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EXPERIMENTAL EVALUATION OF A FORCE IDENTIFICATION TECHNIQUE BASED ON TIME DOMAIN DECONVOLUTION

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Abstract. *This paper addresses the identification of excitation forces from the dynamic responses of vibrating linear systems. A method is investigated based on the inversion of the multi-input-multi-output transfer relation in the time domain, expressed as a convolution integral having the matrix of impulse response functions as its kernel. The main objective is to evaluate the performance and robustness of the method when applied to a real-world structure, tested in laboratory. The paper is organized as follows: after preliminary remarks concerning the methods which have been used for force identification, the basic formulation of the time domain-based deconvolution technique is presented. Then, ill-conditioning of the identification computations is discussed and the Conjugate Gradient algorithm, which is used for obtaining stable force estimates, is presented. The test-structure and experimental procedures are described, followed by the results of various force identification computations, considering different types of excitation forces. Based on these results, discussion and conclusions are presented.*

Key-words: *Force identification, Deconvolution, Impulse response function, Inverse problems, Ill-conditioning*

1. INTRODUCTION

During the phases of design, control and diagnosis of mechanical systems, accurate knowledge of the distribution and magnitude of excitation forces is a crucial issue. The precise determination of the forcing functions can provide a larger confidence in the numeric simulations based on analytic models.

In many situations the forces can be directly measured by using force transducers. However, in a number of other cases, this can be a difficult or unfeasible procedure. For instance, the geometry may be such that one would have to modify the structure in order to mount the load transducers. Engine torque pulse and shaking forces are another example of loads which are difficult to measure since they are generally distributed over the engine. In such cases, indirect force measurement techniques become a valuable alternative. In this context, force identification refers to the process of deriving external forces given a mathematical model of the system and a set of measured structural responses, which can generally be easily acquired from standard vibration tests. The mathematical model can be obtained either by analytical or experimental procedures.

As an inverse problem, the identification of excitation forces is characterized by numerical ill-conditioning of the estimation equations, which leads to a high degree of sensitivity of the solution with respect to disturbances which inescapably affect experimental data (Starkey & Merril, 1989). Thus, numerical instabilities have to dealt with by using the so called *regularization* techniques in order to ensure meaningful force estimates.

Several force identification techniques, operating either in the time domain or in the frequency domain, have been proposed and are documented in the literature (Stevens, 1987). Most of frequency domain methods are based on the inversion of the frequency response function (FRF) matrix for each frequency line in the band of interest. As for the time domain methods, the most widely known is the one named *Sum of Weigthed Accelerations Technique* (*SWAT*) (Bateman et al., 1992). This method is based on the modal equilibrium equations written for the rigid body modes of system. Due to its features, SWAT can only be applied to unconstrained structures and is only capable of providing the resulting forces and moments about the center of mass of the structure, while the actual spatial distribution of the forces remains unknown. A modal approach has also been focused by (Genaro, 1997), enabling to circumvent those drawbacks. Time domain deconvolution has been used by (Kammer, 1996), (Genaro, 1997) and (Silva & Rade, 1999). According to this procedure, the excitation forces are identified by solving a linear system of equations obtained by inversion of the discrete-time multi-input-multi-output convolution integral, which has the matrix of input response functions (IRF's) as its kernel.

Since in previous works the deconvolution method has only been applied to numerically simulated systems, the primary goal of this paper is to perform the appraisal of this method when applied to a real structure tested in laboratory. Particular interest is directed towards the effects of experimental noise, ill-conditioning and regularization techniques and the practical acquisition of the IRF's from experimental responses.

2. TIME DOMAIN DECONVOLUTION TECHNIQUE

Time-domain deconvolution has long been used in a number of technical applications, such as electromagnetic wave propagation (Sarkar et al.,1982) and geophysics (Arya et al.,1978). Applications to the problem of input force identification in the realm of Mechanical Engineering have been performed by (Kammer, 1996), (Genaro, 1997) and (Silva & Rade, 1999). The fundamentals of the method are summarized in the following.

For a linear structure, the multi-input-multi-output transfer relation in the time domain, assuming null initial conditions, is given by the well-known convolution integral (Bendat & Piersol, 1980):

$$\{u(t)\} = \int_0^t [h(t-\tau)]\{f(t)\}d\tau \,, \tag{1}$$

where $\{u(t)\}$, $\{f(t)\}$ and $[h(t)]$ designate, respectively, the vector of time responses (in terms of either displacements, velocities or accelerations), the vector of excitation forces and the matrix of impulse response functions.

Assuming m measurement locations and n excitation forces, Eq. (1) can be developed as follows:

$$\begin{aligned} u_1(t) &= \int_0^t h_{11}(t-\tau) f_1(\tau) d\tau + \cdots + \int_0^t h_{1n}(t-\tau) f_n(\tau) d\tau \\ &\ \vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots \\ u_m(t) &= \int_0^t h_{m1}(t-\tau) f_1(\tau) d\tau + \cdots + \int_0^t h_{mn}(t-\tau) f_n(\tau) d\tau \end{aligned} \tag{2}$$

where $h_{ij}(t)$ designates the response at coordinate i, due to a unit impulse excitation applied at coordinate j.

A discrete-time approximation to Eq. (2) is obtained letting Δt be the sampling interval of the functions $u_i(t)$, $f_i(t)$ and $h_{ij}(t)$. Introducing the notation $u_i(k) = x_i(k\Delta t)$, $f_i(k) = f_i(k\Delta t)$, $h_{ij}(k) = h_{ij}(k\Delta t)$, for k=1 to p, the discrete-time version of Eq. (2), using a zero-order hold, is written:

$$\begin{aligned} u_1(k) &= \left[\sum_{j=0}^{k} h_{11}(k-j) f_1(j) + \cdots + \sum_{j=0}^{k} h_{1n}(k-j) f_n(j) \right] \Delta t \\ &\vdots \qquad\qquad\qquad\qquad \text{for } k\text{=0 to } p\text{-1} \\ u_m(k) &= \left[\sum_{j=0}^{k} h_{m1}(k-j) f_1(j) + \cdots + \sum_{j=0}^{k} h_{mn}(k-j) f_n(j) \right] \Delta t \end{aligned} \tag{3}$$

Equation (3) can also be expressed in matrix form as:

$$\begin{bmatrix} \{u(0)\} \\ \{u(1)\} \\ \vdots \\ \{u(p)\} \end{bmatrix} = \begin{bmatrix} [T(0)] & & & [0] \\ [T(1)] & [T(0)] & & \\ \vdots & \vdots & \ddots & \\ [T(p-1)] & [T(p-2)] & \cdots & [T(0)] \end{bmatrix} \begin{Bmatrix} \{f(0)\} \\ \{f(1)\} \\ \vdots \\ \{f(p)\} \end{Bmatrix} \tag{4}$$

where:

$$[T(k)] = \Delta t \begin{bmatrix} h_{11}(k) & h_{12}(k) & \cdots & h_{1n}(k) \\ \vdots & \vdots & & \vdots \\ h_{m1}(k) & h_{m2}(k) & \cdots & h_{mf}(k) \end{bmatrix} \tag{5}$$

System (4) can be written in a more compact form:

$$\{U\} = [T]\{F\}, \tag{6}$$

with $\{U\} \in \Re^{(p.m)\times 1}$, $\{F\} \in \Re^{(p.n)\times 1}$, $[T] \in \Re^{(p.m)\times(p.n)}$.

Provided that the number of equations is greater than the number of unknowns ($m \geq n$) and that $[T]$ has full rank, system (6) can be solved for the vector containing the discrete values of the excitation functions $\{F\}$. For this, various numerical methods can be used. For example, the normal least-square solution can be calculated as:

$$\{F\} = \left([T]^T [T]\right)^{-1} [T]^T \{U\}, \tag{7}$$

where $[T]^T [T]$ is a symmetric positive definite matrix.

It should be noted that the formulation above can be used for exploring any type of time responses - accelerations, velocities or displacements - provided the corresponding IRF's are used.

The impulse response functions can, in practice, be obtained from experimental data by two different means:

• by computing the inverse Fourier transform of frequency response functions, or;

• by using an identification technique similar to that describe above, assuming that the excitation forces and the corresponding time domain responses are known (Fasana & Piombo, 1996). According to this method, for the identification of a general IRF $h_{ij}(t)$, the system of estimation equations is written:

$$\{h_{ij}\} = [F_j] \{U_i\} , \tag{8}$$

where $\{h_{ij}\}$ is the vector containing the discrete-time values of $h_{ij}(t)$, $[F_j]$ is the matrix formed from the values of the force applied at coordinate *j* and $\{U_i\}$ is the vector containing the values of the response at coordinate *i*.

The latter method has been used in the applications presented in this paper.

3. ILL CONDITIONING AND REGULARIZATION TECHNIQUES

The system of estimation equations (6) is generally ill-conditioned, which implies that small errors affecting $\{U\}$ and/or $[T]$ may lead to large errors on the vector of force estimates $\{F\}$. As argued by some authors (Kammer, 1996), ill-conditioning is expected to be more severe when dealing with non minimum-phase systems, since, in this case, the inverse system is unstable due to the existence of zeros outside the unit-circle. This occurs when the excitation locations do not coincide with the measurement locations.

Some strategies have been propose for stabilizing the computations and reducing the effects of ill-conditioning. Among them, the most widely used are the Singular Value Decomposition and the Tikhonov regularization techniques. In mathematical literature, the singular value decomposition is often mentioned as an effective way to solve ill-conditioned problems (Maia, 1986). According to this procedure, matrix $[T]^T[T]$ in Eq. (7) is converted into a diagonal matrix by a similarity transformation. The eigenvalues of this matrix, below a certain small amount, are set to zero.

According to Tikhonov's method, a matrix $\alpha[I]$, where α is a sufficient small scalar parameter, is added to $[T]^T[T]$ before solving Eq. (7) (Tikhonov, 1963). A major difficulty is the choice of values of α enabling to achieve regularization with a minimum modification of the original system of equations.

In this paper, an alternate approach, based on the iterative Conjugate Gradient Method (CGM), is used for solving system (6). According to this method, it is not necessary to form matrix $[T]^T[T]$. The basic formulation of the CGM is briefly presented the following. A more detailed discussion, including a comparison with other direct and iterative methods intended for solving large systems of linear equations is given by (Sarkar at al., 1981).

According to the CGM, Eq. (6) is solved by choosing a initial guess $\{F\}_0$ for the solution. Then matrices $\{R\}_0$ and $\{P\}_0$ are generated as follows:

$$\{R\}_0 = [T]\{F\}_0 - \{U\} \quad ; \quad \{P\}_0 = -[T]^T\{R\}_0$$

The (k+2)-th estimate is then given by:

$$\{F\}_{k+1} = \{F\}_k + t_k\{P\}_k$$

with:

$$t_k = \frac{\left\|[T]^T\{R\}_k\right\|^2}{\left\|[T]\{P\}_k\right\|^2}$$

$\{R\}_k$ and $\{P\}_k$, appearing in this last equation, are calculate according to:

$$\{R\}_k = \{R\}_{k-1} + t_{k-1}[T]\{P\}_{k-1}$$

and

$$\{P\}_k = -[T]^T\{R\}_k + q_{k-1}\{P\}_{k-1},$$

where:

$$q_{k-1} = \frac{\left\|[T]^T\{R\}_k\right\|^2}{\left\|[T]^T\{R\}_{k-1}\right\|^2}$$

The criterion utilized to stop the iterative process is that the norm of the residual vector must be below a certain prescribed value, i.e.:

$\left\|\{R\}_k\right\| \leq M$ (a prefixed constant).

(Sarkar et al.,1981) maintain that the CGM usually yields good results and is rather insensitive to the choice of the initial guess $\{F\}_0$. Additionally, they argue that the method is well adapted to ill-conditioned problems since in iterative methods the condition number of the system matrix has little influence on roundoff errors. The results of experimental applications presented in this paper seem to confirm those assertions.

4. EXPERIMENTAL SETUP AND PROCEDURES

Vibration tests were performed by using the structure depicted in Figure 1. It is basically a three-story portal frame formed by three aluminum square plates concentrating the major part of the mass, connect by three sets of thin steel beams, each set containing four parallel items. It was verified that, in the low-frequency domain, the system behaves like lightly damped three-degree-of-freedom system, whose natural frequencies are: f_1= 5,87 Hz, f_2=11,56 Hz, f_3= 18,06 Hz. Table 1 gives the main physical and geometrical characteristics of the test-structure. Excitation was applied and responses were measured in the x-direction, only. The relevant excitation and measurement coordinates were supposed to be located at the three plates, numbered as shown in Figure 1.

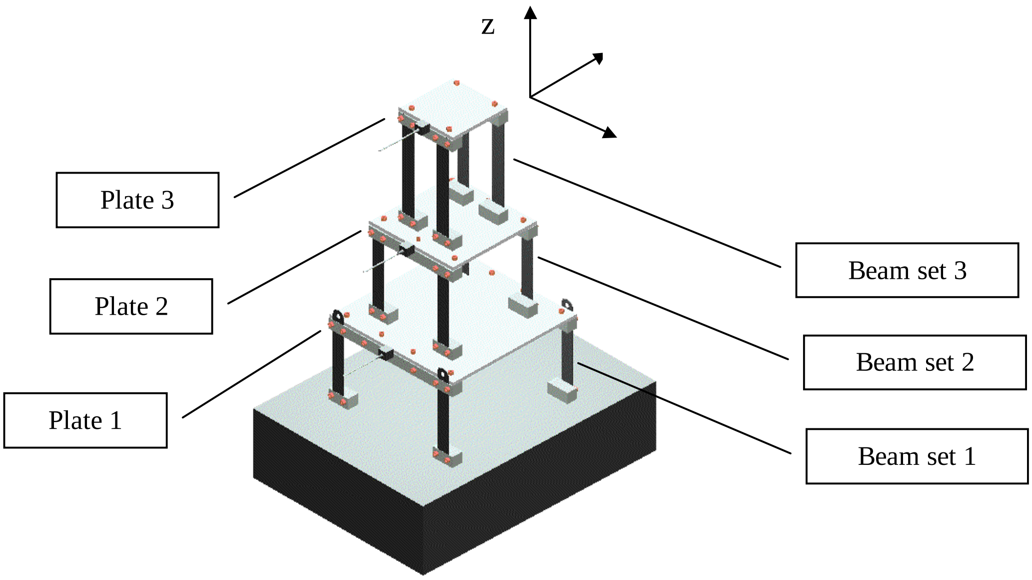


Figure 1. Mechanical system used in the experiments.

Table 1. Characteristics of the test-structure

Component	Mass (kg)	dimensions (cm)
plate 1 (with attachments)	0,930	25×25×0,57
plate 2 (with attachments)	2,230	17×17×0,67
plate 3(with attachments)	3,300	11×11×0,67
beam from set 1	0,016	9,20×2,25×0,1
beam from set 2	0,018	10,20×2,25×0,1
beam from set 3	0,026	14,70×2,25×0,1

The tests were divided into to phases: the first one aimed at the identification of the IRF's, using the technique suggested by Fasana & Piombo (1996). For this, excitation forces were applied at coordinate 1 and the corresponding time responses were simultaneously measured at coordinates 1, 2 and 3. Input signals were measured with the aid of piezoelectric load transducers and the accelerations were measured by using piezoelectric accelerometers, with a time step $\Delta t = 2{,}70\times10^{-3}$ s. Then, equation (8) was successively solved for the IRF's $h_{11}(t)$, $h_{21}(t)$ and $h_{31}(t)$, by using the CGM. This procedure was carried out by using both random and impact excitation. Virtually identical IRF's were obtained in both cases. For illustration, the IRF $h_{31}(t)$ (in terms of accelerations) is shown in Figure 2.

In the second phase, various types of excitation forces were applied at coordinate 1 through an electrodynamic shaker fed by a signal generator. They were measured by using a piezoelectric force transducer. The corresponding acceleration responses were simultaneously acquired at coordinates 1, 2 and 3 through piezoelectric accelerometers. Then, assuming that the excitation forces were unknown and using the IRF's previously identified in the first phase of the tests, the force identification method described in Section 2, combined with the CGM, was used for reconstructing the input forces. Since the exact measured forces were available they could be compared to the identified ones for the purpose of evaluating the accuracy of the force reconstruction procedure.

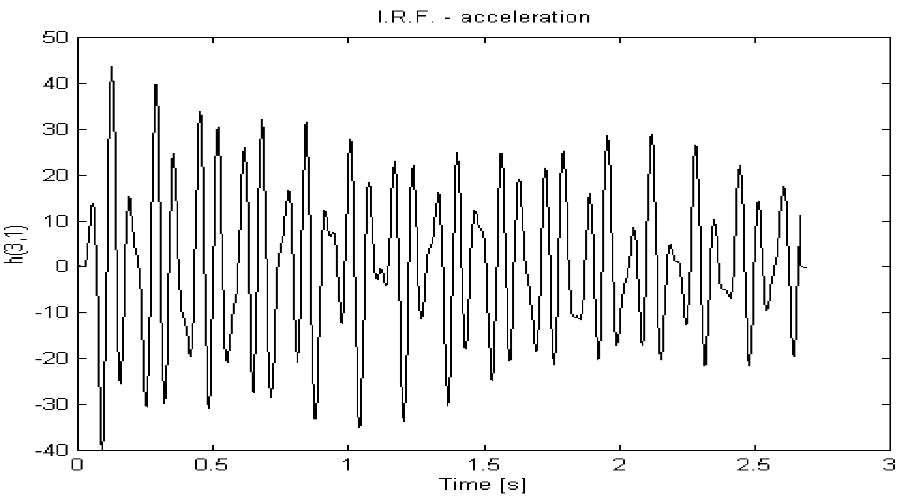


Figure 2 - IRF $h_{31}(t)$ of the test-structure.

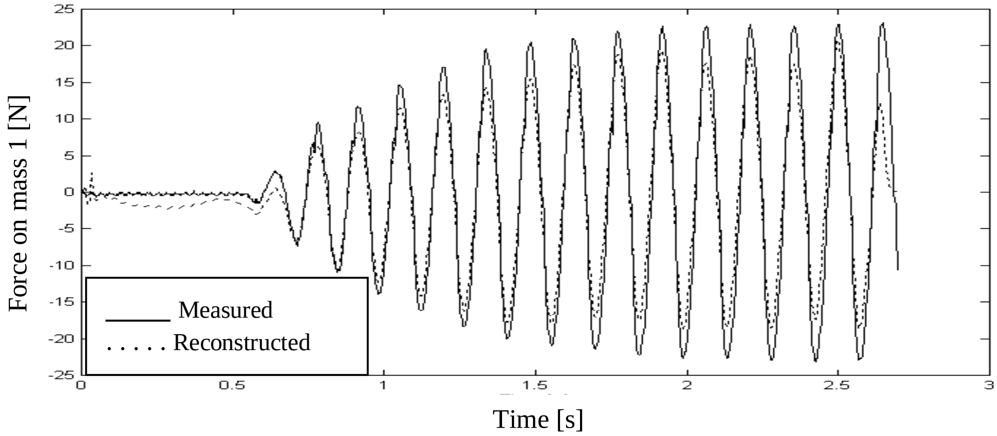
5. FORCE IDENTIFICATION RESULTS

5.1 Identification of a harmonic force

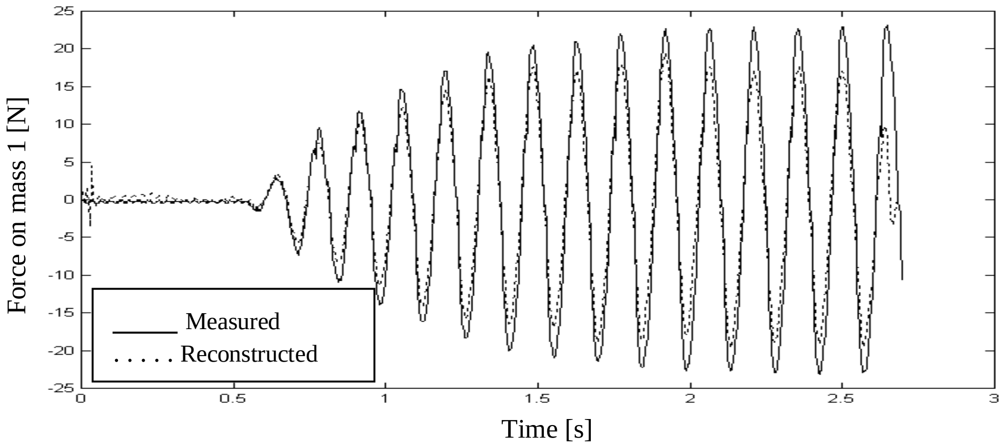
The first identification test aimed at reconstructing a harmonic force of frequency 7 Hz, applied at mass 1. The results of two reconstruction computations are presented in the following. In the first one, the identification was performed by using the acceleration response measured at coordinate 2 whereas in the second the response at coordinate 3 was exploited. The reconstructed forces, compared to the exact (measured) ones, are given in Figure 3.

It should be mentioned that, since the data were acquired immediately after the activation of the shaker, the forcing function is not perfectly harmonic. Instead, typically increasing amplitudes during the first cycles, due to the inertia of the system structure-shaker, are recognized.

As can be seen, both the identification computations provided reasonably accurate force reconstructions. The forcing period is perfectly detected whereas the varying amplitude is retrieved less accurately. It is also important to note the expected agreement between the two reconstructions.



(a)



(b)

Figure 3. Measured and identified forces at coordinate 1.
(a) from response at coordinate 2 ; (b) from response at coordinate 3

5.2 Identification of a low-frequency triangular force

The procedure described in the previous Section was used for the reconstruction of a triangle-type periodic force of frequency 4 Hz applied to mass 1. The identified force, obtained by using the acceleration responses measured at coordinate 3 is compared to its measured counterpart in Figure 4. In analyzing the results, the same comments presented in Section 5.1 can be made.

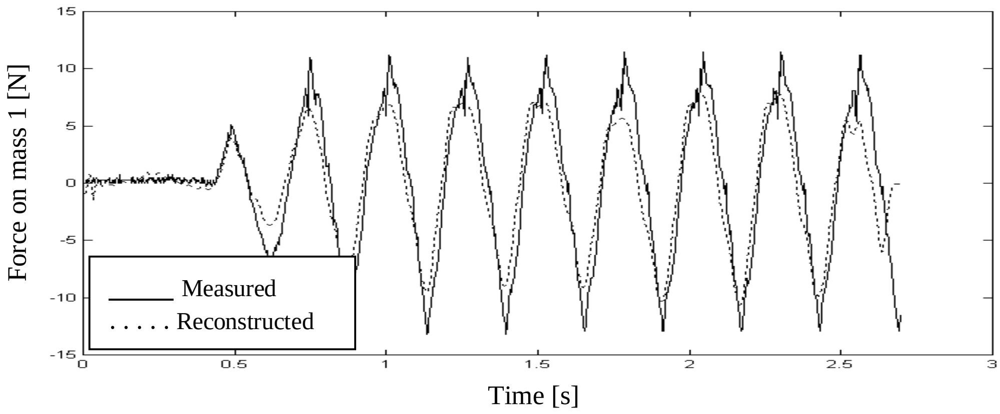


Figure 4. Measured and identified triangular forces at coordinate 1.

5.3 Identification of a fast frequency sweep force

In this test, the excitation force applied at coordinate 1 was a generated by performing a sequence of increasing-decreasing frequency sweeps in the band 2Hz-20Hz. Each cycle of sweep was completed in 0,1 s. The identified force, obtained by using the acceleration responses measured at coordinate 3 is compared to the measured one in Figure 5. As can be seen, the excitation force looks like a zero mean random signal. Here again, it can be said that the identification method was capable of providing a fairly good force reconstruction. Very similar results were obtained by exploring the acceleration responses at the two other coordinates, 1 and 2.

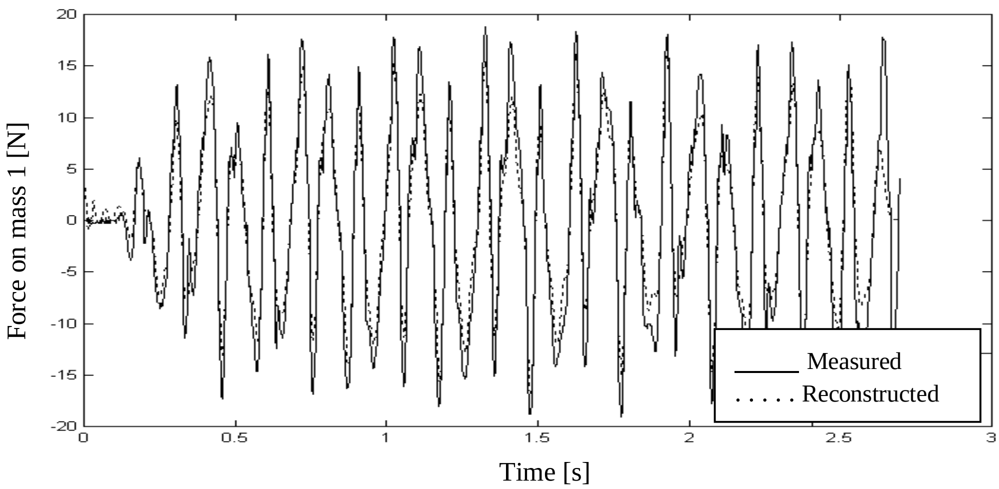


Figure 5. Measured and identified triangular forces at coordinate 1.

6. CONCLUSIONS

A time domain method designed for estimating excitation forces in vibrating elastic systems has been assessed by means of experimental tests. The obtained results demonstrated that the method is capable of providing reasonably accurate force estimates in the presence of experimental noise. Taking into account the fact that the accuracy of the force estimates depends on the precision of the impulse responses functions used in the process, it can be concluded that the fully time-based procedure for the identification of the IRF's was able to provide accurate enough estimates.

Confirming the conclusions of previous studies based on numerically simulated structures, the conjugate gradient method has demonstrated to be a stable and robust procedure for solving the estimation equations, even when dealing with non minimum-phase input-output relations.

Current developments by the authors are focused on the evaluation of the method when applied to more complicated situations, such as the identification of several excitation forces simultaneously. Other types of methods, intended for providing unbiased solutions to noise-contaminated systems of equations, like the instrumental variable method, are currently being incorporated to the force identification procedure.

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