COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

A MODEL UPDATING-BASED METHOD FOR THE OPTIMUM DESIGN OF BEAMLIKE DYNAMIC VIBRATION ABSORBERS

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Abstract. *This paper is focused on a methodology intended for the optimum design of beams as dynamic vibration absorbers (DVA's). It utilizes a substructure coupling technique exploring frequency response functions (FRF's), which enables to calculate the FRF's of the composite structure (primary structure+DVA), given the FRF's of these two components. The FRF's of the primary system can be derived either experimentally or from an analytical model, whereas the FRF's of the DVA are obtained from a finite element model. The FRF's of the compound structure are used to define performance indexes related to the vibration level of the system over a previously chosen frequency band. This index is then optimized, with respect to the design parameters, which are chosen to be the physical and/or geometrical properties of the finite element model of the DVA. The optimization is performed by using Genetic Algorithms (GA's). The first part of the paper is devoted to the formulation of the substructure coupling technique, the definition of performance indexes and a brief review of GA's. A numerical example is then fully described to illustrate the main features of the proposed methodology.*

Key-word: *Dynamic Vibration Absorbers, Damping, Vibration control, Optimization*

1. INTRODUCTION

In its simplest form a DVA is essentially a mass-spring-damper appendage which, once connected to a vibrating system, called the primary system (PS), is capable of absorbing the vibration energy at the attachment point. Since its invention in the beginning of the century (Frahm,1911), this device has been extensively used for in different types of machines and structures. A comprehensive study on the theory and practice of DVA's is given by Koronev & Reznikov (1993). Recent research developments on passive, semi-active and active DVA's are presented by Sun et al. (1995).

According to the basic principle of the DVA's, the absorbing system has to be designed so that its natural frequency when the connection coordinate is constrained not to move

matches the frequency of excitation. It has been demonstrated that, besides the simplest single-degree-of-freedom type, more complex configurations can be used as vibration absorbers, such as multi-degree-of-freedom lumped-parameter (Ram & Elhay,1996) and distributed-parameter systems (Snowdon & Nobile,1980).

Beamlike dynamic vibration absorbers have been studied by some authors lately. Based on Euler-Bernoulli beam theory, Jacquot & Foster (1997) utilized both exact and Rayleigh-Ritz approximate analysis to obtain optimum design curves for beamlike DVA's applied to single-degree-of-freedom primary systems. Damping was introduced in the form of a complex modulus. Considering both single-degree-of-freedom and continuous beamlike primary systems, Snowdon & Nobile (1980) analyzed the performance of several DVA configurations and verified that beamlike DVA's are broadly effective. Viscoelastic beamlibe dynamic absorbers have been considered by Freitas & Espíndola (1993) in the context of a general theory for the optimum design of neutralizer systems.

Although the use of beamlike DVA's has proved to be advantageous, from the practical standpoint, a major difficulty is to select a set of beam parameters (physical and/or geometrical), satisfying design constraints, so that the DVA will have a natural frequency that coincide with the excitation frequency.

In this paper, a methodology for the optimal design of beamlike DVA's, applicable to discrete multi-degree-of-freedom or continuous primary systems with general damping, is proposed. It utilizes a FRF-based substructure coupling technique according to which the FRF's of the composite structure (primary system + DVA's) are expressed in terms of the FRF's of these two components. The FRF's of the compound system are then used to define scalar performance indexes related to the vibration levels of the system over a previously chosen frequency band. These indexes are optimized, with respect to the DVA parameters, which are assumed to be the physical and/or geometrical properties of the FE model of the DVA. The optimization is performed by using Genetic Algorithms (GA's), taking into account design constraints.

One interesting feature of the proposed methodology is related to the fact that it can handle FRF's of the primary system which can be obtained either experimentally from vibration tests or numerically from an analytical model. In the first case, the optimization is not affected by modeling errors, though the effect of experimental noise has to be dealt with.

In the remainder, the formulation of the substructure coupling technique is first presented, followed by the definition of performance indexes and a brief presentation of the basics of Genetic Algorithms. Then, an example based on a numerically simulated structure is shown to illustrate the main features of the proposed methodology.

2. A SUBSTRUCTURE COUPLING TECHNIQUE BASED ON FRF's

In this section the basic formulation of a classical substructure coupling technique exploring FRF's is presented. It has originally been proposed by Crowley et al. (1984) . More recently, it has been assessed by Otte et al. (1991) and has been used by Rade & Steffen (1999) in the context of the optimization of single-degree-of-freedom DVA's.

Given the FRF's of the two substructures A and B, shown in Fig. 1, it is intended to determine the FRF's of the assembled structure C, obtained by coupling A and B through a set of coupling coordinates. For each configuration, the dynamic flexibility relations are:

$$\{X_A(\omega)\} = [H_A(\omega)]\{F_A\} \tag{1}$$

$$\{X_B(\omega)\} = [H_B(\omega)]\{F_B\} \tag{2}$$

$$\{X_C(\omega)\} = [H_C(\omega)]\{F_C\}, \tag{3}$$

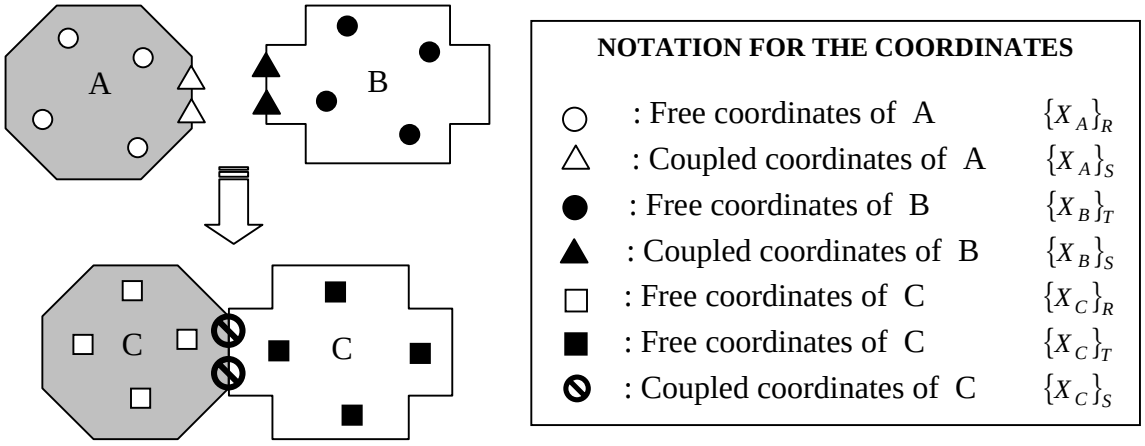


Figure 1 - Coupling of substructures

where: ω designate the forcing frequency, $\{X_A(\omega)\}$, $\{X_B(\omega)\}$ and $\{X_C(\omega)\}$ denote the vectors of harmonic response amplitudes of configurations A, B and C, respectively, $\{F_A\}$, $\{F_B\}$and $\{F_C\}$ are the vectors of the amplitudes of the harmonic excitation forces and $[H_A(\omega)]$, $[H_B(\omega)]$and $[H_C(\omega)]$ designate the FRF (receptance) matrices pertaining to A, B and C, respectively.

Taking into account the partition of coordinates indicated in Fig. 1, Equations (1) to (3) are re-written in the forms (for simplification, dependence on frequency is omitted):

$$\begin{Bmatrix}\{X_A\}_R\\ \{X_A\}_S\end{Bmatrix} = \begin{bmatrix}[H_A]_{RR} & [H_A]_{RS}\\ [H_A]_{SR} & [H_A]_{SS}\end{bmatrix}\begin{Bmatrix}\{F_A\}_R\\ \{F_A\}_S\end{Bmatrix} \tag{4}$$

$$\begin{Bmatrix}\{X_B\}_S\\ \{X_B\}_T\end{Bmatrix} = \begin{bmatrix}[H_B]_{SS} & [H_B]_{ST}\\ [H_B]_{TS} & [H_B]_{TT}\end{bmatrix}\begin{Bmatrix}\{F_B\}_S\\ \{F_B\}_T\end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix}\{X_C\}_R\\ \{X_C\}_S\\ \{X_C\}_T\end{Bmatrix} = \begin{bmatrix}[H_C]_{RR} & [H_C]_{RS} & [H_C]_{RT}\\ [H_C]_{SR} & [H_C]_{SS} & [H_C]_{ST}\\ [H_C]_{TR} & [H_C]_{TS} & [H_C]_{TT}\end{bmatrix}\begin{Bmatrix}\{F_C\}_R\\ \{F_C\}_S\\ \{F_C\}_T\end{Bmatrix} \tag{6}$$

The coupling between the substructures is enforced by imposing the equilibrium of forces and compatibility of displacements at the coupling coordinates, expressed as follows:

$$\{X_A\}_S = \{X_B\}_S = \{X_C\}_S \tag{7}$$

$$\{F_A\}_S + \{F_B\}_S = \{F_C\}_S \tag{8}$$

Introducing Equations (7) and (8) into Equations (4) to (6), after some algebraic manipulations, the following expressions for the sub-matrices appearing in (6) are obtained:

$$[H_C]_{RR} = [H_A]_{RR} - [H_A]_{RS}\left([H_A]_{SS} + [H_B]_{SS}\right)^{-1}[H_A]_{SR} \tag{9}$$

$$[H_C]_{RS} = [H_C]_{SR}^T = [H_A]_{RS}\left([H_A]_{SS} + [H_B]_{SS}\right)^{-1}[H_B]_{SS} \tag{10}$$

$$[H_C]_{RT} = [H_C]_{TR}^T = [H_A]_{RS}\left([H_A]_{SS} + [H_B]_{SS}\right)^{-1}[H_B]_{ST} \tag{11}$$

$$[H_C]_{SS} = [H_A]_{SS}\left([H_A]_{SS} + [H_B]_{SS}\right)^{-1}[H_B]_{SS} \tag{12}$$

$$[H_C]_{ST} = [H_C]_{TS}^T = [H_A]_{SS}\left([H_A]_{SS} + [H_B]_{SS}\right)^{-1}[H_B]_{ST} \tag{13}$$

$$[H_C]_{TT} = [H_B]_{TT} - [H_B]_{TS}\left([H_A]_{SS} + [H_B]_{SS}\right)^{-1}[H_B]_{ST} \tag{14}$$

In the context of the problem examined herein, substructure A can be considered as the primary structure and substructure B can be seen as the DVA's, so that configuration C represents the composite (primary+DVA's) structure. Moreover, assuming that the connection is made through a single coordinate, the number of coordinates of "*S*" type in the equations above is one.

The basic principle of DVA's can be demonstrated by developing Equation (12). For this, all the FRF's appearing in this equation are expressed as ratios of two polynomials in the forcing frequency, as follows (parenthesis are use in place of brackets to indicate scalar quantities, assuming the connection is done at a single coordinate):

$$\left(H_C(\omega)\right)_{SS} = \frac{\left(N_C(\omega)\right)_{SS}}{D_C(\omega)} \tag{15}$$

$$\left(H_A(\omega)\right)_{SS} = \frac{\left(N_A(\omega)\right)_{SS}}{D_A(\omega)} \tag{16}$$

$$\left(H_B(\omega)\right)_{SS} = \frac{\left(N_B(\omega)\right)_{SS}}{D_B(\omega)} \tag{17}$$

In the equations above, the roots of the denominators are associated with the natural frequencies (poles of the FRF's), while the roots of the numerators are associated with the zeros (anti-resonance frequencies) of the FRF's.

Introducing Equations (15) to (17) into Equation (12), one obtains:

$$\left(H_C(\omega)\right)_{SS} = \frac{\left(N_C(\omega)\right)_{SS}}{\left(D_C(\omega)\right)_{SS}} = \frac{\left(N_A(\omega)\right)_{SS}\left(N_B(\omega)\right)_{SS}}{\left(N_A(\omega)\right)_{SS}\left(D_B(\omega)\right)_{SS} + \left(N_B(\omega)\right)_{SS}\left(D_A(\omega)\right)_{SS}} \tag{18}$$

According to Equation (18), the FRF of the compound structure will have, among its zeros, the roots of the polynomial $\left(N_B(\omega)\right)_{SS}$. Note that the zeros of $\left(N_A(\omega)\right)_{SS}$ remain unchanged. Moreover, it can be demonstrated that the roots of $\left(N_B(\omega)\right)_{SS}$ are identical to the

natural frequencies of substructure B when the connection coordinate is grounded (Rade & Silva,1999). Thus, if the absorbing structure B is designed so that one of the zeros of $(H_B(\omega))_{SS}$ coincide with the forcing frequency, vibration amplitudes will vanish at the connection coordinate. However, it may be more convenient to select the DVA parameters so as to guarantee minimum vibration amplitudes over a finite frequency band. This procedure is addressed in the next section.

3. OPTIMUM DESIGN OF DVA's OVER A FREQUENCY BAND

3.1 Performance indexes

In the previous section it was shown how the FRF's of the assembled system can be expressed in terms of the FRF's of the primary system and the FRF's of the DVA. Once the frequency band of interest and the coordinates at which vibrations are to be attenuated have been defined, these FRF's can be used to form performance indexes related to the vibration levels of the compound system in the selected frequency band. Then, these indexes are optimized for the optimal choice of DVA parameters. Assuming that de DVA is modeled by finite elements, the design variables are chosen to be the physical and/or geometrical characteristics of the model. Constraints can be imposed on the values of these variables to meet design constraints.

For illustration, let us consider the case where a harmonic force F_i is applied at coordinate *i* and the response at the coordinate *j* is to be minimized over a frequency band $\omega_L \le \omega \le \omega_U$, by attaching a DVA to either coordinates *i* or *j*. In this situation, some performance indexes can be defined as, for example:

$$J_1(\{P\}) = \frac{1}{p}\sum_{j=1}^{p} Q_k \; abs\left[H_{ij}(\omega_k, \{P\})\right] \tag{19}$$

$$J_2(\{P\}) = max\left\{abs\left[H_{ij}(\omega_j, \{P\})\right]\right\} \tag{20}$$

where *p* is the number of frequency lines in the band of interest, $\{P\}$ designates the vector of design variables and Q_j, *j=1 to p* denote weighting factors ascribed to each frequency line.

Various numerical optimization procedures can be used for the optimization of the select performance index, including the traditional gradient-based non-linear constrained algorithms (Vanderplaats,1984) and Genetic Algorithms (Goldberg,1989). The latter were used in the numerical application described in this paper and is briefly reviewed in the next section.

3.2. Genetic Algorithms – an overview

In this section, the fundamentals of GA's and their basic operators are briefly reviewed. For a detailed description, the reader should refer to Goldberg (1989).

Genetic algorithms are based on the principles of Darwin's evolutionary law, i.e., they are structured random search techniques that mimics the concept of natural selection (Holland,1975).

In its simplest form, GA's comprise three operators: *selection, crossover and mutation*. Two other operations are also considered as operators by some authors: coding and evaluation of the fitness function.

The purpose of coding is to put the variables into a genetic design space. In this work a

binary codification is used to represent each variable V_i as a *b*-bit binary number, which approximates 2^b discrete numbers in the range of the variables, according to:

$$V_i = V_i^{max} + \frac{bin}{2^b - 1}\left(V_i^{max} - V_i^{min}\right)$$

where V_i^{min} and V_i^{max} are the lower and upper bounds of the *i*-th continuous variable and *bin* is an integer number between zero and 2^b-1.

Considering the minimization of an objective function, during the evaluation operation, a proper fitness index is assigned to each candidate set in such a way that the lower the value of the objective function associated to an individual candidate, the higher the fitness index given to it. The algorithm starts with an initial population randomly generated over the whole search space. Each member of the population can be seen as a *chromosome* or a binary string. After codification and initial evaluation, genetic algorithms work iteration by iteration on this population of strings, in a way similar to natural population growth, where each generation is evolved into another through reproduction, making use of the three operators.

The *selection* process is responsible for the choice of which individual, and how many copies of it, will be passed to the next generations. An individual is selected if it has a high fitness value, and the choice is biased towards the fittest members. A way to do that is to use a proportional selection procedure, with the number of copies given by $n = f_i / f_{avg}$, where f_i is the fitness index of the *i*-th individual and f_{avg} is the average fitness index of the whole population.

Crossover takes two strings (parents) from the mating pool and performs a randomly exchange in some portions between them to form a new string (children). The crossover occurs with a probability p_c.

In a binary coding scheme, *mutation* involves switching individual bits along the string, changing a zero to one or vice-versa. This operator keeps the diversity of the population and reduces the possibility that the GA's find a local minimum or maximum instead of the global optimal solution, although this is not ever guaranteed. The mutation occurs with a probability p_m.

4. NUMERICAL EXAMPLE

To illustrate the main features of the proposed method, a numerical application to a structural system simulated by finite elements (FE) is presented. The primary structure is depicted in Fig. 2, consisting in a bidimensional clamped-free frame whose FE model counts 40 Bernoulli-Euler beam elements, each one containing 2 nodes and 3 degrees-of-freedom per node. The total number of degrees-of-freedom of the model is 120. The physical and geometrical characteristics of the primary system are given in Table 1 and the values of its first four natural frequencies are given in Table 2.

It is intended to minimize the vibrations in the y direction at node A, in a frequency band containing the first natural frequency, by attaching a beamlike ADV (BCD) at that node in the way shown in Fig. 2. For the design of the ADV, an initial FE, containing 20 Bernoulli-Euler bidimensional beam elements whose characteristics are given in Fig. 3 and Table 3 is used.

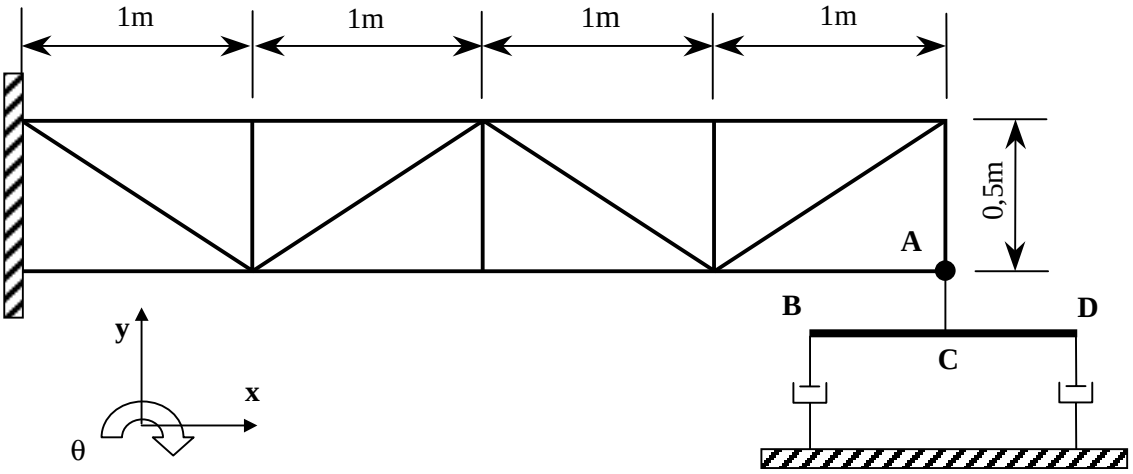


Figure 2 – Characteristics of the FE model of the compound system

Table 1. Physical and geometrical characteristics of the FE model of the primary system

Property	**Value**
material density	7800 kg/m^3
Young modulus	2,1×10^{11} N/m^2
cross section dimensions	2,5 cm × 2,5 cm
cross-section moment of inertia	3,25×10^{-8} cm^4
mass	73,0 kg

Table 2. Values of the natural frequencies of the primary system

Mode	**Natural frequency [Hz]**
1	28,50
2	77,25
3	81,20
4	86,80

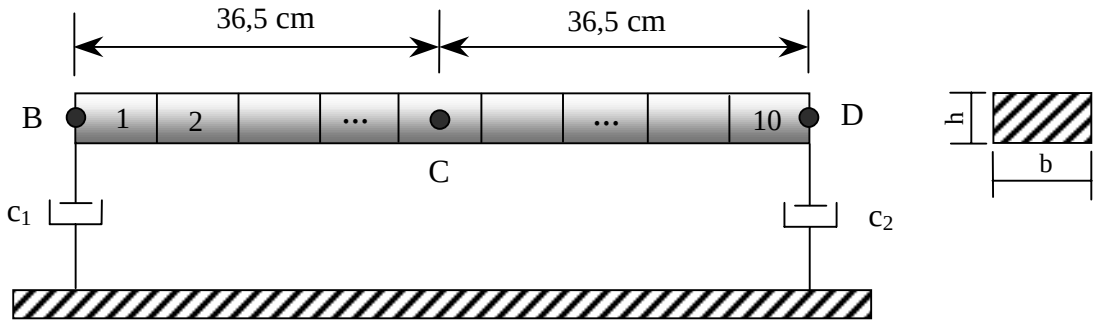


Figure 3 – Characteristics of the FE model of the DVA, divided into 10 macro-elements

Table 3. Physical and geometrical characteristics of the initial FE model of the DVA

Property	Value
material density	$\rho = 7800$ kg/m^3
Young modulus	$E = 2{,}1\times10^{11}$ N/m^2
cross section dimensions	$b\times h = 36{,}6$ cm $\times$ 11,4 cm = 417,2 cm^4
cross-section moment of inertia	$I_z = bh^3/12 = 4518{,}7$ cm^4
viscous damping coefficients	$c_1 = 100$ Ns/m ; $c_2 = 100$ Ns/m
total mass	$m_B = 2{,}54$ kg

For the optimum design of the DVA, the following strategy is adopted: each pair of neighboring elements of the model of the beam are first regrouped into a single macro-element. These macro-elements are numbered consecutively from left as shown in Fig. 3. To each macro-element is assigned a design variable α_i^h, i =1 to 10, which is a dimensionless multiplicative factor intended to modify the height of the cross section of the initial model. Clearly, by applying this factor both the mass and the bending stiffness of the macro-elements are changed. Dimensionless factors α_i^c, i =1 to 2, to be applied to the initial values of the coefficients of viscous damping, given in Table 3 are considered as design variables also. Thus, the problem has a total number of 12 unknown design variables.

The main features of the optimization computations are the following:

• **frequency band**: *[22 – 33 Hz]* , containing the first natural frequency of the primary structure.

• **target FRF**: $H_{AA}(\omega)$ (driving point FRF related to the vertical motion at node A).

• **design constraints**:

- $m_B \leq 3{,}70$ kg (total mass of the DVA must be less then 5% of the mass of the primary structure)
- $0{,}5 \leq \alpha_i^h \leq 3$, i =1 to 10
- $0{,}5 \leq \alpha_i^c \leq 3$, i = 1 to 2

• **performance index**: $J = max\{20\ log_{10}(\ abs\ [H_{AA}(\omega)] / 1\times10^{-6}\ )\ \}$

• **Genetic algorithms**:

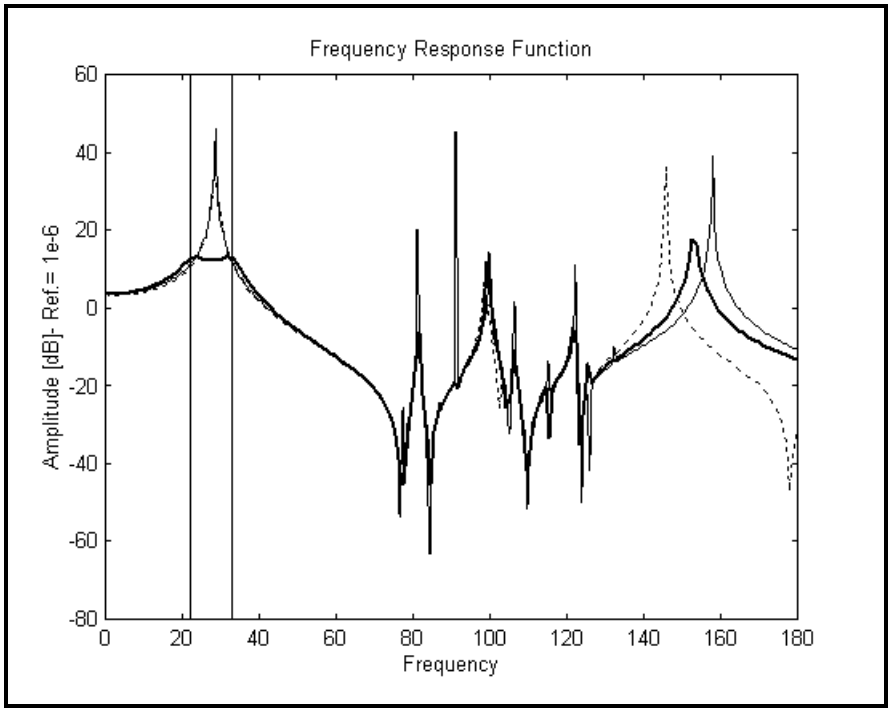
- *population size: 80*
- *mutation probability: 1%*
- *crossover probability: 90%*
- *maximum number of generations allowed: 150*

The optimum values of the design variables obtained are given in Table 4 and the effect of the attachment of the DVA on the amplitudes of the FRF $H_{AA}(\omega)$ can be evaluated in Fig. 5, where the target frequency band is limited by vertical lines. As can be seen, the

attachment of the DVA leads to a significant reduction of the vibration level in the frequency band of interest, with the complete removal of the first resonance peak.

Table 4. Optimum values of the design variables

Design variable	Optimum value	Design variable	Optimum value
α_1^h	1,44	α_7^h	1,91
α_2^h	2,39	α_8^h	2,60
α_3^h	1,33	α_9^h	2,28
α_4^h	1,14	α_{10}^h	0,82
α_5^h	0,50	α_1^c	2,91
α_6^h	1,58	α_2^c	3,00



—— without DVA ; - - - - Initial DVA ; ━━ Optimal DVA

Figure 5 – Amplitudes of the FRF $H_{AA}(\omega)$

5. CONCLUSIONS

A methodology for the optimum design of beamlike dynamic vibration absorbers has been proposed and illustrated with a numerical example. It has been verified that the method can be conveniently used in practical design, since it is able to tackle the optimization problem in a broad sense, taking into account general design constraints. Although the study was limited to beamlike vibration absorbers, the methodology can well be extended to the design of distributed parameter vibration of any geometry.

It has all been verified that the method is computationally since, during the optimization

procedure, the FRF's of the primary system are computed just once, while all the model updating computations are confined to the DVA, whose FE model possesses a moderate number of degrees-of-freedom.

Numerical applications have demonstrated that in spite of being more time consuming than traditional gradient-based methods, Genetic Algorithms have demonstrated to be a very efficient and robust optimization procedure.

In the sequence of this work the following aspects are currently being addressed: a) the incorporation of structural damping in the FE model of the DVA; b) the design of DVA's intended for the simultaneous attenuation of vibration over several discontinuous frequency bands; c) the use of experimentally derived FRF's of the primary system.

ACKNOWLEDGMENTS

S.S. Cunha Jr. gratefully acknowledges the support of the Brazilian Ministry of Science and Technology through the award of a CNPq scholarship.

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