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DIRIGIBILITY STUDY APPLIED TO HUMAN POWERED TRICYCLES USING CHAINSET TRANSMISSION AND CONTINUOUSLY VARIABLE TRANSMISSION

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Abstract: *This work is basically a dirigibility study applied to HPT - Human Powered Tricycles. The behaviour of the vehicle is simulated in a track with compacted floor, contend curves, straight line, up and down grades. A simplified mathematical model is used to model the system (vehicle + cyclist). As the system of equations obtained is extremely coupled and non-linear, it is necessary to use numeric procedures to solve the equations and to obtain the dynamic behaviour of the system during its path in the racetrack. The influences of the track in its stability, the behaviour of the vehicle using chainset transmissions and CVTs (Continuously Variable Transmission) are also analysed.*

Keywords: *Chainset Transmission, CVT, Energy Expenditure, Pedaling Frequency.*

1. INTRODUCTION

The HPT is something close to a conventional bicycle, see Fig. 1, with two steering wheels in front and a driving wheel behind. The position of the cyclist is reclined in the vehicle, with the legs ahead of the body (Werninghaus, 1997). This vehicle is designed for great distances in racetracks with compacted floor and its main advantages when compared with a conventional bicycle are: the cyclist's larger comfort and safety, smaller front area, lower center of gravity, and smaller effort in the pedals (Beck *et al.*, 1996 and Becker *et al.*, 1998).

As one of the main objectives of this type of vehicle is to provide a larger comfort to its user, the use of CVTs instead of chainset transmissions, usually applied in bicycles and tricycles, is proposed. The CVT provides a frequency of pedaling practically constant during the whole path (Forti, 1997). The mathematical model of the vehicle is made through the analysis of its free-body diagram and the use of the Newton-Euler's equations (Becker, 1997 and Becker *et al*., 1998). A study about longitudinal and lateral stability of HPT can be found in Becker *et al*. (1998).

Figure 1 – Photo of the Human Powered Tricycle and the cyclist.

2. DYNAMIC ANALYSIS

The dynamic analysis is based on the previous works about stability of tricycle vehicles (Chang & Lee, 1990, Raman *et al.*, 1995, and Becker *et al*., 1998). The model used to represent the behavior of the tires is a lineal model, based on Huston *et a*l. (1982). Through Newton - Euler's equations, the system of differential equations that governs the behavior the tricycle is obtained. To obtain these equations, the following simplifications are assumed:

- The HPT is modeled as a rigid body, damping and stiffness are not considered in the model of the vehicle;
- Mass center of the HPT is low enough that movements in pitch and roll axes can be neglect;
- The HPT is represented as a triangular plan and,
- The HPT is considered symmetrical with relation to the longitudinal axis.

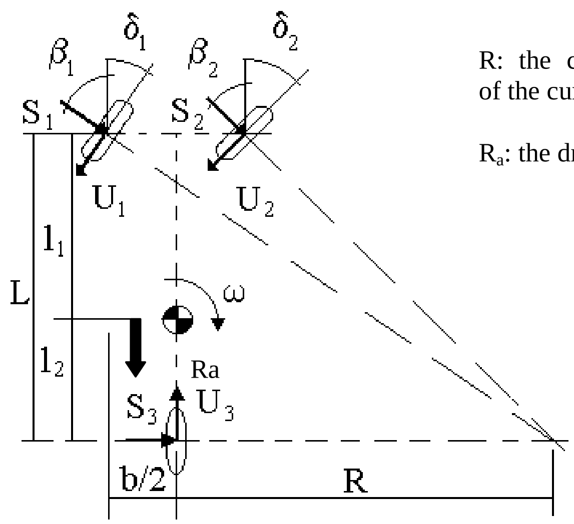


Figure 2 – Free-body diagram of the Human Powered Tricycle.

After these simplifications, the motion of the HPT can be considered as a rigid body motion. Only movements in the longitudinal, lateral and yaw axes are considered. The cyclist supplies to the system, through the pedals, the force (F_p) that discounted the rolling resistance, results in the longitudinal force (U_i). The friction between wheels and soil is directly proportional to the normal reaction (P_i). The direction of the motion of the vehicle is given through the steering of the front wheels of an angle $\delta_i(t)$ and the use of the Arckemann geometry. When the vehicle turns, lateral forces of soil reaction (S_i) appear perpendicularity to the plans of the wheels (W_i) in theirs contact points with the soil. The lateral forces are functions of the slip angles (ψ_i). These angles are defined as the angles between the plans of the wheels and the current direction of motion of the HPT. And the momentum of inertia, represented by I_Z.

2.1 Lateral Forces

The lateral forces applied in the wheels (S_i) are a function of the slip angles (ψ_i) and the steering angles (δ_i) (Becker, 1997):

$$S_1 = C_{\psi F}\left(\delta_1 - \frac{V_Y + l_1\,\omega_Z}{V_X + \frac{b}{2}\omega_Z}\right) \quad S_2 = C_{\psi F}\left(\delta_2 - \frac{V_Y + l_1\,\omega_Z}{V_X - \frac{b}{2}\omega_Z}\right) \quad S_3 = -C_{\psi T}\left(\frac{V_Y - l_2\,\omega_Z}{V_X}\right) \tag{1}$$

2.2 Longitudinal Forces

The longitudinal forces in the system are: driving force and dissipation forces (rolling resistance and drag forces). The driving force F_m is directly proportional to the force applied in the pedals by the cyclist, as it can be observed below in the Fig. 3.

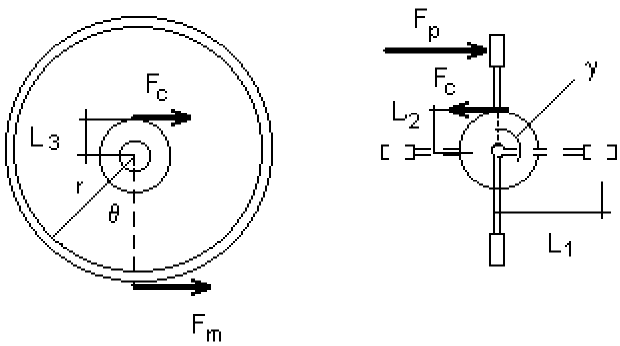


Figure 3 - Relation between the force applied on the pedal (F_p) and the horizontal component of the force in the rear wheel (F_m).

For chainset transmissions, L_3 and L_2 vary to change the transmission ratios. For CVTs, L_3 and L_2 are fixed and represent the parameter denominated *secondary reduction* that fastens the work zone of the transmission. In the practice, losses occur during the transmission of the force from the pedals to rear wheel, Eq. (8). For chains system, the losses are between 3 and 5% (if the chains are in good conditions and perfectly aligned). These loses interfere the efficiency of the system (ef) as in the respective equation.

$$F_m = F_p\left(\frac{L_1 rt}{r}\right)ef \tag{2}$$

Where: $L_3/L_2 = rt$

In this paper, an efficiency of 95% was adopted for CVTs and for chainset transmissions (Forti, 1997).

$$F_m = \left[\left[C_1 + C_2 \sin\left(2\,rt\,\theta - \pi/2\right) \right] \left(\frac{l_1\,rt}{r} \right) ef \right] \quad (3)$$

Where: $C_1 = \dfrac{P}{Freq_{pedal}\ Length_{pedal}\ 2\pi}$ and $C_2 = \dfrac{C_1}{2\pi}$

The value of C_2 is based on the studies of Soden & Adeyefa (1979). However, generally, the cyclist doesn't feel this variation due to the inertia of the system. Just when the resistance forces are considerable, for example the ascent of a ramp, the variation of the force in the pedals and the speed of the HPT during a cycle of pedaling are felt by the cyclist.

Frequently, in the practice, the magnitude of the dissipation forces can be represented in certain intervals of speed as:

$$F_d = a \,.\, v^{\,n} \quad (4)$$

The roll resistance depends on several factors: the deformations of the tires and of the racetrack; the friction of the bearings and the friction of the tires on the racetrack; the adhesion of the tires to the racetrack and the irregularities of the racetrack. Of all factors, the deformations of the tires and of the racetrack have larger influence. In this paper, to simplify, the follows equations was adopted, for the roll resistance (Becker *et al.* 1998):

$$R_1 = \frac{\mu m g l_2}{2L} \qquad R_2 = \frac{\mu m g l_2}{2L} \qquad R_3 = \frac{\mu m g l_1}{L} \quad (5)$$

Where: μ – roll resistance coefficient.

The drag force of an object moving slowly through a flowed is an example of viscous force. The drag force, in this particularly case called resistance of the air (R_a), ρ, Ar and ν, represent respectively, the density of the air, the front area to the motion and the motion speed.

$$R_a = \frac{1}{2} C \rho A r v^2 \quad (6)$$

Prampero *et al.* (1979), knowing that the Reynolds' number is practically constant in the speed zone used in the cycling (Pugh, 1971) adopted a constant value for the cyclist's posture in the bicycle and for the density of the air, obtaining the follow equation:

$$\frac{1}{2} C\ \ Ar\ \ \rho = 0.2 = k \quad (7)$$

Substituting Eq. (7) in Eq. (6):

$$R_a = k V_X^{\ 2} = 0.2\, V_X^{\ 2} \quad (8)$$

As the front area of the tricycles is smaller when compared with conventional bicycles, the Eq. (8) is used in this paper to represent the resistance of the air as an approach of the real value for tricycles. As this way, the longitudinal forces on the wheels can be represented as:

$$U_1 = \frac{\mu\ m g l_2}{2L} \qquad U_2 = \frac{\mu\ m g l_2}{2L} \qquad U_3 = Fm - Ra - \frac{\mu m g l_1}{L} \tag{9}$$

This way, these equations will be introduced in the system of the differential equations that governs the motion of the HPT. The Newton - Euler's equations:

$$\sum F_X = ma_X = m(\dot{V}_X - V_Y\,\omega_Z) = U_3 - U_1\cos\delta_1 - U_2\cos\delta_2 - S_1\cos\beta_1 - S_2\cos\beta_2 - P_{X\alpha} \tag{10}$$

$$\sum F_Y = ma_Y = m(\dot{V}_Y + V_X\,\omega_Z) = S_3 + S_1\,\text{sen}\beta_1 + S_2\,\text{sen}\beta_2 - U_1\,\text{sen}\delta_1 - U_2\,\text{sen}\delta_2 + P_{Y\alpha} \tag{11}$$

$$\sum M_Z = I_Z\,\dot{\omega}_Z = -l_1(U_1\,\text{sen}\delta_1 + U_2\,\text{sen}\delta_2) - l_2 S_3 + l_1(S_1\,\text{sen}\beta_1 + S_2\,\text{sen}\beta_2) + \ldots \\ \frac{b}{2}(U_2\cos\delta_2 - U_1\cos\delta_1 + S_2\cos\beta_2 + S_1\cos\beta_1) \tag{12}$$

Where $P_{X\alpha}$ and $P_{Y\alpha}$ are the weight vector (W) projections on the slide slop, in other words $P_{X\alpha} = mg\ sin\alpha\ cos\delta$ and $P_{Y\alpha} = mg\ sin\alpha\ sin\delta$

The system of differential equations obtained is extremely coupled and non-linear. So, numeric methods are necessary to solve these equations. In this paper, a program using the software Visual Basic 5.0 was developed for such procedure. The use of Visual Basic software provides a faster program with better interface between software and user, when compared with MatLab.

3. CHARACTERISTICS OF THE TRANSMISSIONS

The transmission continually variable (CVT), type *Ball Variator*, Fig. 4, with transmission ratios varying from 1:3 to 3:1. It can be noticed that none of the transmissions for available bicycles and tricycles in the Market, has such extensive work zone (9:1). However, a fixed extra transmission ratio between the CVT and the rear wheel is used to avoid that the cyclist would pedal in a high frequency and, then improve the comfort. So, as each ratio value of extra fixed transmission varies the transmission ratio of the CVT, this parameter is used to compare the behavior of the CVT as a function of the different work zones. This relation of external extra fixed transmission to CVT was denominated: *secondary reduction* (Forti, 1997).

Table 1 - Work zone for the ratio value - CVTs.

Continuously Variable Transmissions	
Secondary Transmission	Extreme of the Ratio Transmission
0.65	1.95~0.2167

Chainset Transmissions - with relation to the Shimano's chainset transmissions (Shimano, 1995), 21 of the 24 possible transmission ratios were chosen for the simulations.

Another group of 15 speeds was obtained starting from the one of 21 speeds subtracting some ratios that had very close values. Already the groups of 5, 3 and 1 speeds were removed of the central part of the group of 21 speeds (all these possibilities are available in the program).

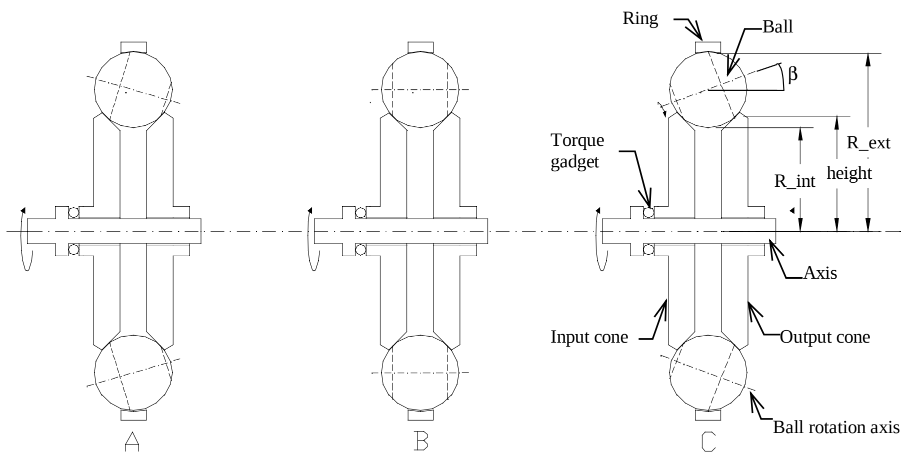


Figure 4 - View of the CVT type *Ball Variator*: (A) reduction, (B) relation 1:1 and (C) amplification.

Table 2 - Work zones for values of the transmission ratios of the chainset transmissions.

Chainset Transmissions	
Number of Speeds	**Transmissions Ratios Values**
21	1.23-0.923-0.889-0.808-0.778-0.692-0.667-0.65-0.609-0.538-0.522-0.5-0.461-0457-0.444-0.391-0.389-0.348-0.333-0.304-0.261
15	1.23-0.923-0.889-0.808-0.778 0.692-0.65-0.609-0.538-0.5-0.461-0.444-0.391-0.333-0.391-0.333-0.261
5	0.923-0.778-0.65-0.522-0.457
3	0.778-0.65-0.522
1	0.65

4. SIMULATIONS

The simulations take in consideration an ideal system for transmission. That means that the changes of speeds for the chainset transmissions and the adjustment of the CVT are instantaneous. In the practice it is known that exists a delay between the changes of speeds in the chainset transmissions, and also a tendency in decreasing the intensity of the force in the pedals during these changes. Already in CVTs the delay of the adjustment depends of the inertia of the mobile parts of the transmission system.

4.1 Strategies for Simulation

Then, it was adopted the following strategies for the numeric process control: For Continuously Variable Transmissions (CVT): The simulation always begins with the transmission ratio in the larger value position; The program tried to maintain the frequency of pedaling in 60 rpm most part of the time. In the practice this can be made using some kind of mechanism to control the velocity (Chironis, 1965), or electrically through sensors and motors. When the system was in the minimum transmission ratio, in the beginning or during any instant of the simulation, the power supply was cut if the force in the pedals was superior as twice the person's weight. Because, if it happened, the cyclist would be making an enormous force on the pedals, and consequently a great effort (Soden & Adeyefa, 1979); When the greater transmission ratio was reached, ends inferior of the intervals, it opted for leaving of supplying potency in frequency of 75 rpm.

For Chainset Transmissions (CST): The simulation always begins with the transmission ratio in the position of larger value; One zone of pedaling was adopted: between 45 and 75 rpm. This zone is the most common for recreational cyclists. The limits of this zone are used as parameters for the speed changes. The zone possessed the medium point in 60 rpm that corresponds the frequency that should be maintained by CST most of the time; When the system was in the minimum transmission ratio, in the beginning or during any instant of the simulation, the power supply will be cut if the force in the pedals was superior as twice the person's weight, see Fig. 5. Because, If it happened, the cyclist would be making an enormous force on the pedals, and consequently a great effort (Soden & Adeyefa, 1979); When the greater transmission ratio was reached, ends inferior of the intervals, and the frequency of pedaling reached the end superior of the respective zone, it left him and to supply potency to the system.

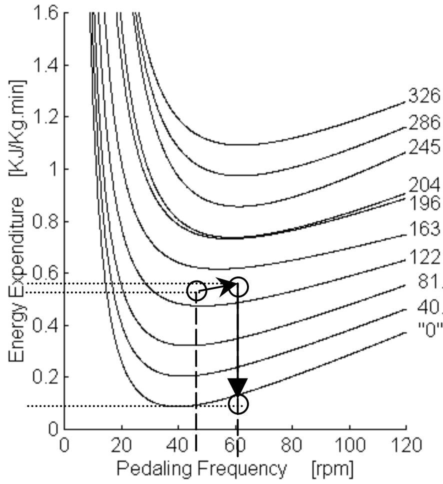


Figure 5 – Energy Expenditure vs. Pedaling Frequency for various Input Power.

It is necessary comment, that any cyclist should have an incredible ability and a fantastic memory to use the speeds of chainset transmissions with 21 and 15 speeds manually as simulated by the program. But, a CVT, with automatic position control, would be adjusted every moment, without the need of the cyclist's intervention.

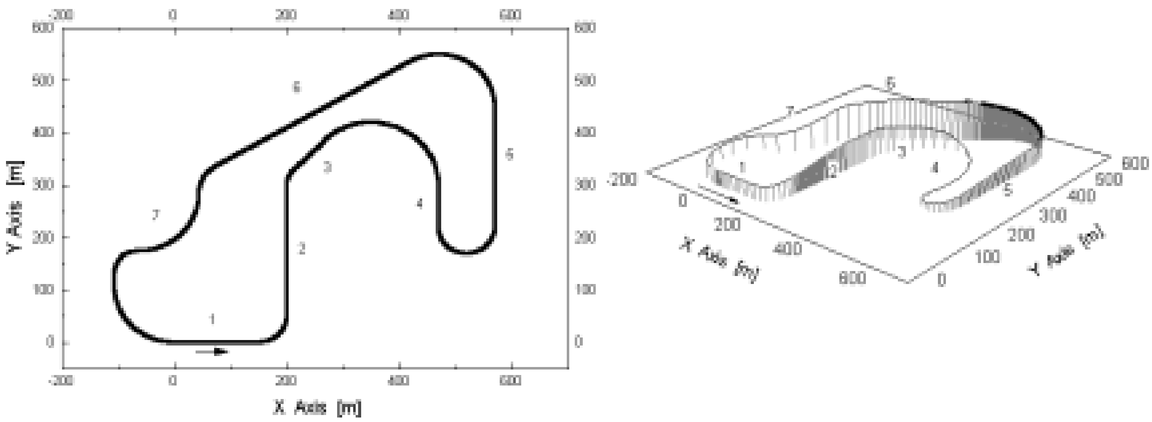
4.2 Characteristics of the Vehicle

The tricycle vehicle HPT used in the simulations has the following constructive characteristics:

Track (b):	0,85 m	l_1:	0.825 m
Wheel base (L):	1,10 m	l_2:	0.275 m
Roll resist. coefficient (μ):	0.025	Input Power (P):	163 and 245 W
Height of mass center (h):	0,50 m	Chainset Transmission:	21 speeds
Vehicle mass:	30 Kg	CVT secondary ratio:	0,65
Cyclist mass:	70 Kg	Momentum of inertia (I_z):	2.93 kg m^2
Weight distribution:	25% (front) 75% (rear)	Diameter of the wheels:	0,48 m front 0,68 m rear

4.3 Characteristics of the Racetrack

The racetrack used to simulate the HPT behavior has 2,437 m and it is showed below. The track is compacted floor, with curves, straight line, up and down grades. The Fig. 5-a shows the race track projection on XY plane and, the Fig. 5-b, a 3D-image of the racetrack.



(a) (b)

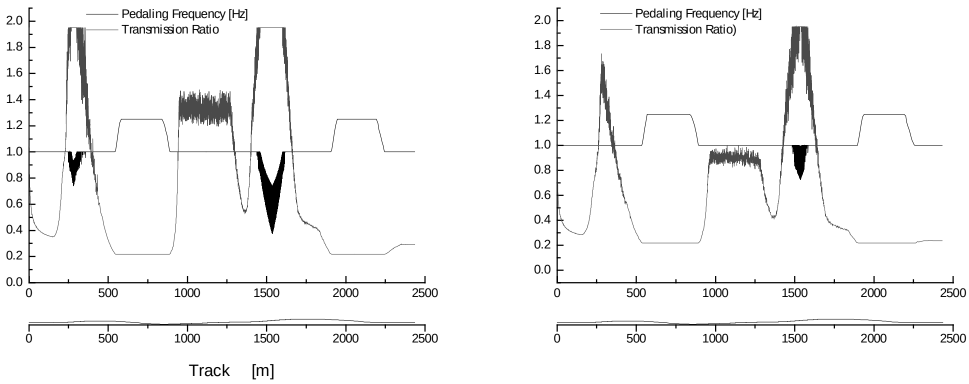
Figure 5 – (a) Racetrack on XY plane and (b) 3D-image of the racetrack.

4.4 Results

The influences of the transmission type on the pedaling frequency and energy expenditure during the whole path in the racetrack are showed on Fig. 6 to Fig. 9 (for 163 and 245 Watts of Input power).

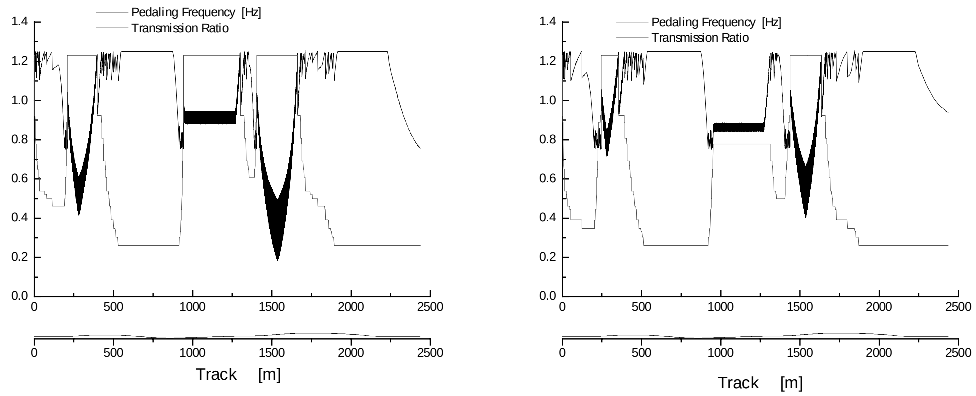
Table 3 – Time to finish the path.

Transmission Type	Input Power	Time
CVT	163 Watts	15 min. 14 sec.
	245 Watts	10 min. 50 sec.
CST	163 Watts	15 min. 41 sec.
	245 Watts	11 min. 02 sec.



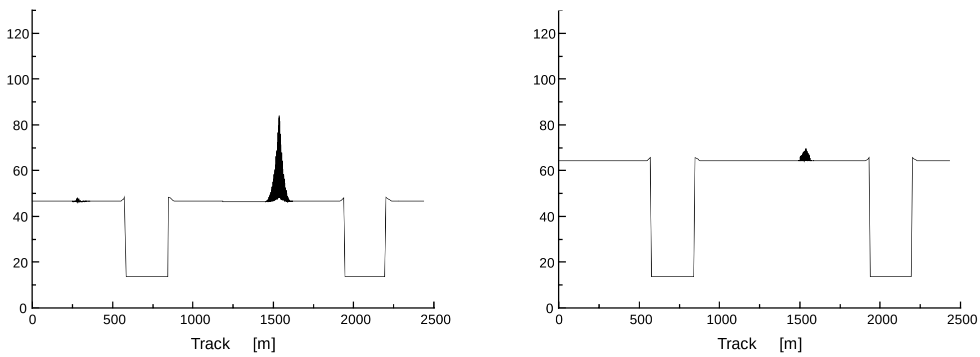
(a) (b)

Figure 6 - Pedaling frequency and transmission ratio for a tricycle with a CVT (3:1:3): (a) 163 Watts and (b) 245 Watts.



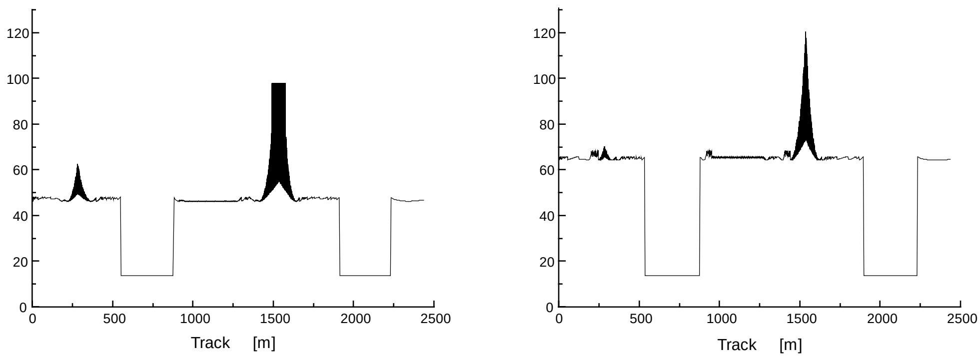
(a) (b)

Figure 7 - Pedaling frequency and transmission ratio for a tricycle with a CST (21 speeds): (a) 163 Watts and (b) 245 Watts.



(a) (b)

Figure 8 - Energy expenditure for a tricycle with a CVT (3:1:3): (a) 163 Watts and (b) 245 Watts



(a) (b)

Figure 9 - Energy expenditure for a tricycle with a CST (21 speeds) (a) 163 Watts and (b) 245 Watts.

5. CONCLUSIONS

The transmission type influences the pedaling frequency, energy expenditure and time to finish the path. Especially for the racetrack simulated, it is possible to affirm that the CVT had a better performance than the CST and improved the user comfort (less oscillations on pedaling frequencies and on energy expenditure). As “said” before, any cyclist should have an incredible ability and a fantastic memory to use the speeds of a chainset transmission with 21 or 15 speeds manually. A CVT, with automatic position control, would be adjusted every moment, without the need of the cyclist's intervention. As future works, it is possible to suggest the development of a mechanism to be used in CVTs for bicycles and tricycles, more studies about energy consume, the use of damping and stiffness effects on vehicle chassis to improve its performance in curves and the use of others racetracks to analysis the influence of the track on CVT vs. CST performance.

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