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THE LIMIT ANALYSIS APPROACH FOR ORTHOGONAL CUTTING

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Abstract. *The main objective of this work is to propose a numerical model to analyse orthogonal metal cutting processes. Metal cutting is a manufacturing process used to remove unwanted workpiece material to achieve the desired dimensional accuracy. In orthogonal metal cutting the direction of relative movement between the wedge-shaped cutting tool and workpiece is perpendicular to the straight cutting edge. It is adopted the model of limit analysis including finite elements and an adaptive mesh refinement process.This strategy is efficient to study the relations between the shear angle and the ranke angle. The proposed model is used to study plane strain problems.*

Keywords: *Limit Analysis, Numerical Methods, Orthogonal Cutting.*

1. INTRODUCTION

Investigators in the metal cutting field have attempted to develop a fairly analysis of the cutting process which gives a clear understanding of the forces, stresses and strains involved and which enables the prediction of the important cutting parameters, complementing the experimental datas. Many times, a previous theoretical analysis is used to improve the experiment parts assemble. This approach can be used in the tool life estimate and in the cut power calculus trying to optimizate the process.

Advances in computer technology have been enhanced the use of the numerical methods to study metal forming processes (Tyan and Yang 1992). The finite element analysis has been widely applied in these process studies, mainly in complex shapes, where the mathematical treatment becomes rapidly complicated.

The orthogonal cut process is considered a common and widely utilized process for many industries because it gives quickness and a workpiece high quality surface finish. Several models to describe the process have been developed; some have been fairly successful in describing the process, but none can be fully substantiated and definitely stated to be the correct solution. Thus, while none of the analyses can precisely predict conditions in a practical cutting situation, the analyses are worth examining because they can qualitatively explain phenomena observed and indicated the direction in which conditions should be changed to improve cutting performance.

In the analytic approach, proposed to plain strain conditions, there is confliting evidence about the nature of the deformation zone in metal cutting. In other words, what is the stress behavior along this deformation zone? And along the tool-chip interface? What is the influence between the shear angle ϕ and the friction angle β? Must we consider contact and friction condictions ? This has led to two basic schools of thought in the approach of the analysis. Many workers, such as Piispanen, Merchant, Kobayashi and Thonsen, have favored the thin-plane (or thin-zone) model. Others such as Palmer and Oxley, and Okushima and Hitomi, have based analysies on a thick deformation region(Johnson and Mellor, 1973, Armarego and Brown, 1969 and Tyan and Yang 1992).

Some of these questions can be answered by the very simplificated frictionless model used in the present work. Obviously, the tool-chip interface friction is one of the most important parameters in this process, however, some characteristics of the global process behavior can *a priori* be analysed without this parameter consideration. For instance, the nature of the deformed zone, the stress behavior along the shear plane and the tool-chip contact length .

In this work development, the main goal is to verify the viability of modeling an orthogonal metal cutting, with the limit analysis approach associated with the finite element method, exploring an adaptive mesh refinement strategy to localize plastic regions(Borges *et al*, 1998). Plain strain problems are also being studied for that accomplishment. This preliminary study dispenses the friction boundary condition, so it is only done a qualitativy analysis. The model presented herein may point out the way of improving the inadequated aspects of the current models as, for example, the controlled contact length model. From this first numeric aproximation we intend to develop a model to introduce more real cut conditions, including friction.

2. LIMIT ANALYSIS

The metal conforming for steady-state process, utilizes an Eulerian formulation; in other words, the model description is refered to the actual configuration of the body defined by the conforming tool or cast. For orthogonal cutting this configuration is not obvious and it is the main issue discussed by many authors in order to propose analytical or numerical models. See next section for more details.

The load acting under the workpiece that assures a steady–state velocitiy field is related to a static and plastic admissible stress field. These flow conditions state the development of a plastic flow under constant loading and are the same that characterize the incipient plastic collapse phenomenon experimented for elastic-ideally plastic materials.

Under the assumption of proportional loading, the limit analysis problem consists in finding a load factor α such that the body undergoes plastic collapse when subject to the reference loads F uniformly amplified by α. In turn, a system of loads produces plastic collapse if there exists a stress field in equilibrium with these loads, which is plastically admissible and related by the constitutive equations to a plastic strain rate field being kinematically admissible. Thus, the limit analysis problem consists in finding $\alpha \in I\!R$, a stress field $T \in W'$, a plastic strain rate field $D^p \in W$ and a velocity field $v \in V$ such that

$$D^p = \mathcal{D}v, \qquad v \in V \tag{1}$$

$$T \in S(\alpha F) \tag{2}$$

$$T \in \partial X(D^p) \tag{3}$$

The meaning of these relations and the used notation is explained in the following. Equation (1) imposes that the collapse plastic strain rate is related to a kinematically admissible velocity field v by means of the tangent deformation operator $\mathcal{D}$.

The symbol $S(\alpha\ F)$ in (2) denotes the set of all stress fields in equilibrium with the given system of forces $\alpha\ F$, that is satisfying the Principle of Virtual Power

$$\int_{\mathcal{B}} T \cdot \mathcal{D}v\ \ d\mathcal{B}\ =\ \alpha\ (\int_{\mathcal{B}} b \cdot v\ d\mathcal{B} + \int_{\Gamma_\tau} \tau \cdot v\ d\Gamma\ ) \quad \forall\, v \in\ V \tag{4}$$

where b and τ are body and surface loads respectively, and Γ_τ the region of Γ where tractions are prescribed

The constitutive relation describing an elastic ideally-plastic material is written in (3). The symbol $\partial X(D^p)$ denotes the subdifferential of the plastic dissipation function X, that is, the set of all stress fields such that(Borges *et al*,1996).

$$X(D^{p*}) - X(D^p)\ \ \geq\ \ \int_{\mathcal{B}} T \cdot (D^{p*} - D^p)\, d\mathcal{B} \quad \forall D^{p*} \in W \tag{5}$$

For these materials the dissipation function is related to the set P of plastic admissible stress fields, by

$$X(D^p) = \sup_{T^* \in P} \int_{\mathcal{B}} T \cdot \mathcal{D}v\ \ d\mathcal{B} \tag{6}$$

Frequently the set P is defined as

$$P = \{T \in W' \mid f(T) \leq 0 \quad in\ \mathcal{B}\} \tag{7}$$

where the inequality above is then understood as imposing that each component f_k, which is a regular convex function of T, is non-positive. Then, at any point of $\mathcal{B}$, equation (3) is equivalent to the normality rule $D^p = \nabla f(T)\,\dot{\lambda}$, where $\nabla f(T)$ denotes the gradient of f, and $\dot{\lambda}$ is the $\widehat{m}-$vector field of plastic multipliers. At any point of $\mathcal{B}$, the components of $\dot{\lambda}$ are related to each plastic mode in f by the complementarity condition $\dot{\lambda} \geq 0$, $f \leq 0$ and $f \cdot \dot{\lambda} = 0$ (these inequalities hold componentwise).

The classical extremum principles of limit analysis, that is the kinematical, statical and mixed formulations, can be derived from the optimality conditions (1-3). The discretized versions of these formulations lead to a single type of finite dimensional problem, which can be cast in four strictly equivalent forms, namely the statical, mixed and kinematical discrete formulations, and the set of discrete optimality conditions (Borges *et al*, 1996).

Discret Model The discrete limit analysis problem consists in finding a load factor $\alpha \in I\!R$, a stress vector $T \in I\!R^q$, a velocity vector $v \in I\!R^n$ and a plastic multipliers vector $\dot{\lambda} \in I\!R^m$, such that the system represented by a deformation matrix $B : I\!R^n \rightarrow I\!R^q$ and a convex function $f(T) \in I\!R^m$, undergoes plastic collapse for some load being proportional to a given force vector $F \in I\!R^n$. It is assumed that all rigid motions are ruled out by the kinematical constraints, so that the kernel of matrix B only contains the null velocity vector.

The discretized version of the limit analysis formulation leads to a finite dimensional problem that can be seen as a discrete version of the Eqs.(1-3), that is,

$$B\,v - \nabla f(T)\,\dot{\lambda} = 0 \tag{8}$$

$$B^T\,T - \alpha F = 0 \tag{9}$$

$$F \cdot v = 1 \tag{10}$$

$$f_j(T)\dot{\lambda}_j = 0 \qquad f_j(T) \leq 0 \qquad \dot{\lambda}_j \geq 0\ , \qquad j = 1, \cdots, m \tag{11}$$

A Newton-like strategy for solving this discrete problem is described by Borges *et al* (1996), and is not discussed here. In the same way, no particular emphasis is placed on the adaptive strategy used . Details about this procedure is presented in Borges *et al* (1998).

For limit analysis, the locking characteristics are important in plane strain and in solids with symmetry of revolution. It happens because when using Mises plastic function the exact velocity field is isocoric. These aspects are not discussed in the present work, but it is worth to be mentioned, that the mixed triangular element used is specially created to face the lock problem.This element has six nodes utilized for the C^o-quadratic interpolation of geometry and velocities and it has also three nodes, at vertices, for the discontinuous linear interpolation of the deviatoric stressses. Piecewise constant interpolation for mean stress is adopted (Borges *et al.*,1996).

3. ORTHOGONAL CUT MODEL

An orthogonal metal cutting process for a controlled contact tool is shown in Fig 1. An Eulerian reference coordinate is used to describe the steady state motion of the workpiece relative to a stationary cutting tool.

The model consists in a workpiece of thickness H moving towards a stationary tool at a constant speed while a non-deformed chip thickness t (the cutting depth) is being cut away; in the same way, a deformed chip thickness t_c is machined. A layer of large shear deformation occurs along the plane AB (the shear plane) inclined at an angle ϕ (shear angle) to the horizontal line; α is the tool rake angle and β is the friction angle between the resultant force R and the normal to the rake face. The width of the chip is assumed to be large as compared with the cutting depth t and the chip thichness t_c. This assures the two dimensional plane strain model. The controled contact tool model is adopted by many researchers (Armarego and Brown, 1969 and Tyan and Yang 1992) and is adopted herein. In that case the tool-chip contact lenght (Ad) is previously settled and this lenght is named ℓ. Since a controlled contact tool is used, a full contact of chip with the rake surface is assumed and for the frictionless model the stress shear is zero in this region.

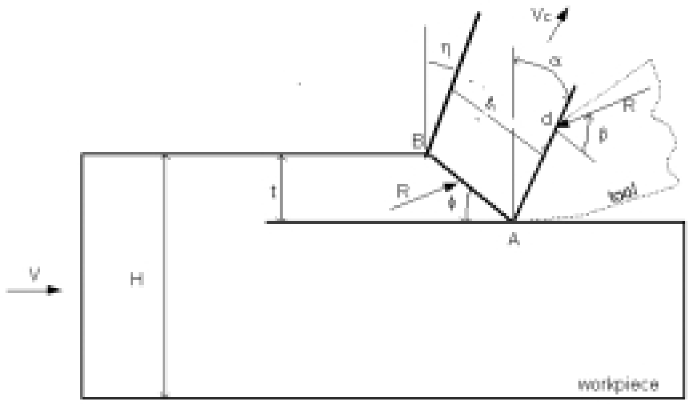

Figure 1: Orthogonal cut scheme.

The geometry of the chip is described by the chip stream angle η and the chip thickness t_c. For the general case, those parameters are variables of the problem. However, in the limit analysis by finite elements, an *a priori* knowledge of geometry of the chip is required . For problems with friction more attention needs to be done to this characteristic of the problem, but for the frictionless study, based on Tyan and Yang, (1992), it is assumed that the chip stream angle η is equal to the rake angle α and the chip thickness t_c is equal to the the cutting depth t. The deformed zone position, defined by the angle ϕ, assumed known in the analytical approach, is obtained by a limit analysis procedure in association with an adaptive mesh strategy.

About the material behavior some basic assumptions are made . First of all, the effects of strain rate and temperature are not considered; the tool is assumed rigid and the workpiece is modelled to be infinitely ductile. The last hypothesis is adequate with the continuous chip formation model adopted. For most metals the work-hardening rate falls to small values for large strain and so it reaches a near constant saturation stress. The high strain rates that accompany the machining operation are said to raise the yield strength of the material and make it approximate the idealized plastic material (Johnson and Mellor, 1973). So a Von Mises perfect plastic workpiece in the sense of assymptotic yield behavior is assumed.

4. NUMERICAL APPLICATIONS

Cutting processes have been analysed in this section considering three differet values for the ranke angle α: $\alpha = 10^0$, $\alpha = 30^0$ and $\alpha = 80^0$. For each angle, three diferent cutting detphs are used : t = 0.1H, t = 0.2H, t = 0.3H. For the tool-chip contact length it is used $\ell = 0.2H$. This value of ℓ is proposed by Tyan and Yang 1992.

The finite element meshes were obtained with an adequated adaptive refined process for limit analysis. With this strategy the region of localized plastic deformation is captured and also the shape of the shear plane is estimated. Observing the figure 2 can be concluded that depending on the value of cutting detph and of the ranke angle, the plastic region is a thick-plane and not a thin-plane. For instance, for $\alpha = 80^0$ and cutting detph $t_\ell = 1.5$ is thick-plane, and for $\alpha = 30^0$ and cutting detph $t_\ell = 0.5$ is a thin-plane.

Figure 3 shows the cutting forces for the analysed cases. The relation between the force and the cutting depth presents the same close to linear behavior obtained by Tyan and Yang(1992).

The other parameter for important comparison is the normal stress distribution along

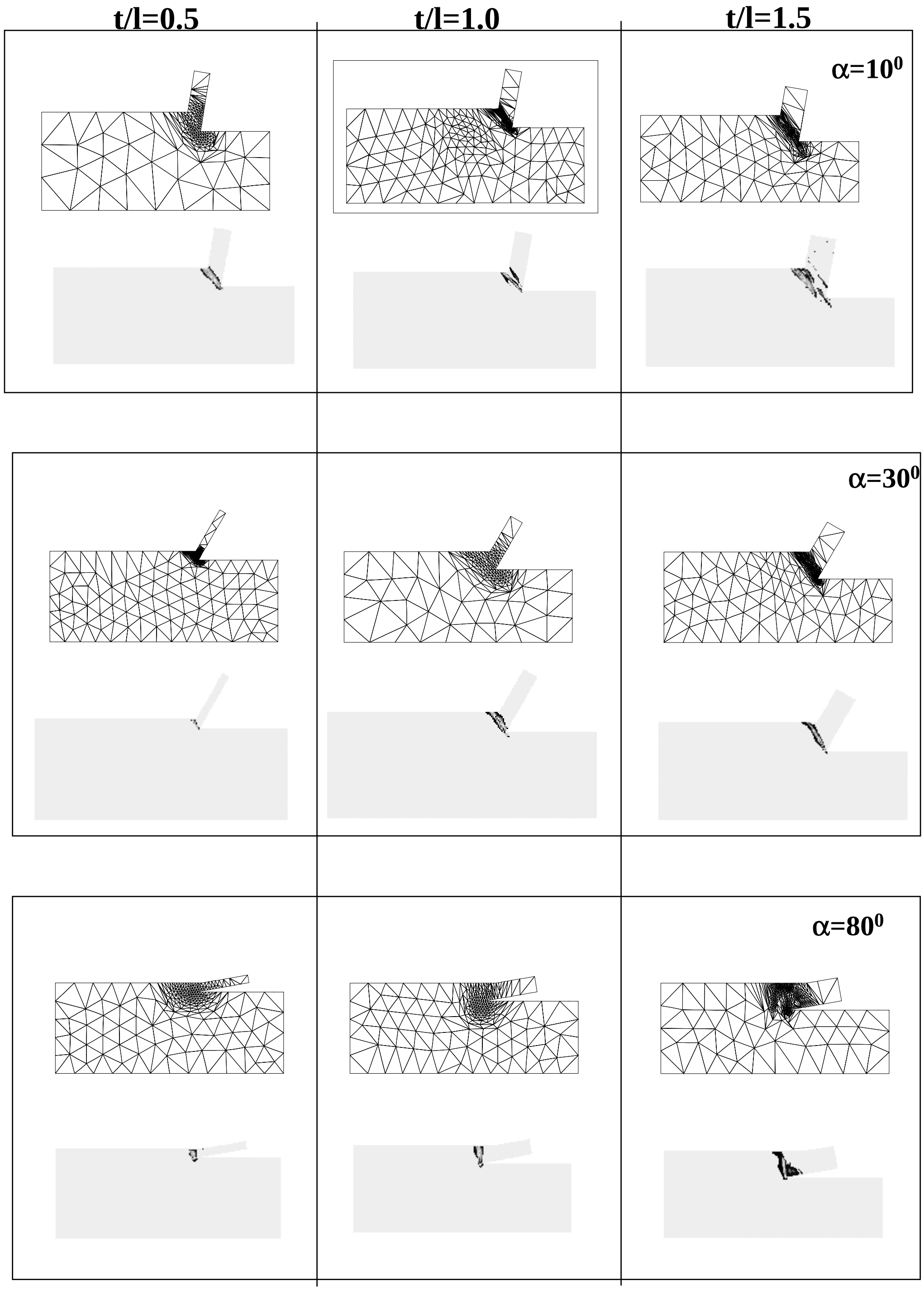

Figure 2: Orthogonal Cut - Finite Element Mesh and Plastic Deformation.

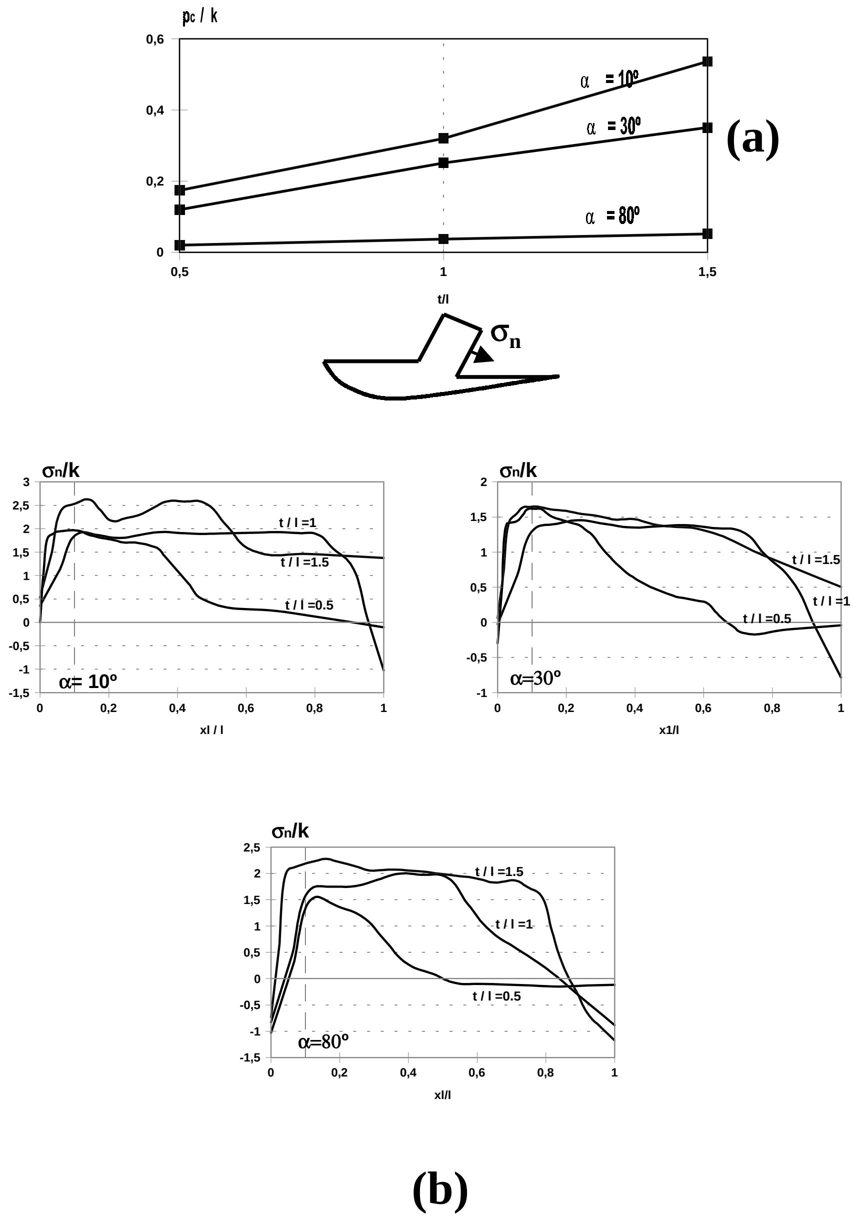

Figure 3: Orthogonal Cut - Cutting Force and Normal Stress under Tool.

the Ad interface shown in Fig. 3b. Even for frictionless case, a theoretical approach considers that the normal stress decreases from a maximum value in A and zero in d and, as proposed by Merchant, this decreased function is linear. This linear behavior is not observed in the numerical cases studied. In most of then a region of nearly constant normal stress followed by a high decreased stress rate is identified. In the analysis of the graphics we did not consider the values of normal stresses at points localized until 0.1ℓ, because this region is very close to the point A where there are high discontinuities in stress. The same behavior obtained in this work is assumed by other researcheres in their theories (Johnson and Mellor, 1973).

The most important result detected in analysing the graphics of normal stress is that the model of controlled contact lenght can not precisely predict the tool-chip contact condiction. For instance near $x_\ell = \ell$ the normal stress is not zero, as expected. In some cases the normal stress is positive showing that the contact length is shorter than necessary. In another case the computed normal stress is negative, indicating that we previewed a high contact length for this case. So, to improve the numerical model of orthogonal cutting is important, not only to consider the friction condition, but also incorporate an unilateral boundary contact condition. This permits to identify the real tool-chip contact length which is a very important project parameter for practical applications.

5. FINAL REMARKS

A numerical method was introduced to study an orthogonal cutting process. The frictionless model is not a real practical condition but make possible to identify the way that the model can be improved to overcome some gaps in the theoretical approaches proposed in the classical literature. In the next works the friction and an unilateral boundary contact condition to the tool-chip contact interface will be incorporated.

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