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IDENTIFICATION OF DYNAMIC LOADS: A COMPARATIVE ANALYSIS BETWEEN TIME AND FREQUENCY DOMAIN TECHNIQUES

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Abstract. *Force identification has become a topic of interest in the modal and vibration testing community since early 1980's. Similarly to most identification techniques, the knowledge of input forces (and possibly moments) acting on a given structure from response measurements represents na inverse problem in mechanics. Inverse problems are notoriously known for their inheirent numerical ill conditioning since they represent a deconvolution operation in the time domain. Two methods have been used to estimate the force from measured motions, the pseudo-inverse technique and the SWAT method. The pseudo-inverse technique is a frequency domain based dorce identification method that requires the knowledge of the measured structure's responses (usually accelerations) as well as the structure's frequency function (FRF) model. It requires na inversion of the system's FRF matrix for all spectral lines in the frequency range of interest. This inversion process usually offers numerical difficulties at some frequencies. The SWAT (Sum of the Weighted Accelerations Technique) is a time domain technique basead on the principle of motion of the mass center. It employs the measured time domain acceleration signals in the identification process and requires that suitable weighting factors be defined in order to give goods estimates od the resulting force acting on the structure (and possibly its line of action). The main objective of this paper is to compare these two force identification techniques, describe their main advantages as well as main pitfalls in the identification of some special types of excitation signals commonly employed in modal and vibration testing.*

Keywords: *pseudo-inverse, SWAT, modal testing, force identificaiton.*

1. INTRODUCTION

The identification of the excitation forces that act on a given structure under study plays an important role in the modal and vibration testing context. Knowledge of the input forces that cause some measured vibration is important due to a number of reasons. First, one might be interested in validating a given finite element model of the structure under test [EWINS, 1984]. Then, if some experimental data is available, a set of forces can be calculated and it can be used as inputs to the FE model. Second, one might be interested in obtaining a set of input forces that will be applied to the structure under test in the laboratory environment [McONNELL, 1995] in order to reproduce or predict the strucuture's dynamic behaviour in the field environment. Third, a given component or assemblage may be failing in the field environment and the knowledge of the excitation forces is important in order to determine the cause of the failure and possibly the structure's modes of failure.

Force identification represents an inverse problem in mechanics, since it usually requires that the input force(s) be determined from measured responses and the structure's dynamic characteristics. Inverse problems are notouriously known to offer numerical difficulties in predicting the input forces. Force prediction algorithms can be formulated in the time and frequency domains. When using the data in the time domain, the SWAT (Sum of the Weighted Accelerations Technique) [PRIDDY, T. G and GREGORY, D.L., 1986] is frequently employed. This force identification technique is based on the principle of motion of the mass center used in rigid body dynamics. It requires the knowledge of the structure's measured response in the time domain and determines a resulting force acting on the structure as well as its line of action.

The pseudo-inverse technique [VAROTO, 1996] operates in the frequency domain and requires the knowledge of the structure's Frequency Response Functions (FRFs) as well as the measured responses at several locations in the frequency domain. A set of pseudo-forces is obtained from this technique and under some special circumstances [VAROTO, 1996] these forces will resemble the true input forces that act on the structure.

Both the SWAT method and the pseudo-inverse technique presents their own features, and the objective of this paper is to perform a comparison of both in the identification of impulsive typed of dynamic loads. A simple experiment is performed on a free free beam and the data obtained is used in the prediction of the applied impulsive force to the beam.

2. BASIC THEORY OF FORCE IDENTIFICATION

In this section the basic theory concerning the pseudo-inverse technique and the SWAT method is reviewed.

2.1. Pseudo-Inverse Technique

The frequency domain equations of motion of a linear structure for n_1 response measurements and n_2 external input forces is given as [EWINS, 1984]

$$\{X(\omega)\}_{n_2 x1} = [H(\omega)]_{n_2 xn_1} \{F(\omega)\}_{n_2 x1} \tag{1}$$

where $\{X(\omega)\}$ is the frequency domain acceleration response vector ,

$\{F(\omega)\}$ is the frequency domain input forces vector,

$[H(\omega)]$ is the structure's accelerance FRF matrix.

The pseudo-inverse equation is obtained from Eq.1 by solving for the input forces. Thus, the following result is obtained

$$\left\{F(\omega)\right\} = [H(\omega)]^{+}\left\{X(\omega)\right\} \tag{2}$$

onde $[H(\omega)]^{+}$ designates the pseudo-inverse of the structure's FRF matrix, and it is given in a least squares sense as

$$[H(\omega)]^{+} = ([H(\omega)]^{T}[H(\omega)])^{-1}\ [H(\omega)]^{T}$$

Hence, in order to get the input forces, the pseudo-inverse technique requires that the structure's FRF matrix be inverted for all points in the frequency range. This inversion process usually presents some numerical difficulties at some specific frequencies where one or more singular values of the FRF matrix drops to a minimum, and the FRF matrix becomes nearly singular [VAROTO, 1996]. In order to overcome this numerical difficulty, the inversion of the structure's FRF matrix is performed in a least squares sense as shown above. In this case, the number of measured motions is greater than the number of unknown input forces ($n_1>n_2$). Another alternative to improve the conditioning of the pseudo-inversion process is the use of regularization algorithms, such as the singular value decomposition [ELLIOTT, 1988] in order to solve for the unknown input force vector. Another disadvantage of the pseudo-inverse technique is the fact that it requires the knowledge of the locations on the structure where the unknown input forces are applied, since the FRF matrix contains the driving point as well as transfer point FRFs connecting the points of load application to the points where the response accelerations are measured.

Despite the numerical problems that can arise when employing the pseudo-inverse technique, it presents the advantage of identifying a unique set of input forces for each set of measured acceleration responses. In some cases, specially when the location of the unknown forces is known, the identified forces will result close to the true forces.

2.2. The SWAT Technique

The SWAT (**S**um of the **W**eighted Accelerations **T**echnique) [PRIDDY et al, 1988] is a time domain based force identification technique that employ weighted acceleration signals in order to get an estimate of the resulting force that act on a flexible structure. Its working strategy is based on the principle of motion of the mass center used in rigid body dynamics. In some cases this method also gives an indication of the line of action of this resulting force. This technique has been primarily used in the identification of impact signals [PRIDDY et al, 1986]. The key for succeding in the application of this technique relies on the choice of suitable weighting factors for the measured acceleration signals in order to solve for the resulting input force. Clearly, this technique presents the disadvantage of allowing the identification of a single force, while the pseudo-inverse technique gives a set of forces. However, it requires no matrix inversion as it is required in the pseudo-inverse technique, what facilitates the computations. Another key issue in employing the SWAT method is the choice of suitable acceleration measurement points on the structure. Figures 1 and 2 illustrates the technique. Figure 1 shows a hypotetical structure subjected to a resulting force F(t) and instrumented at N points where the acceleration response signals are measured.

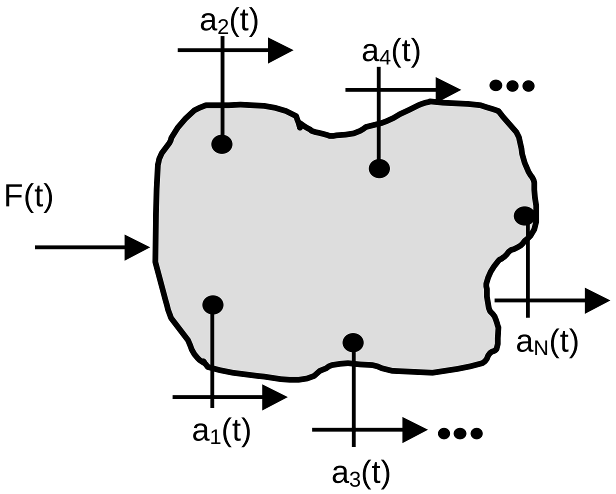

Figure 1: Instrumented structure for acceleration measurements

Figure 2 shows the flow chart of measured acceleration signals being affected by suitable weights and summed to give the resulting force that acts on the structure.

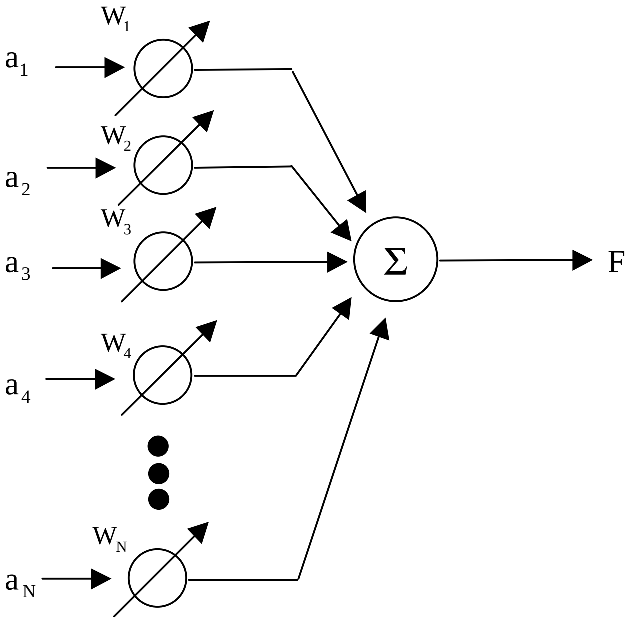

Figure 2: Flow chart of SWAT thechnique

In the time domain, the equations of motion of the structure under test is given as by the well known discrete model

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \tag{3}$$

where $[M]$ is the mass matrix;

$[C]$ is the viscous damping matrix,

$[K]$ is the stiffness matrix ;

$\{f(t)\}$ is the vector of unknown applied forces ;

$\{x(t)\}$ is the displacement response vector in the geometric coordinates ;

The response vector can be expressed as

$$\{x(t)\} = [\Phi]\{q(t)\} \tag{4}$$

where [Φ] is the mode shapes matrix and {q(t)} is the vector of generalized coordinates. Substitution of Equation 4 into the Eq. 3 and pre-multiplying both sides of the resulting equation by $[\Phi^r]^T$ gives the following resulting equation

$$\{\Phi_i^r\}^T[M][\Phi]\{\ddot{q}(t)\} + \{\Phi_i^r\}^T[C][\Phi]\{\dot{q}(t)\} + \{\Phi_i^r\}^T[K][\Phi]\{q(t)\} = \{\Phi_i^r\}^T\{f(t)\} \tag{5}$$

By making use of the orthogonality relationships and considering that the generalized stiffness and damping forces are negligible for a rigid body, Eq. 5 leads to the following result

$$m_i^r \ddot{q}_i(t) = \{\Phi_i^r\}^T\{f(t)\} \tag{6}$$

where m_i^r is the generalized mass corresponding to the r^th^ mode shape vector $\{\Phi_i^r\}$, that is given as

$$m_r = \{\Phi_i^r\}^T[M]\{\Phi_i^r\} \tag{7}$$

Hence, the following result is obtained from Eqs. 6 and 7

$$\{\Phi_r^T\}[M][\Phi]\{\ddot{q}(t)\} = \{\Phi_r^T\}\{f(t)\} \tag{8}$$

Defining a weighting vector as

$$\{w\}^T = \{\Phi_r^T\}[M] \tag{9}$$

and performing a back substitution of Eq. 9 into Eq. 8 the following result is obtained

$$\{w\}^T\{\ddot{x}(t)\} = \{\Phi_r^T\}\{f(t)\} \tag{10}$$

Equation 10 is a basic version of the SWAT technique. The determination of the weighting vector is done by post-multiplying both sides of Eq. 9 by the mode shape matrix

$$\{w\}^T[\Phi] = \{\Phi_r^T\}M[\Phi] \tag{11}$$

The right hand side of Eq. 11 is a column vector with zeroes at all entries except at the rigid body entry. Thus we have a generalized form given as

$$[\Phi]^T\{w\} = \begin{bmatrix} m_r \\ \cdots \\ 0 \end{bmatrix} \qquad (12)$$

Solution of this last linear system of equations gives the required weights for the reconstruction of the input force given in Eq. 10. It should be noticed that the number of weights must be at least the same number of mode shape vectors considered in the discretization problem in order to get a complete and determined solution.

3. EXPERIMENTAL DETERMINATION OF IMPACT FORCES ON FLEXIBLE BODIES

In order to verify the feasibility of the SWAT method and the pseudo-inverse technique, a simple experimental analysis was performed on a cold rolled steel beam (1000 x 25 x 6.25 mm). The beam was suspended by flexible cords in order to simulate the free free boundary conditions. A single impact excitation was used to drive the beam's mid point. The input excitation signal is measured by a B&K 8200 piezoelectric force transducer and conditioned by a B&K 2626 signal conditioner. The acceleration response signals are measured by small piezoelectric accelerometers (B&K model 4375) and conditioned by the B&K 2626 signal conditioner. The beam's output acceleration is measured at nine different locations. Input and output signals are fed into a four channel Tektronix 2630 spectrum analyzer that will process the data.

A common feature of the SWAT and pseudo-inverse techniques is that they require that the output accelerations be simultaneously measured. This represents a difficulty when a few number of measurement channels is available. In order to overcome this problem, the pendulum impact system shown in Figures 3a and 3b was built. Figure 3a shows the beam and the excitation pendulum while Fig. 3b shows details of the impact pendulum. The pendulum was used in order to produce repeatable impacts to the beam so that a few output channels could be measured at a time

Figure 4 shows an ilustration of the data acquisition system used to measure the input and output signals.

3. 1 Results for impact identification

Figure 5a shows the force identification results using the SWAT technique. In this case, the weighting factors were obtained as given by Eq. 12. A significant discrepancy in the amplitude of the identified force when compared with the measured force. Figure 5b shows the identified force obtained for a different set of weights. In this case, the weights have the same value. Again, the SWAT method yelded an identified force greater than the measured force.

Figure 6 shows the result for the pseudo-inverse technique. In this case, the beam's FRFs were measured in order to obtain the FRF matrix to be inverted . A better estimate of the excitation pulse is obtained in this case in comparison with the experimentally measured signal. When compared with the results obtained from the SWAT method, the pseudo-inverse technique yelded better results as well.

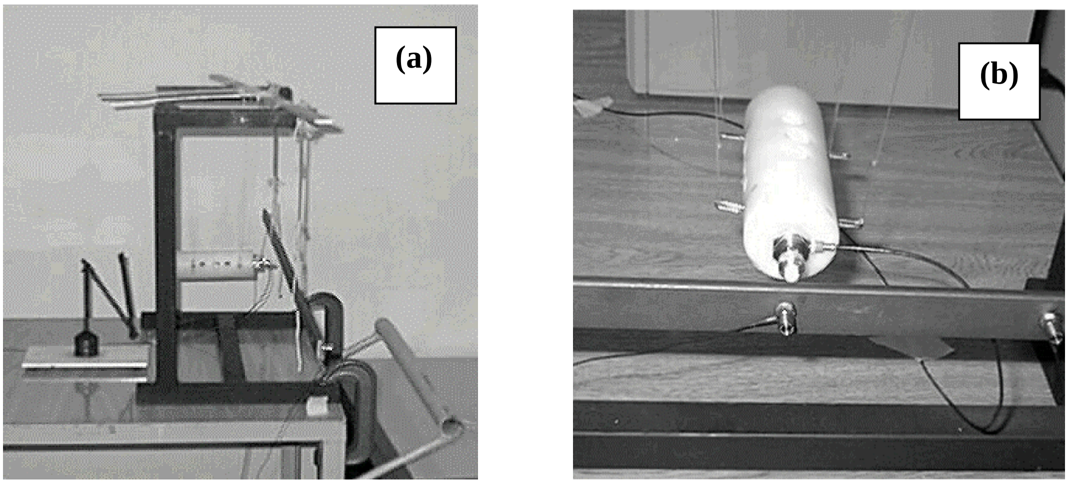

Figure 3 – Beam setup for experimental tests

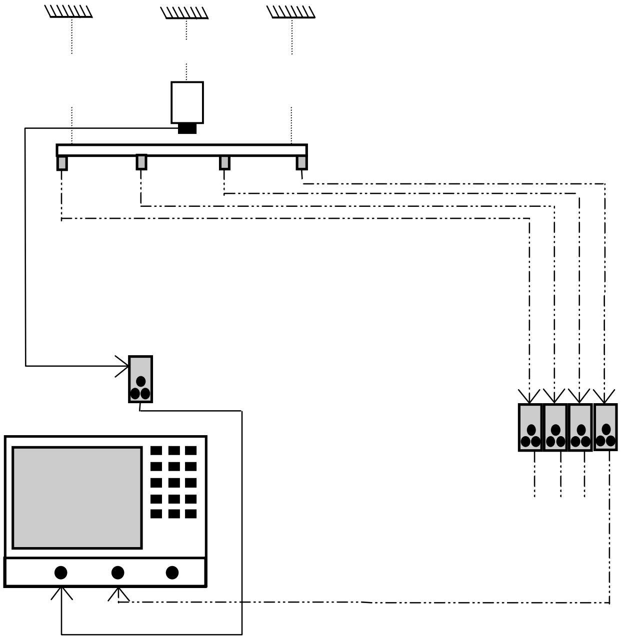

Figure 4 – Ilustration of the data acquisition system

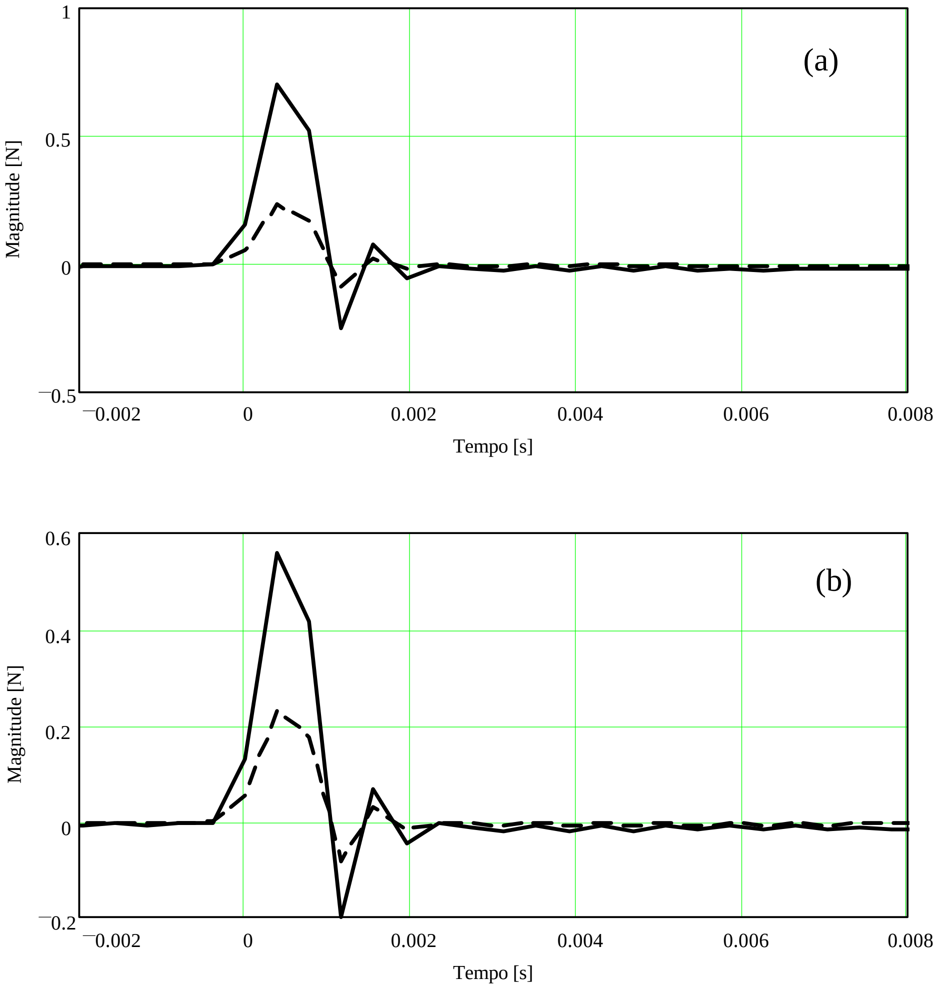

Figure 5 – Measured and identified forces from SWAT method. Solid line: experimental; Dashed line: identified

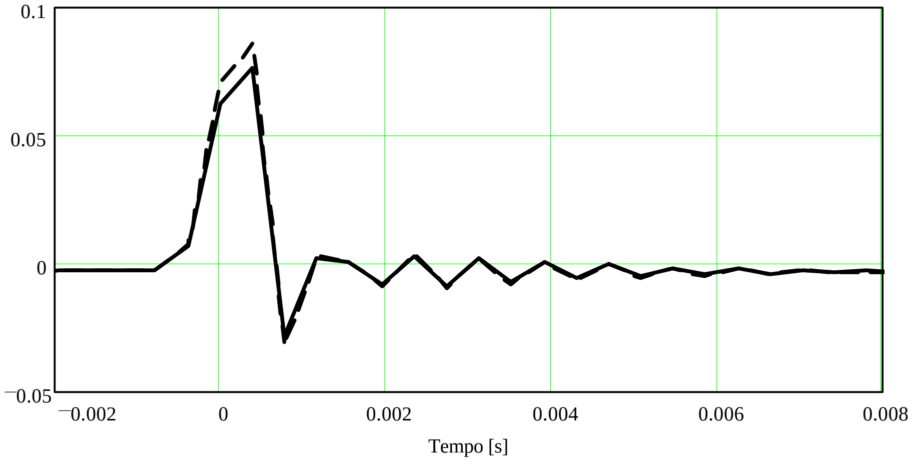

Figure 5 – Measured and identified forces from pseudo-inverse method. Solid line: experimental; Dashed line: identified

4. CONCLUSIONS

Some amplitude distortions were observed in the results from the SWAT method and that were greatly reduced in the pseudo-inverse technique. Some possible reasons for this behaviour can be a non suitable choice of weighting factors when employing the SWAT technique and possibly a lack of repeatibility of the impacts introduced to the beam by the excitation pendulum. The pseudo-inverse technique yelded a good estimate of the excitation force with minor amplitude discrepancies. Further work is being performed by the authors in order to aprimorate the technique as well as extend some ideias in the context of the identification of multiple input forces

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