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# DYNAMIC AND CONTROL OF MULTIBODY SYSTEM WITH FLEXIBLE APPENDAGES

**ROBERTO GARCIA NEGRÃO, LUIZ CARLOS SANDOVAL GÓES**
Instituto Tecnológico de Aeronáutica - ITA-CTA
Divisão de Engenharia Mecânica-Aeronáutica - IEMP
12228-900 - São José dos Campos - SP, Brazil
**ÁLVARO MANOEL DE SOUSA SOARES**
Divisão de Engenharia Mecânica
Universidade de Taubaté– UNITAU –Taubaté _ S.P., Brazil

***Abstract****. The objective of this work is to describe the design and the implementation of an experiment to study the dynamics and the position control of an unconstrained multi-link flexible structure. The experimental apparatus was designed to be representative of a flexible space structure such as a satellite with multiple flexible appendages. We describe the analytical modeling and the model validation studies carried out through experimental modal testing and parametric system identification studies in the frequency domain. An experimental position control with collocated sensor and actuator is also described. The control was implemented in real time in the MATLAB.*

***Keywords :*** *Flexible structures, Modal analysis, Identification, Control of structures.*

## 1. INTRODUCTION

This paper presents the analytical modeling together with model validation studies carried out through experimental modal tests and parametric system identification studies in the frequency domain of a multibody flexible structure prototype. The real time position control was implemented in MATLAB.

The experimental setup, show in the figure 1, was assembled at ITA-IEMP Dynamics Laboratory with the aim to investigate the dynamics and the position control of flexible structures representative of aerospace structures such as a satellite with flexible appendages. The experimental setup is composed of two flexible aluminum beams coupled to a central rigid hub. The hub is mounted on a steel disc supported on a gas bearing, in an attempt to minimize the static friction and to simulate the structure's slew motion in space conditions. The steel disc is linked to a brushless DC motor which gives the necessary excitation to the structure. The direct-drive torque actuation avoids the introduction of spurious non-linear effects such as dry friction and backlash in the gear transmission system.

The instrumentation and measurement subsystems consist of collocated and non-collocated sensors and their respective signal conditioning systems. An extensometric accelerometer, is used to monitor the vibration displacement of the beam tip. Two full strain-gage bridges are used to measure the elastic deformation at two known positions along the arms. The collocated sensors consist on a tachometer and a potentiometer both fixed to the motor axis.

A Dynamic Signal Analyzer is used to estimate the experimental frequency response function between the actuator input signal and each one of the sensors output. The experimentally determined modal response functions together with their coherence signals are used to carry out parametric system identification studies in the frequency domain. MATLAB subroutines were used to validate the analytical model described below, by comparing the theoretical transfer functions with the experimentally determined ones. A schematic view of the experimental set up is shown in figure 1.

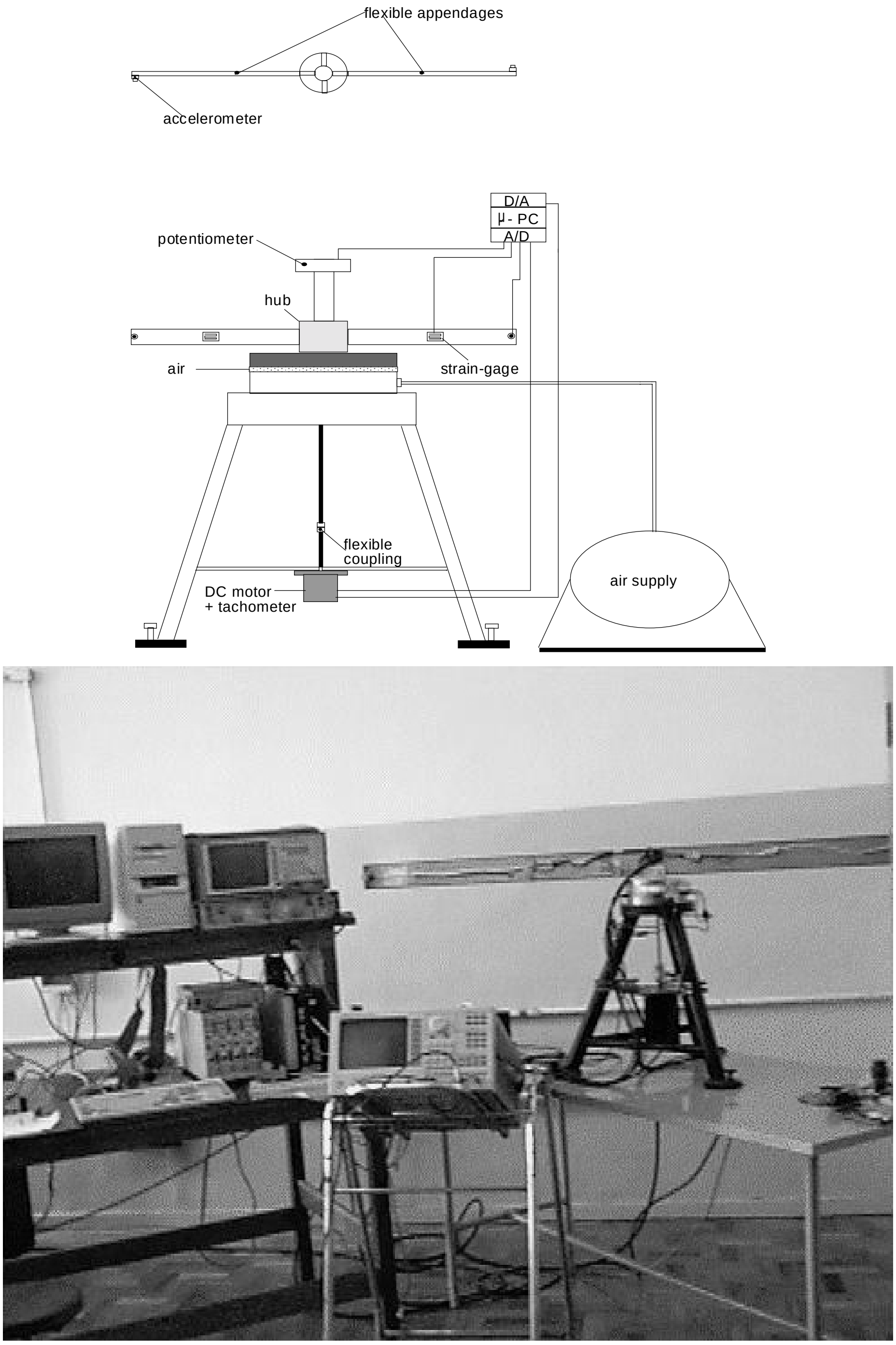



Figure 1- Experimental Setup

## 2. THE ANALYTICAL MODEL

The generalized Lagrangean approach is used to derive the analytical model of the unconstrained multi-link flexible structure, where the unconstrained characteristic results from the natural motion without external influences, i.e, all the structure is allowed to vibrate and its solution involves both the inertia of the rigid and the flexible parts (Barbieri & Özgüner, 1988). In this study we assume that the elastic deformation on the beams are symmetric with respect to the hub, consequently it is necessary to model only the elastic displacement of one of the arms (Junkins and Kim, 1993). The position of a generic point on the beam is written on a local body fixed coordinate system, as shown in the figure 2, is :

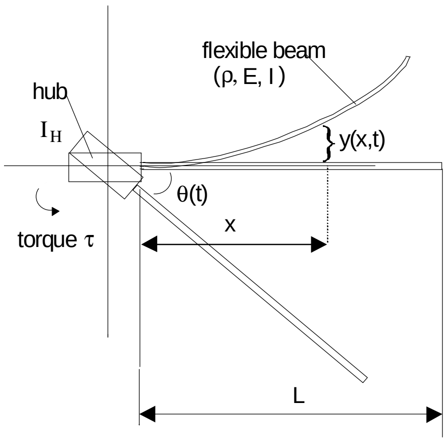



Figure 2. Coordinate system

$$\underline{R} = (r + x)\underline{C}_x + y(x,t)\underline{C}_y \tag{1}$$

where $\underline{C}_x, \underline{C}_y$ *and* $\underline{C}_Z$ are unit vectors of a reference system fixed on the body. The velocity of a point on the deformed beam is written as :

$$\dot{\underline{R}} = \frac{d}{dt}(\underline{R}) = \frac{d}{dt}(\underline{R})_{body} + \underline{\omega} \times \underline{R} \tag{2}$$

where ω is the angular velocity ($\omega = \dot{\theta}\,\underline{C}z$) of the hub. The kinetic energy of the system is the sum of the kinetic energy of the hub, the arms and the tip mass.

$$\mathrm{T} = \mathrm{T}_{hub} + \mathrm{T}_{beam} + \mathrm{T}_{boundary\backslash} \tag{3}$$

with

$$T_{hub} = \frac{1}{2} I_{hub} \dot{\theta}^2 \tag{4}$$

$$T_{beam} = \int_o^L \rho\, \dot{\underline{R}}^2 dx \tag{5}$$

$$T_{boundary} = \frac{1}{2} m_t \dot{\underline{R}}^2 (L) \tag{6}$$

where $I_{hub}$ is the hub inertia, ρ is the linear mass density of the beam, L is the appendages length and $m_t$ is the mass of the accelerometer located at the tip of the beam. Thus we have the following expression for the kinetic energy :

$$T=\frac{1}{2}I_{hub}\dot{\theta}^2+\frac{1}{2}\int_0^L\rho(x\dot{\theta}+\dot{y})^2\,dx+\frac{1}{2}m_t(L\dot{\theta}+\dot{y}(L))^2 \tag{7}$$

The potential energy of the distributed parameter system do not take into account the shear deformation and the rotary inertia of the beam and is given by the following expression:

$$V=\int_0^L EI\left[\frac{\partial^2 y(x,t)}{\partial x^2}\right]^2 dx \tag{8}$$

The Lagrangian of the system, is written as the total kinetic energy minus the potencial energy of the structures and the nonconservative work done by the applied torque are respectively:

$$L=T-V \quad ; \qquad \delta W_{nc}=\tau\delta\theta \tag{9}$$

From Góes et al. (1998) and Negrão (1998), we have the equation of motion for the rigid body mode of the system:

$$\left(I_{hub}+\rho\int_0^L x^2dx+m_tL^2\right)\ddot{\Theta}=\tau \tag{10}$$

and the equation for the elastic mode of the system is:

$$\frac{\left(-\left(I_{hub}+I_{beam}+m_t\,L^2\right)\theta_j\right)}{\left(\int_0^L\rho\phi_j^2\,dx+m_t\phi_j^2(L)+I_{hub}\theta_j^2\right)}\ddot{\Theta}+\ddot{\eta}_j+\omega_j^2\eta_j=F_{\eta_j}; \qquad i,j=1,2,... \tag{11}$$

where

$$I_{beam}\equiv\int_0^L\rho\,x^2\,dx \tag{12}$$

$$\theta_i=-\frac{\rho\int_0^L(x)\phi_i(x)dx+m_t(L)\phi_i(L)}{I_{hub}} \quad ; \qquad i,j=1,2,... \tag{13}$$

$\phi_j(x)$ are the eigenfunctions

and the frequency equation, is written as:

$$\begin{aligned}&\frac{2}{I_{hub}\lambda^3}[I_{hub}\lambda^6(-1-cos(\lambda L)cosh(\lambda L))+(\gamma\lambda L\rho-\lambda^5Lm_t)sin(\lambda L)\\&+(\gamma I_{hub}\lambda^3-\lambda^3\rho)cosh(\lambda L)sin(\lambda L)+(L\lambda^5m_t+\gamma L\lambda\rho)sinh(\lambda L)\\&+(\lambda^3\rho-\gamma I_{hub}\lambda^3)cos(\lambda L)sinh(\lambda L)-2\gamma\rho sin(\lambda L)sinh(\lambda L)=0\end{aligned} \tag{14}$$

with: $\gamma=\frac{m_t\omega^2}{EI}$

Applying the equations mentioned before, the following matrix equation is obtained for the first three modes:

$$\mathrm{M}\ddot{\mathrm{q}} + \mathrm{Kq} = \mathrm{F} \tag{15}$$

$$M = \begin{bmatrix} I_T & 0 & 0 & 0 \\ I_j & 1 & 0 & 0 \\ I_j & 0 & 1 & 0 \\ I_j & 0 & 0 & 1 \end{bmatrix} \text{ and } K = \begin{bmatrix} 0 & 0 \\ 0 & diag[\,\omega_1^2 \ldots \omega_3^2\,] \end{bmatrix} \tag{16}$$

where:

$$I_T = I_{Hub} + I_{beam} + m_t\, l^2 \quad ; \; I_j = \frac{\left(-\left(I_{Hub} + I_{beam} + m_t\, l^2\right)\theta_j\right)}{\left(\int_0^l \rho\phi_j^2\, dx + m_t\phi_j^2(l) + I_{Hub}\theta_j^2\right)} \quad ; \; \mathrm{j=1,2,3} \tag{17}$$

$$q = [\Theta \quad \eta_1 \quad \eta_2 \quad \eta_3]^T, \quad ; \; F = [\tau_m \quad \phi_1'(0)\tau_m \quad \phi_2'(0)\tau_m \quad \phi_3'(0)\tau_m]^T \tag{18}$$

Now it is simple to get the state-space representation of the system in the form:

$$\underline{\dot{X}} = \underline{\underline{A}}\,\underline{X} + \underline{B}\underline{u} \tag{19}$$

where the $\underline{\underline{A}}$ e $\underline{B}$ matrix are:

$$\underline{\underline{A}} = \begin{bmatrix} \underline{0} & \underline{\underline{I}} \\ \underline{\underline{M}}^{-1}\underline{\underline{K}} & \underline{0} \end{bmatrix} \quad ; \quad \underline{B} = \begin{bmatrix} \underline{0} \\ \underline{\underline{M}}^{-1}\underline{F} \end{bmatrix} \tag{20}$$

In order to obtain the analytical transfer functions, we need to define the observation matrix, $\underline{\underline{C}}$, that describe the measured signals in terms of the state variables. This matrix is obtained from the model of the available sensors. As described in the section before, the instrumentation system is composed by four sensors: an extensometric accelerometer, a potentiometer, two strain-gage bridges and a tachometer. The accelerometer is located at the free tip of the beam and, its signal is conditioned by a pre-amplifier and a double integrator filter with a global coefficient of sensitivity given by Ga, in V/cm units. Thus, we can write:

$$e_{ac} = Ga(\,L\theta + y(\,L,t\,)) \tag{21}$$

Rewriting the integrated accelerometer equation, as in (Negrão, 1998):

$$e_{ac} = Ga[L \quad \phi_1(L) \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0][\theta(t) \quad \eta_1(t) \quad \eta_2(t) \quad \eta_3(t) \quad \dot{\theta}(t) \quad \dot{\eta}_1(t) \quad \dot{\eta}_2(t) \quad \dot{\eta}_3(t)]^T \tag{22}$$

The potentiometer provides a voltage proportional to the angular position of the hub, $e_p = G_p\theta(t)$. The full extensometer bridge gives a signal proportional to the axial strain of the beam ($\varepsilon_s$), which can be related with the elastic deformation y(x, t), at the point were it is located by the eq. (23),

$$\varepsilon_s\big|_x = \left[\frac{e}{2}\right]\left(\frac{\partial^2 y}{dx^2}\right)\Bigg|_x \tag{23}$$

where *e* is the thickness of the beam. Rewriting the extensometer equation as:

$$\varepsilon_s = \left[\frac{e}{2}\right]\left[0 \quad \frac{d^2\phi_1(x_1)}{dx^2} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right][\theta(t) \;\; \eta_1(t) \;\; \eta_2(t) \;\; \eta_3(t) \;\; \dot{\theta}(t) \;\; \dot{\eta}_1(t) \;\; \dot{\eta}_2(t) \;\; \dot{\eta}_3(t)]^T \tag{24}$$

where $x_1$ is the position where the extensometer are located on the beam. The tachometer gives a signal proportional to the angular velocity of the hub, $e_t = \dot{\theta}(t)$, which combined with the other sensor equations, gives the observation vector $\underline{Y} = \underline{\underline{C}} \,.\, \underline{X}$, where

$$\underline{Y} = [\, e_{ac} \quad e_p \quad e_s \quad e_t \,]^T \tag{25}$$

and,

$$\underline{\underline{C}} = \begin{bmatrix} L & \phi_1(L) & \phi_2(L) & \phi_3(L) & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{e}{2}\left[\frac{d^2\phi_1(x_1)}{dx^2}\right] & \frac{e}{2}\left[\frac{d^2\phi_2(x_1)}{dx^2}\right] & \frac{e}{2}\left[\frac{d^2\phi_3(x_1)}{dx^2}\right] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

## 3. THE ANALYTICAL TRANSFER FUNCTIONS

To obtain the analytical transfer functions, we used the physical parameters listed in table 1, for the unconstrained multi-link flexible system.

Table 1. Model parameter of the unconstrained flexible beams

| | | | |
|---|---|---|---|
| Aluminum density | ρ | $2.7950\ 10^{3}$ | Kg/m³ |
| Aluminum Young's modulus | E | $6.8900\ 10^{10}$ | N/m² |
| Beams width | Eb | $4.1200\ 10^{-3}$ | M |
| Beams height | Hb | $8.0780\ 10^{-2}$ | M |
| Beams length | L | $9.7150\ 10^{-1}$ | M |
| Beams cross-section area | A | $3.3281\ 10^{-4}$ | $m^2$ |
| Beams moment of inertia | I | $4.7070\ 10^{-10}$ | $m^4$ |
| Beams mass moment of inertia | $I_b$ | $2.8430\ 10^{-1}$ | Kg $m^2$ |
| Hub mass moment of inertia | $I_{hub}$ | $7.6749\ 10^{-1}$ | Kg $m^2$ |
| Hub radius | r | $9.0000\ 10^{-2}$ | M |

Applying the Laplace transform into eq. (20) with zero initial conditions and using the model parameters listed in table 1, we can obtain the analytical transfer functions for each sensor. The Bode plots of the open loop system are obtained by substituting ($s=j\omega$) in the Laplace transfer functions shown below:

$$\underline{Y}(s) = \underline{\underline{C}}.(s\underline{\underline{I}} - \underline{\underline{A}})^{-1}.\underline{B}.U(s) \tag{27}$$

## 4. MODEL VALIDATION AND PARAMETRIC IDENTIFICATION

Comparing the analytical and experimental frequency response functions we can observe some discrepancies between them. These errors are due mainly to sensor noise and unmodelled sensor dynamics. Nonetheless, the experimental transfer functions clearly show some sharpen peaks in the spectrum, which can be associated with the vibration modes of the unconstrained flexible system. In order to check this assumption a more accurate comparison can be done between the predicted and the experimentally determined modal frequencies. Table 2 below shows a comparison between the analytical and the experimental results, which suggests a reasonable agreement between these analysis.

Table 2. Comparison between analytical and experimental results

| Mode nº | Analytical (Hz) | Experimental (Hz) | error % |
|---|---|---|---|
| 1 | 12.51 | 12.0 | 4.25 |
| 2 | 31.11 | 33.25 | 5.72 |
| 3 | 62.86 | 59.5 | 4.97 |

In the figures (3)-(6), we show a comparison between the analytical and experimental model:

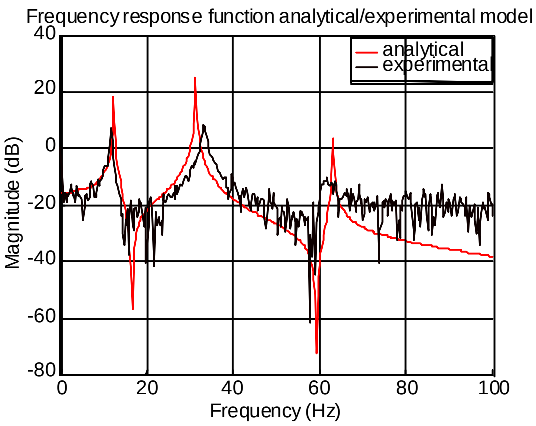



Figure 3. Analytical and experimental curves - strain-gage

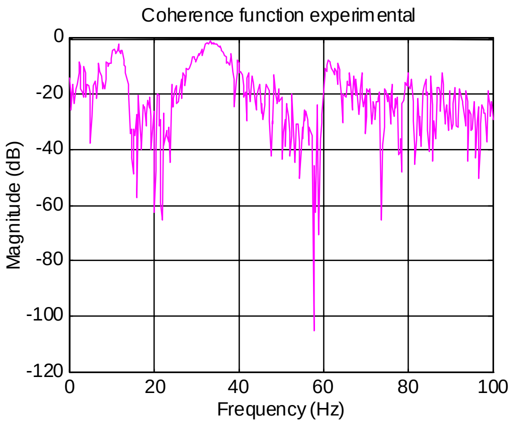



Figure 4. Experimental coherence function – strain-gage

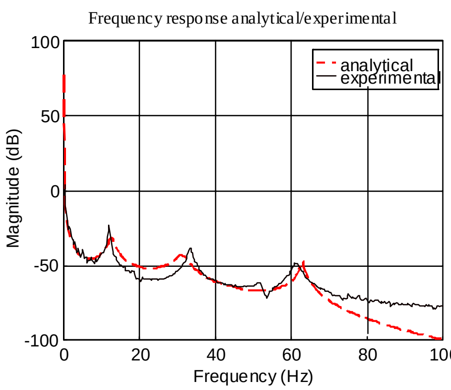



Figure 5. Analytical and experimental curves - accelerometer

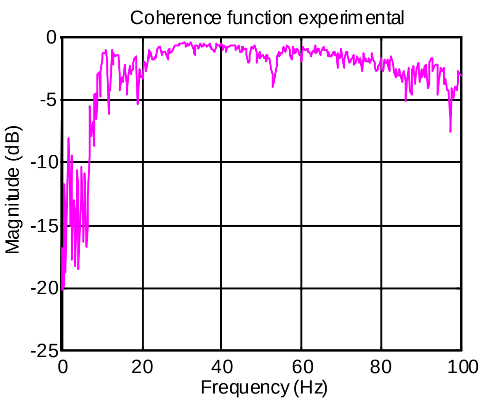



Figure 6. Experimental coherence function – accelerometer

## 5. POSITION CONTROL

Position control of mechanical systems with structural flexibility has been an important research topic in recent years. We show experimental results of a position control using potentiometer and tachometer-feedback control (collocated sensor feedback). The control strategy consists on the determination of the potentiometric, $k_{pot}$, and tachometric $k_{taco}$, gain constants, according to the desirable positions of the closed-loop poles of the system. This positional loop was implemented to control the angular position of the hub in real time, with only an indirect active control of the vibrations. The scheme of experimental position control is shown in the Figure (7).

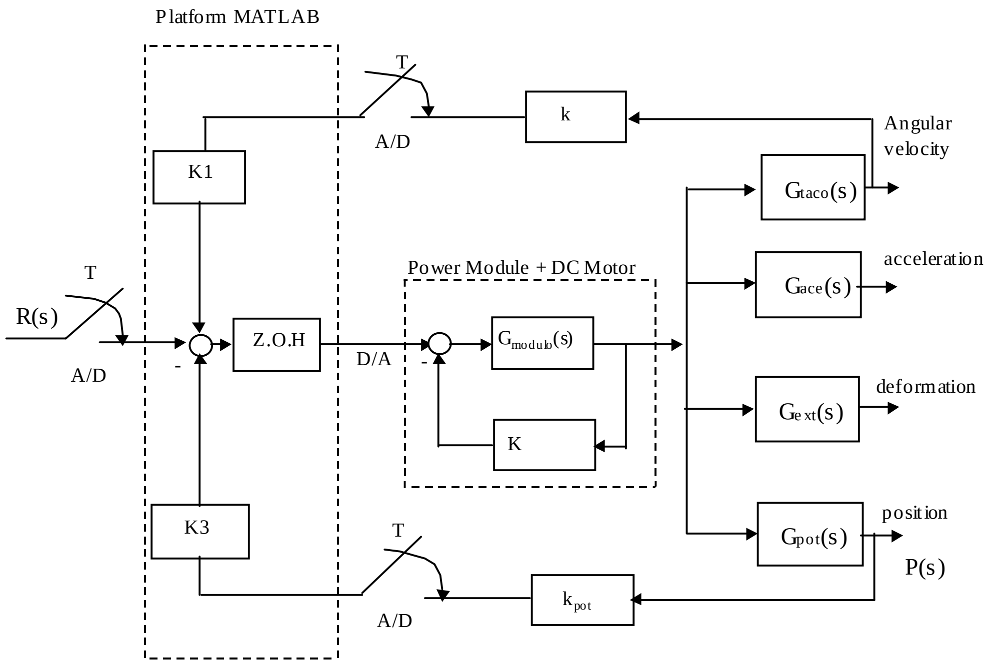



Figure 7- Control scheme

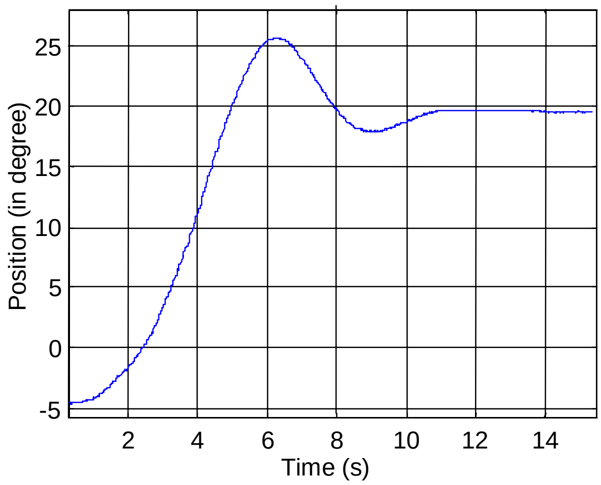
The parameters of the experimental control are shown in table 3 and the experimental results are shown in the Figures (8) and (9).

Table 3 Parameter for experimental control

| $K_1$, $K_3$ | $k_{taco}$ | $k_{pot}$ | Position (in degree) |
|---|---|---|---|
| 0,017 | 37 | 1 | 18,6 |

As one can see in the Figures (8) and (9), the positional control is efficient. The final position was reached in 10 seconds with an overshoot of 25%. This was the best performance that could be achieved without excitation of the higher vibrational modes of the beam. Further increase of the open-loop gain drives the system to instability. This work is still in progress, and we are implementing a real-time control using the platform program MATLAB/ SIMULINK. We also intend to implement others kind of control strategy including the

LQG/LTR, which due to the system inaccuracies, could be proven to be more robust to the unmodelled dynamics and sensor noise.



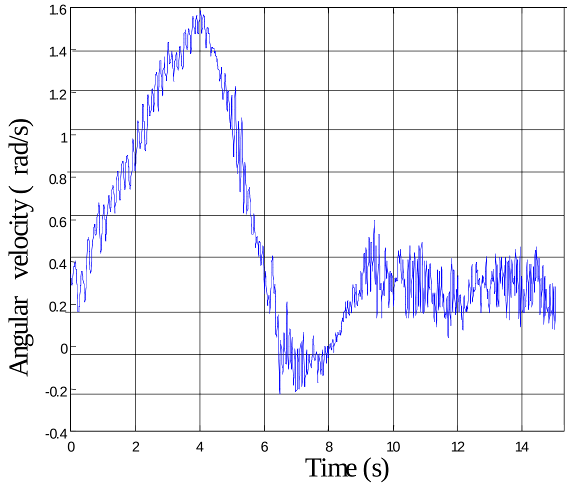




Figure-8 Angular position for step reference

Figure-9 Angular velocity for step reference

## 6. CONCLUSIONS

This paper reports preliminaries results obtained with an experimental apparatus with multiple flexible bodies. The model was derived using the Lagrangean approach and its discretization was done with the Assumed Modes Method. The model validation and identification studies were done, first matching the analytical and experimental frequency response functions of a SIMO class model, and second through a parametric identification in frequency domain of the experimentally determined system transfer functions.

Comparing the analytical and experimental frequency response functions we can note some discrepancies between them. These errors are due to sensor noise and unmodelled sensor dynamics. A more accurate comparison between models can be done using the results shown in the table 2, that contain the numerical values of the modal parameters. This comparison suggest a good agreement between the analytical and the experimental models. The parametric identification results showed the same difficulties pointed by Cannon and Schmidt (1984) and Miu (1991) that refers to possible instabilities caused by the non-collocation of sensors and actuators.

To have some insight in the control area, we implemented a potentiometer and tachometer-feedback control. The experimental control results shown that the controller reach the desirable angular position. This work is still in progress and using MATLAB to implement control we intend to implement other control strategy, such as robust control. Due to the system inaccuracies a robust control synthesis like LQG/LTR should be more suitable for this system (Soares, Goes and Souza, 1996).

## REFERENCES


Barbieri, E. & Ösgüner, Ü. , 1988, Unconstrained and Constrained Mode Expansions for a Flexible Slewing Link,  Journal of Dynamic Systems, Measurement, and Control, 110, 4, 416-421.

Cannon, R. H. Jr. & Schmitz, E., 1984, Initial Experiments on the End-Point Control of a Flexible One-Link Robot , The International Journal of Robotics Research., 3, 3, 62-75.

Góes, L. C. S., Negrão, R. G., Rios Neto, W., 1998. Modeling and Control of Multibody System with Flexible Appendages., eds Balthazar, J. M., Gonçalves, P. B. and Clayssen, J. (Editors), Nonlinear Dynamics, Chaos, Control and Their Applications to Engineering Sciences. Vol 2: Vibrations with Measurements and Control, published by Brazilian Society of Mechanical Sciences - ABCM, Brazilian Society of Computational and Applied Mathematics - SBMAC and Society for Industrial and Applied Mathematics - SIAM, pp. 75-91.

Inman, D. J., 1989, Vibration With Control, Measurement, and Stability, Prentice-Hall International Inc., USA.

Junkins, J. L. and Kim, Y., 1993, Introduction to Dynamics and Control of Flexible Structures, AIAA Educational Series, USA.

Miu, D. K., 1991, Physical interpretation of transfer function zeros for simple control system with mechanical flexibility, Journal of Dynamic System, Measurement, and Control, 113, September, 419-424.

Negrão, R. G., 1998, Dinâmica e Controle de Um Sistema Mecânico com Apêndices Flexíveis, Master Thesis, Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil.

Soares, A. M. S. & Souza, L. C. G. & Goes, L. C. S., 1996, Modal Analysis of a Multibody System with Flexible Appendages, proceedings of the Second International Conference on non-linear dynamics, chaos, control and their applications in engineering sciences (ICONE 96), São Pedro, SP, Brazil.