is the location of the beam on the face of CC3, after the internal reflection in CC3. The beam reflects on the mirrored table at (☆) and leaves parallel to the *z* axis.

This beam is then reflected to PPM, in which is deflected sideways, then goes on to (☆) on the mirror of position 3 and on to (●) on the face of CC3.

In position 4, the dot (●) shows the location of the beam on the face of CC3 after the internal reflection and the star (☆), superimposed on the (+) shows the exit beam at the desired ideal location on the mirrored table, leaving the table parallel to the *Z* axis.

The same description can be applied to Fig 3b, in which tilting around the vertical axis is considered.

As these schemes show, limited rotations of the table do not cause any tilting and displacement of the exit beam from ideal condition without tilting. If Δs is the shift of the beam 1 on CC3, caused by tilting , it doubles as shown in 2 and 3 positions : in the case of a Double Shaker having as reflecting surfaces four spots 4,5 mm diameter, a 2mm diameter beam can be displaced only up to 1,25 mm corresponding to Δs = 0,62mm. If the distance between the interferometer body and the mirrors is 300 mm, the allowable tilt angle is 200 arc seconds. Some particular working characteristics must be considered:

- The beams reflected towards CC3 are inclined, so they need a corner cube to be back reflected parallel to each other.
- When the input beam enters perpendicularly to the entrance face of the cube PBS, the beam reflected by PBS to PPM also is perpendicular.

If we position a penta-prism so that its faces are perpendicular to the incidence plane of this beam, all inside reflections occur in the same plane. This allows the use of the penta prism, attached to the 90° prism, since this PPM prism has the following properties (not pertaining to the corner cube) fundamental for obtaining the stability of the exit beam:

- it back reflects the beam parallel to the incoming beam, with a separation spacing which is constant. This separation can be obtained equal to 15 mm through the exact definition of the geometrical dimensions of the penta-prism/prism retroreflector
- the optical path inside PPM is constant

Using a corner cube or a 90° prism instead of the PPM, does not produce the exit beam insensitivity to shaker table tilting.

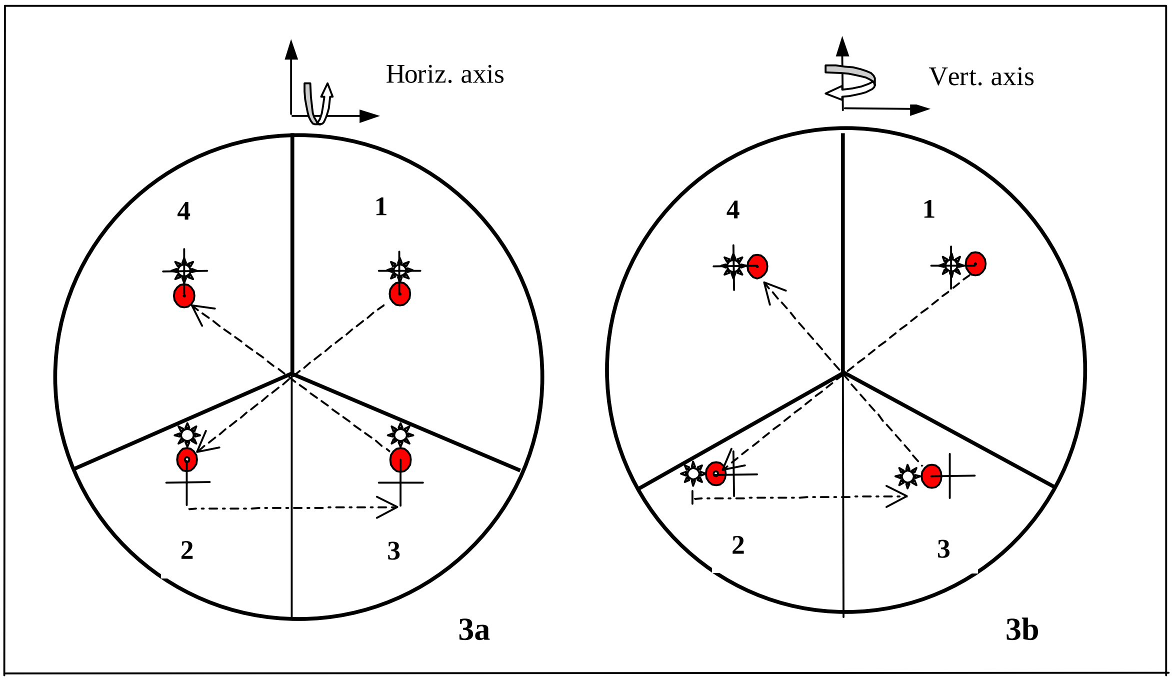

Fig. 3. Positions of the beam spots on the table and the corner cube.

3.2 The eight beam interferometer.

In the differential version, both beams of the interferometer are active, with four symmetrical reflections on each side of the shaker table. In order to couple the two interferometer parts , the polarisation of the direct beam and of the beam coming from the first part, are both 90° rotated by means of two half wave plates, as shown in Fig. 4.

This makes it possible the deviation of the direct beam to the second interferometer body and the correction for small polarisation rotations introduced by parasitic reflections.

With this interferometer the interference period λ is proportional to $\lambda/16 \cong 40$ nm for red light. Compensation for tilting is maintained.

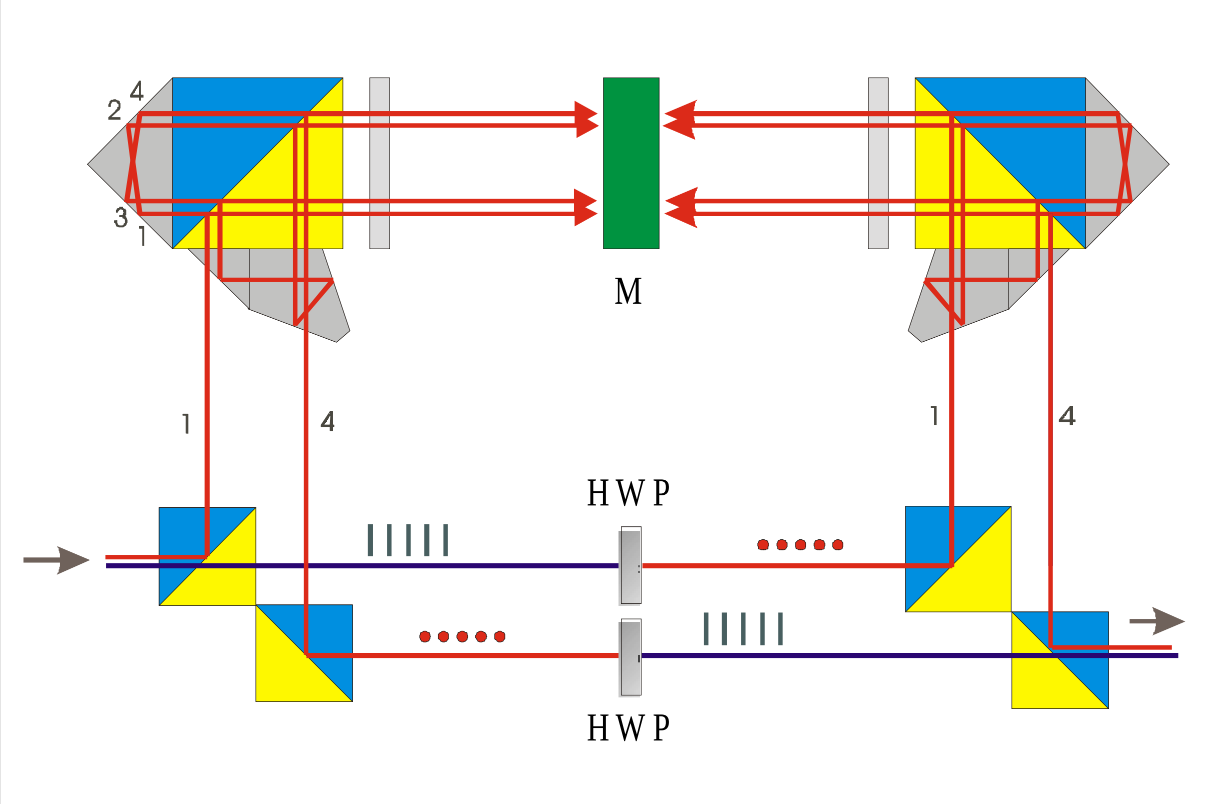

Fig. 4. Overall scheme of the 4+4 reflection differential interferometer: M= shaker table with two mirrored surfaces; HWP is a Half Wave Plate.

4. SOURCES OF SYSTEMATIC ERRORS

The absolute determination of the vibration amplitude requires the accurate definition of the disturbance quantities. Even if the effects of tilting are compensated, a secondary effect on the measurement uncertainty is due to a lack of symmetry of the four beam spots around the transducer. Other possible sources of systematic errors in the interferometric measurement are to be considered only if the desired calibration uncertainty is better than 100 ppm.

They are described in the following.

4.1 **Laser source**.

Wavelength stability: 1ppm.

Intensity stability: short term stability is high; however measuring methods are insensitive to intensity variation.

Beam deviation: a low deviation of the exit beam from the laser occurs in the warming up phase. At the exit of the optical fibre, which can be considered a spatial filter, this effect is negligible. The maximum difference of 1ppm between measurements carried out at the beginning and after the warm up period can be estimated.

Wave front distortion: in principle the distortion of the wave front does not influence the interference signal , if the mirror moves parallel to the reference surface without shifting. In the interferometer scheme this is true, since the incoming and exit beam are parallel. More dangerous is the presence of interference patterns due to spurious reflections from the optical surfaces and residual components of the crossed polarisations, because of a non perfect quality of optics. Distortions of the optical signal of the order of some nanometers have been detected in an horizontal double pass interferometer (Basile, Becker and alii, 1993) used for displacement measurement: this kind of distortion in negligible in very low amplitude vibration measurements

Beams superposition: in case of oblique superposition of interference beams, a residual optical fringe in the interference field of two beams (2mm diameter) corresponds to an inclination angle between two wave fronts of about $1x10^{-4}$ rad. The systematic error associated is lower than $1x10^{-8}$.

4.2 Optics

Optics quality can influence the optical path, because of inhomogeneity of the material, when beams are tilted. Experimentally it would be possible to evaluate this error, together with the Abbe error, including them in the estimation of tilt compensation.

Optical surfaces. A $\lambda/10$ planarity can be considered a good compromise to have locally an ideal flat surface.

4.3 Characteristics of the optical components

Cube polarising beam splitter. This cube does not require special manufacturing, since its alignment is obtained through the adjustment of the base2. Anti reflection coating is required for three faces, if the corner cube, penta prism and quarter wave plate are not cemented.

Corners cubes. The deviation between entrance exit beams should be lower than
2 arc sec.

Penta prism and mirror. The tolerances for the penta prism and the 90° prism attached to it are very strict. The separation between entrance and exit beams must be equal to 15mm ± 0,1mm and their parallelism must be better than 10 arc sec.

Small cubes polarising beam splitter. Only a good parallelism between the entrance and exit face of the second cube polarising beam splitter, recombining the reference and measuring beams, is required: 5 arc sec.

5. CONCLUSIONS

The important application of multi-reflection interferometers to shakers, to extend the frequency range in the calibration of accelerometers has been underlined. The optical schemes of two interferometers with four and eight reflections have been shown. Their insensitivity to tilts and the stability of the interfering beams make possible their application to simple shakers or double shakers (where both sides of the table can be mirrored) , even in the case of severe geometrical conditions. The important characteristics of a particular retroreflector prism, composed by a penta-prism and a 90° deflecting mirror, have been shown: the constant spacing between the incoming and reflected beam and the constant internal optical path of the beam.

This preliminary theoretical part of the program is to be followed by an experimental part.

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