COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

PROPOSAL AND APPLICATION OF A MATHEMATICAL MODEL FOR VOLUMETRIC ACCURACY ANALYSIS OF CMMs

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ABSTRACT

In this research an approach to volumetric determination of CMMs, which permits interpolation between measured points, has been developed. Mathematical and statistical techniques, such as Response Surface Methodology, have been used to represent the relationship between each volumetric error component (Ex, Ey and Ez) at a point, within the measuring volume, and the co-ordinates of the point (X_i, Y_i, Z_i). After fitting mathematical models that represent the volumetric error components, it is possible to construct the volumetric error map of a CMM under test. In this paper, the measured data obtained by applying a modular space frame on a LK CMM are used to fit mathematical models that represent each volumetric error component. Also, the parametric errors of that machine are derived from the volumetric error data. In addition, the graphical representation of the volumetric error data and that of the parametric errors related to the LK CMM are presented.

Keywords Response Surface, Co-ordinate Measuring Machines

1. INTRODUCTION

Over the past two decades, many methods for assessing the accuracy of co-ordinate measuring machines have been proposed and applied. Basically, the methods can be classified into three groups of common techniques. They are: transfer standard technique; parametric calibration technique or synthesis method and kinematic reference standard technique.

Transfer standard technique. This is a technique by which a space structure of accurately known dimension, such as ball plate and tetrahedron, is measured by the machine. The error is defined as the deviation of the machine measurement from the true dimensions (Kunzmann et al., 1993; Zhang, 1991). The transfer standard technique presents some limitations. Firstly, it is difficult to manufacture mechanical artefacts that have the following properties: lightweight, high thermal stability, easy calibration and non-expensive. Secondly, various sizes of the standards are required for different sizes of machines, and the problems of storage, handling as well as transporting may also arise (Pahk, 1991). Despite such limitations, the transfer standard technique has the advantage of getting measuring data very similar to the way by which the CMMs perform their measuring tasks. This technique is very useful, in particular, when the CMMs incorporate software compensation system of geometric errors.

Parametric calibration technique or synthesis method. This is a technique in which the structure of the machine is considered as a kinematic model, and then analysed using the rigid body kinematics. Each error component, such as yaw, roll, pitch, straightness, squareness and positioning error, is measured by conventional measuring equipment, for example, laser interferometer, electronic level, straight edge and square. In this technique, the parametric errors can be combined to give the volumetric error components by using a geometric model based upon rigid body kinematics (Burdekin, 1981; Pahk, 1991; Huang, 1995). The parametric technique has the advantage of providing information for the error diagnosis of the machine. However, this technique is time consuming, requires expensive equipment and special skill to operate that equipment, for instance, laser interferometer system. In addition, the error obtained from the parametric calibration technique may not represent the real error components at the probe position. That can happen, especially, when the CMMs have a software compensation system. This is because those error data may be changed or reduced due to the software compensation (Pahk, 1991).

Kinematic reference standard technique. This is a technique based on measuring volumetric errors with some type of kinematic reference standard such as a ball bar (Bryan, 1982; Kunzmann, 1983; Burdekin, 1991). The kinematic reference standard technique is particularly simple for acquiring data. However, it is difficult to cover all of the measuring volume as well as it is difficult to interpolate between the measured points.

It is worth noting that none the existing techniques, described above, meet all requirements in terms of consumed time, simplicity to use and measure, diagnosis of errors, etc. Therefore, a critical need exists in order to overcome disadvantages that existing techniques, to assess accuracy of CMMs, present. In this regard, it is necessary to developed a new technique that is capable of assessing the performance of modern CMMs that incorporate software compensation systems. Also, the technique must be able to carried out both verification and calibration of any type of CMM. Additionally, a new technique should require a minimum number of mechanical transfer standard and should be simple to use and measure.

2. PROPOSED NOVEL SPACE FRAME TO MEASURE VOLUMETRIC ERRORS OF CMMs

In order to measure the volumetric errors of CMMs a novel form of space frame was designed and manufactured in this research (Silva, 1997). This space frame has the form of a tetrahedron which contains a sphere at each apex. The base of the tetrahedron comprises a ball plate that contains three high accuracy spheres. Each tetrahedron contains three magnetic ball links. A simple magnetic ball link comprises a link connecting magnetically to two spheres whose sphericity is better than 0.1μm.. One sphere is located on the ball plate and the other one is at a space point where three links are connected together, as shown in figure 1. This space frame will be referred to as a modular space frame as different configurations of tetrahedron can be obtained easily and rapidly. The number of possible configurations for a given three ball plate is a function of the number of available links of different length and is given by the following equation:

$$N = \frac{n!}{(n-3)!} \qquad (1)$$

where,

N= number of possible configurations.
n!= factorial of n.
n= number of available links of different length.

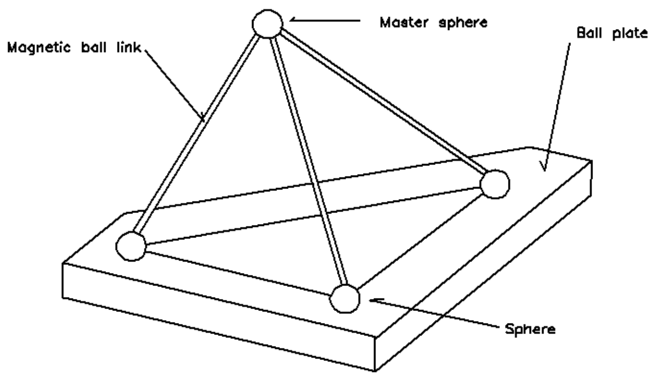

Figure 1- Typical modular space frame

A mechanical artefact before being used to measure the volumetric error of a CMM must be calibrated. Unlike other types of mechanical artefacts, the modular space frame developed in this research is very simple to be calibrated. That is because the modular space frame itself needs not be calibrated. The calibration process only involves in calibrating the elements that define the modular space frame, i.e. magnetic ball links and ball plate. Then, a computer program calculates the co-ordinates of the apices of the tetrahedron by using the calibrated data. A calibration technique based on a laser interferometer system, which is capable of calibrating both magnetic ball links and ball plate, was developed and applied in this research (Silva, 1996).

3. BACKGROUND TO MATHEMATICAL MODELLING OF VOLUMETRIC ERRORS OF CMMs

In this research an approach to volumetric accuracy analysis of CMMs has been developed. The approach consists of a general mathematical model to represent the volumetric errors of CMMs. In addition, the proposed approach is capable of deriving the parametric error components from the volumetric error data. Response surface methodology has been applied to fit a mathematical model to represent the volumetric errors of CMMs. Response surface methodology, or RSM, is a collection of mathematical and statistical techniques that are useful for the modelling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimise this response (Montgomery, 1991). In practice, most response surface methodology problems can be established by utilising either a first order model such as:

$$g(X,\beta) = \beta_o + \beta_1 + \beta_2 + ... + \beta_k X_k \qquad (2)$$

or a second order model such as:

$$g(X,\beta) = \beta_o + \sum_{i=1}^{k}\beta_i X_i + \sum_{i=1}^{k}\beta_{ii} X_{ii}^2 + \sum_{i=1}^{k}\sum_{i=1}^{k}\beta_{ij} X_i X_j \quad (3)$$

Then, a response Y can be written as

$$Y = \beta_o + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k + \varepsilon \quad (4)$$

or

$$Y = \beta_o + \sum_{i=1}^{k}\beta_i X_i + \sum_{i=1}^{k}\beta_{ii} X_i^2 + \sum_{i=1}^{k}\sum_{j=1}^{k}\beta_{ij} X_i X_j + \varepsilon \quad (5)$$

Generally, the assumption of common error variance σ^2 is made, suggesting the use of ordinary least squares for the estimation of coefficients β's. In this research, matrix formulas for the method of least squares have been applied.

Fitting and analysing response surface is greatly facilitated by the proper choice of an experimental design. Experimental design is referred to the process of planning the experiment in such a way that appropriated data, which can be analysed by statistical methods, will be collected resulting in valid and objective conclusions (Montgomery, 1991). In this research, the experimental design used to collect data is defined by the points generated by the modular space frame mentioned in section 2.

3.1 Fitting mathematical models to represent the volumetric accuracy

The general equation that represents each volumetric error component (Ex, Ey, Ez) can be written either by using a first-order mathematical model, that is,

$$E_k(X_1, X_2, X_3) = \beta_{ko} + \beta_{k1} X_1 + \beta_{k2} X_2 + \beta_{k3} X_3 \quad (6)$$

or by using a second-order mathematical model, that is,

$$\begin{aligned} E_k(X_1, X_2, X_3) = {} & \beta_{ko} + \beta_{k1} X_1 + \beta_{k2} X_2 + \beta_{k3} X_3 + \beta_{k4} X_1^2 + \beta_{k5} X_2^2 + \beta_{k6} X_3^2 \\ & + \beta_{k7} X_1 X_2 + \beta_{k8} X_1 X_3 + \beta_{k9} X_2 X_3 \end{aligned} \quad (7)$$

where,
k=x, y or z

In both equation (6) and (7) the coefficients β's are to be estimated by the method of least squares where the basic formula is given by the following equation:

$$\beta_k = (XX^T)^{-1} X^T Y_k \quad (8)$$

where,

k=x, y or z and it is related to X, Y and Z direction , respectively.
$\mathbf{Y}_k$= the vector of error component Ex, Ey or Ez in X, Y or Z direction, respectively.
X^T= Transposed of matrix **X**
X= the matrix of independent or predictor variables X_1, X_2 and X_3 .

A computer program has been developed, in this research, to calculate the coefficients β_{ki} of equations 6 and 7. The performance of this computer program was verified by attributing synthetic data to the coefficients $\beta_{ki.}$ The vector of error component Y_k was simulated by considering the co-ordinates of the points generated by the modular space frame and the synthetic values of the coefficients β_{ki}. Then, the computer program calculated the coefficients β_{ki} following the equation 8 and using the simulated values of the vector Y_k along with the matrix **X** that contains the co-ordinates of the points generated by the modular space frame. The calculated coefficients were very close to the input synthetic values in both the first and second-order mathematical model as can be seen in tables 1 and 2, respectively. The small difference between the synthetic and calculated coefficients might have been caused by rounding errors.

Table 1 - Synthetic and calculated coefficients β_{ki} (equation 6)
(First-order mathematical model)

	β_0	β_1	β_2	β_3
Synthetic coefficients	4.000	-3.000	2.000	-5.000
Calculated coefficients	4.002	-3.001	2.001	-5.002

Table 2 - Synthetic and calculated coefficients β_{ki} (equation 7)
(Second-order mathematical model)

	β_0	β_1	β_2	β_3	β_4
Synthetic coefficients	-5.000	1.000	3.000	2.000	4.000
Calculated coefficients	-4.999	0.997	2.996	2.004	3.999
	β_5	β_6	β_7	β_8	β_9
Synthetic coefficients	1.000	2.000	-4.000	-2.000	-3.000
Calculated coefficients	1.002	2.003	-3.999	1.998	-2.999

The approach developed in his research was applied on a LK CMM. It is a computer numerically controlled co-ordinate measuring machine of the moving bridge type. The machine has XYZ travels of 600x500x400 mm, respectively. Basically, the machine construction comprises: a granite surface table which has a matrix of threaded holes (M10) which is used to locate and clamp components to be measured; X axis guideway that consists of a granite straight edge bonded on the CMM table; Y axis guideway and a Z axis spindle which are both made from ceramic material. The volumetric error data obtained from that practical application are used to establish mathematical models to represent the volumetric error components of the LK CMM. Also, based on the background described by Box (1978) the adequacy of the fitted mathematical models is performed.

3.2 Results of the utilisation of RSM to represent the volumetric error components of the LK CMM.

Two cases have been considered to represent the volumetric error component in the X direction. Initially a first-order mathematical model was fitted. Next, a second-order mathematical model was developed. In both cases the residuals (Y_i - Y_{if}) were plotted against the fitted value, Y_{if}, obtained from the fitted mathematical model. Those plots are shown in figures 3 and 4. By analysing the residual plots shown in figures 3 and 4, it was found that the second-order model more adequately represents the measured data. The analysis of variance concerning the second-order mathematical model, table 3, allows the F-test for significance of regression to be established. The F ratio calculated is F=26.76 and from the F distribution table the 95% point F(9,26,0.95) is equal 2.27. Since the calculated F exceeds the critical F value in the Table 3, that is F=26.76 > 2.27, the overall regression is statistically significant. Therefore, the second-order mathematical model has been selected to represent the volumetric error component in the X direction. The regression equation can be written as follows:

$$\begin{aligned} Ex(X_1,X_2,X_3) = & -4.6296 - 3.0748X_1 - 2.8401X_2 - 3.4202X_3 \\ & +1.4212X_1^2 - 2.5820X_2^2 - 0.0453X_3^2 \\ & +0.8896X_1X_2 - 2.1331X_1X_3 - 1.6848X_2X_3 \end{aligned} \quad (9)$$

where,

X_1, X_2 and X_3 are coded variables

Table 4 presents the analysis of variance concerning the second-order model related to Y direction. The F ratio calculated is F=14.50 and from the F distribution table the 95% point F(9,26,0.95) is equal 2.27. Once the calculated F is greater than the critical F value in the table, that is F=14.50 > 2.27, the fitted regression model is statistically significant. Thus, the second-order mathematical model has been selected to represent the volumetric error component in the Y direction. The regression equation is given as follows:

$$\begin{aligned} Ey(X_1,X_2,X_3) = & \ 4.4817 - 0.3694X_1 + 2.5662X_2 + 2.7662X_3 \\ & +0.9425X_1^2 + 1.2064X_2^2 - 1.3123X_3^2 \\ & -2.3131X_1X_2 + 1.8896X_1X_3 + 0.6940X_2X_3 \end{aligned} \quad (10)$$

where,

X_1, X_2 and X_3 are coded variables.

Table 5 establishes the analysis of variance related to second-order model related to Z direction. The F ratio calculated, shown in table 5, is 12.94. The 95% point F(9,26,0.95), which is obtained from F distribution table is 2.27. Since the calculated F exceeds the critical F value in the table, that is F=12.94 > 2.27, the overall regression is statistically significant. Thus, the second-order mathematical model has been selected to represent the volumetric error component in the Z direction. The regression equation is defined as follows:

$$\begin{aligned} Ez(X_1,X_2,X_3) = & -5.8200 - 1.3586X_1 - 2.4692X_2 + 2.5301X_3 \\ & +5.7018X_1^2 + 0.0191X_2^2 + 0.2181X_3^2 \\ & +2.7005X_1X_2 - 4.7821X_1X_3 + 0.1136X_2X_3 \end{aligned} \quad (11)$$

where,
X_1, X_2 and X_3 are coded variables.

Table 3 - Analysis of Variance - ANOVA (Ex)

Source of variance	Sum of squares	Degrees of freedom	Mean square	Fcal
Regression	328.1676	9	36.4631	26.76
Residual	35.4264	26	1.2626	
Total	364.5149			

Table 4 - Analysis of Variance - ANOVA (Ey)

Source of variance	Sum of squares	Degrees of freedom	Mean square	Fcal
Regression	173.6991	9	19.2999	14.50
Residual	34.6074	26	1.3311	
Total	208.5578			

Table 5 - Analysis of Variance - ANOVA (Ez)

Source of variance	Sum of squares	Degrees of freedom	Mean square	Fcal
Regression	99.3816	9	11.0424	12.94
Residual	22.1921	26	0.8535	
Total	121.5282			

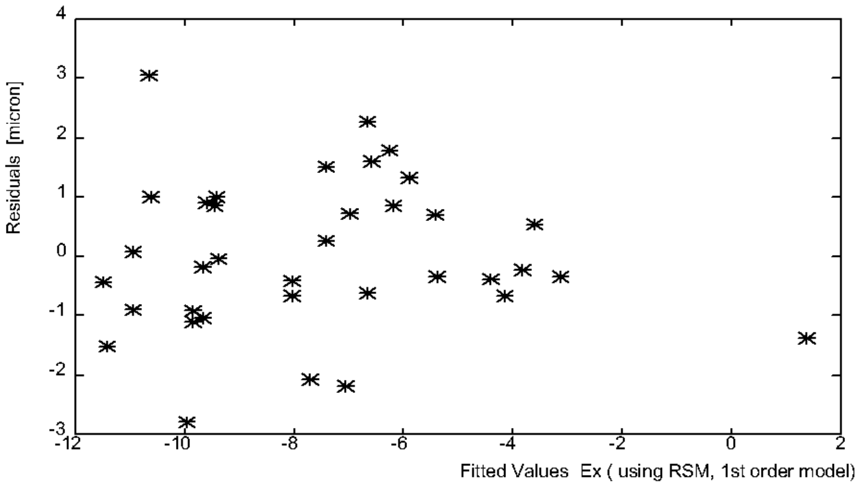

Figure 2 - Residuals plotted against fitted values, Ex (First order mathematical model)

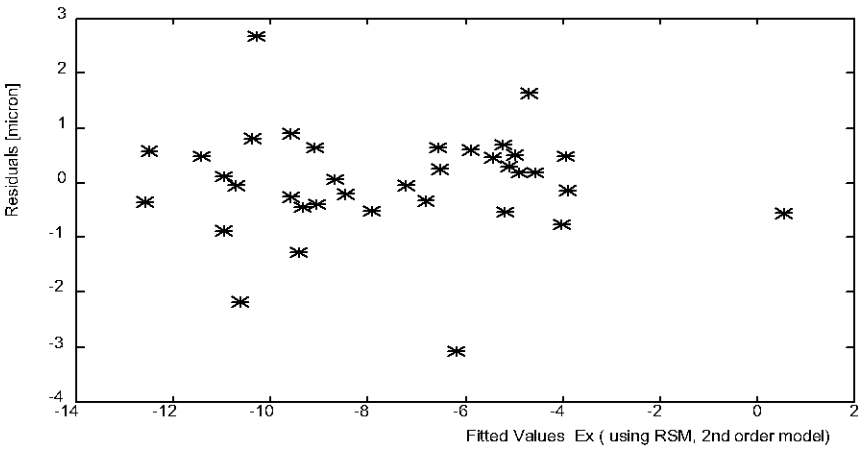

Figure 3 - Residuals plotted against fitted values, Ex (Second order mathematical model)

3.3 Representation of the volumetric error components of CMMs

Once the equations that represent the volumetric error have been established, it is possible to construct the volumetric error map of a CMM under test. For example, by using the equations 9, 10 and 11 the volumetric error components Ex, Ey and Ez can be calculated at any point within the measuring volume defined by the modular space frame. Thus, the volumetric error map can be obtained by using the regression equation that define the volumetric error component and the co-ordinates (X_i,Y_i, Z_i) of points that define a particular grid on a select reference plane within the measuring volume.

Response surfaces that represent the volumetric error components can be also plotted as contour lines or contour diagram. This feature facilitates visualisation of the shape of a response surface. In practice, the contour lines are obtained by slicing off portions of the response surface at various horizontal levels and then projecting the outlines of the slices onto the base which is defined by the selected reference plane (Box, 1978). Contour lines enable one to identify which region of the selected reference plane the maximum and minimum values of the volumetric error component are found. Also, by considering a line on the axis of movement it is possible to derive parametric errors, such as positioning and straightness error, from the contour line diagrams.

Figure 4 shows the response surface which represents the volumetric error component in the X direction at plane Z=0. Figure 5 shows the contour lines that have been obtained from the figure 4. From figure 5 it is possible to derive the positioning error, $\delta_x(X)$, along the X axis at Y=0. That is indicated by the intersection points between the contour lines and the X axis. Also, from figure 5 the straightness error, $\delta_X(Y)$, can be derived. This error is obtained by considering the intersection points between the contour lines and the Y axis.

Figure 6 shows the response surface that represents the volumetric error component in Y direction at plane Z=0. Figure 7 shows the contour lines that have been obtained from the figure 6. By using figure 7 it is possible to derive the positioning error, $\delta_y(Y)$, along the Y axis at X=0. That is indicated by the intersection points between the contour lines and the Y axis. Also, from the figure 7 the straightness error, $\delta y(X)$, can be derived. This error is obtained by considering the intersection points between the contour lines and the X axis.

The response surface that represents the volumetric error component in Z direction at plane Z=0 is shown in figure 8. The contour lines that have been obtained from this figure are shown in figure 9. The straightness error, $\delta z(X)$, can be derived from figure 9. This error is obtained by considering the intersection points between the contour lines and the X axis. Also, from the figure 9 the straightness error, $\delta z(Y)$, can be derived. This error is obtained by considering the intersection points between the contour lines and the Y axis.

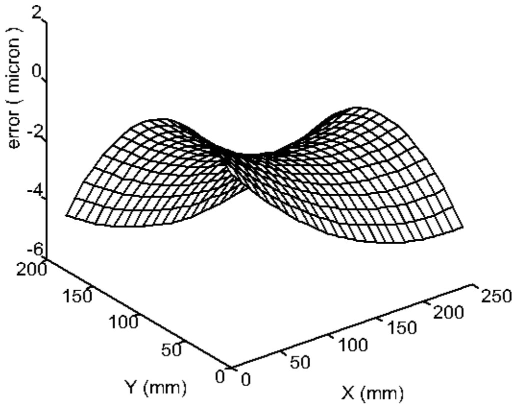

Figure 3 - Volumetric error component in the X direction (at plane Z=0)

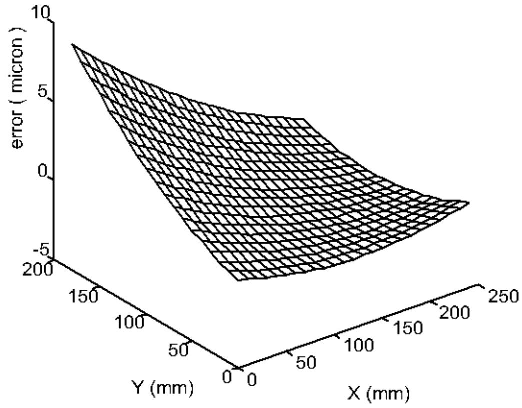

Figure 5 - Volumetric error component in the Y direction (at plane Z=0)

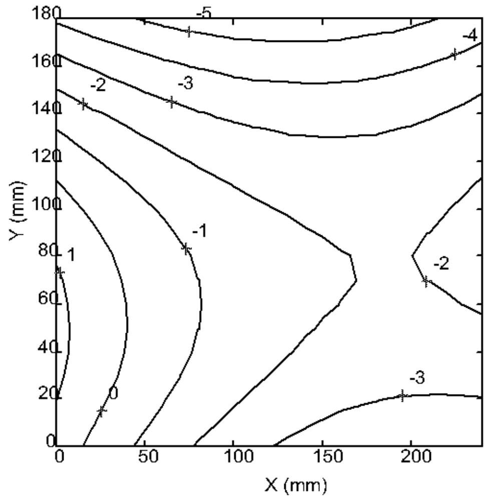

Figure 4 - Contour lines of the volumetric error component in the X direction (at plane z=0)

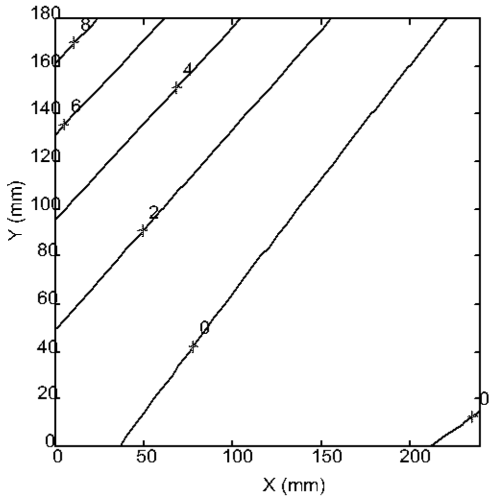

Figure 6 - Contour lines of the volumetric error component in the Y direction (at plane Z=0)

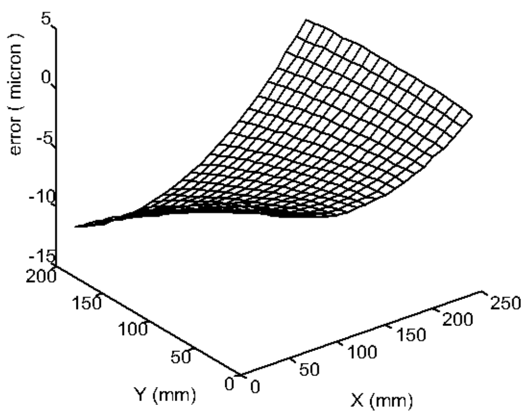

Figure 7 - Volumetric error component in the Z direction (at plane Z=0)

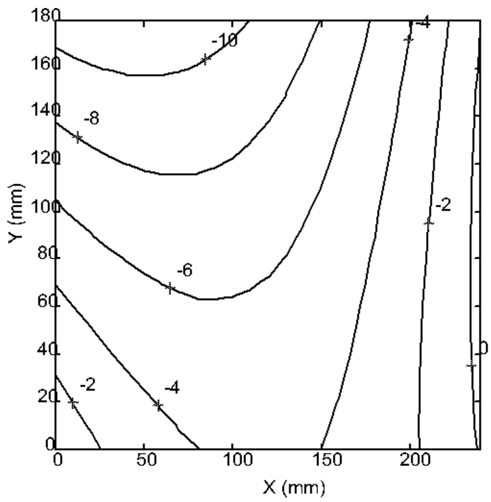

Figure 8 - Contour lines of the volumetric error component in the Z direction (at plane Z=0)

4. CONCLUSION

The technique described in section 3 provides an efficient and practical method for establishing the volumetric accuracy analysis of CMMs. This is because the contour line diagrams facilitate the understanding of the shape of the response surface of each volumetric error component. The proposed technique, to mathematical modelling of volumetric errors of CMM, provides an efficient approach for measurement and analysis of the volumetric accuracy of any type of CMM. It enables one to identify which region of the selected reference plane the maximum and minimum values of the volumetric error component are found.

5 REFERENCES

Box, G. E., Hunter, W. G. and Hunter, J. S., 1978, Statistics for Experimenters. J. Wiley, New York.

Bryan, J. B., 1982, A simple method for testing measuring machines and machine tools, Part 1: Principles and Applications. *Precision Engineering*, Vol. 4, n. 2, pp. 61-68, Part 2: Construction Details. *Precision Engineering*, Vol. 4, n. 3, pp. 125-138.

Burdekin, M. and Jywe, W., 1991, Application of CONTISURE for the verification of the contouring performance of precision machines. *Proceedings of the 6th International Precision Engineering Seminar*, Braunschweig, Germany, pp. 107-123.

Burdekin, M. and Voutsadopoulos, 1981, Computer aided calibration of the geometric errors of multi-axis co-ordinate measuring machines, *Proceedings Institution Mechanical Engineers*, Vol. 195, n. 20, pp. 231-239.

Huang, P. S. and Ni, J., 1995, On line error compensation of co-ordinate measuring machines. *International Journal of Machine Tools and Manufacture*, Vol. 35, n. 5, pp. 725-738.

Kunzmann, H., Trapet, E. and Waldele, F., 1993, Concept for the traceability of measurements with co-ordinate measuring machines. *Proceedings of the 7th International Precision Engineering Seminar*, Kope, Japan, pp. 40-52.

Kunzmann, H., Waldele, F., 1983, On testing co-ordinate measuring machines (CMM) with kinematic reference standards (KRS), *Annals of the CIRP*, Vol. 32, n. 1, pp. 465-468.

Montgomery, D. C., 1991. Design and Analysis of Experiments, 3rd ed., J. Wiley, New York.

Pahk, H. J. and Burdekin, M., 1991, Evaluation of the effective parametric errors in co-ordinate measuring machines using the locus of stylus on the horizontal plane. *Proc Instn Mech. Engrs*, Vol. 205, pp. 123- 138.

Silva, J. B. A., 1997, Volumetric accuracy measurement and analysis of co-ordinate measuring machines, Ph. D. thesis, Department of Mechanical Engineering, UMIST,UK.

Zhang, G. X. and Zang, Y. F., 1991, A Method for machine calibration using 1-D ball array, *Annals of the CIRP*, Vol. 40, n. 1, pp. 519- 522.