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# AN ASSESSMENT OF GENETIC ALGORITHMS AS APPLIED TO SOME INVERSE PROBLEMS IN ELASTODYNAMICS

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***Abstract.*** *Genetic algorithms (GA's) are optimization and search procedures based on Darwin's natural selection principles. The main purpose of this work is to perform an evaluation of the GA's when applied to some model-based parameter identification problems in elastodynamics. Such problems are formulated as optimization problems in which the objective functions represent the difference between the experimentally observed dynamic behavior and its analytical counterpart. The following inverse problems are focused: localization and evaluation of the extent of modeling errors from real eigenvalues and eigenvectors and identification of physical parameters of linear and nonlinear mounts from time responses. Based on both numerically simulated and experimentally tested systems, the GA's are appraised in terms of the accuracy of the solutions and robustness with respect to random noise contaminating the data.*

***Key-words:*** *Genetic algorithms, parameter identification, inverse problems, model updating*

## 1. INTRODUCTION

Genetic algorithms have been found to be an interesting computation strategy for treating a number of problems which can be formulated as optimization problems. As opposed to traditional algorithms, which seek the solution starting from a single point of the search space, GA's operate simultaneously with a large number of points, thus increasing the chances of finding the global minimum of the objective function. Furthermore, GA's do not require the knowledge of the gradients of the objective function, whose computation is generally a source of computational burden and numerical inaccuracies.

Within the scope of Mechanical Engineering, damage detection, health monitoring of structures and finite element model updating are some of the topics that have received much attention lately. This problems are generally formulated as inverse parameter identification problems and solved by minimizing cost functions representing the differences between experimental and analytical dynamic characteristics, either in time, frequency or modal domains. The optimization problems have classically been tackled by using traditional gradient-based algorithms (Vanderplaats, 1984). Nevertheless, the capability of these algorithms to provide accurate solutions is restrained by some inherent features of the parameter identification problem, namely: a) the highly complex nature of the objective functions, which are generally non linear in the parameters and exhibit local minima besides the global minimum; b) numerical ill-conditioning, due to a lack of information engendered by the use of incomplete experimental data; c) the presence of experimental uncertainties (noise) corrupting the objective functions. Due to its basic features, GA's have demonstrated to be less sensitive to these drawbacks and this is the main motivation for the increasing number of applications of these algorithms to various parameter identification problems. Friswell et al. (1996) used GA's combined with a standard eigensensitivity method to localize and estimate the extent of damage in vibrating structures. Yap and Zimmerman (1998) focused on finding a new genetic coding for fault identification. Worden and Deacon (1996) addressed the problem of identification of parameters of supports exhibiting nonlinear behavior. An assessment on the use of GA's for solving various kinds of model-based inverse problems in structural dynamics, using different types of experimental data and residual functions has been recently made by Silva (1999).

In this paper, the performance of GA's is evaluated by considering two kinds of inverse problems. The first one is the localization and evaluation of the extent of modeling errors (which can also be interpreted as structural damage), based on residual functions formed from the natural frequencies. The second problem is the estimation of the physical parameters of non linear mounts, by exploring residual functions based on the time-domain responses. For the first problem, the evaluation is made through numerical applications to a real structure, tested in laboratory, whereas for the second one, examples based on numerically simulated mechanical systems are presented.

## 2. GENETIC ALGORITHMS – AN OVERVIEW

In this section, the fundamentals of GA's and their basic operators are briefly reviewed. For a detailed description, the reader should refer to Goldberg (1989).

Genetic algorithms are based on the principles of Darwin's evolutionary law, i.e., they are structured random search techniques that mimic the concepts of natural selection (Holland, 1975).

In its simplest form, GA's comprise three basic operators: *selection, crossover and mutation*. Two other operations are also considered as operators by some authors: coding and evaluation of the fitness function.

The purpose of coding is to put the variables into a genetic design space. In this work a binary codification is used to represent each variable $V_i$ as a $b$-bit binary number, which approximates $2^b$ discrete numbers in the range of the variables, according to:

$$V_i = V_i^{min} + \frac{bin}{2^b - 1}\left(V_i^{max} - V_i^{min}\right), \tag{1}$$

where $V_i^{min}$ and $V_i^{max}$ are the lower and upper bounds of the $i$-th continuous variable and *bin* is an integer number between zero and $2^b$-1.

Considering the minimization of a given objective function, during the evaluation operation, a proper fitness index is assigned to each candidate set in such a way that the lower the value of the objective function associated to an individual candidate, the higher the fitness index given to it. The algorithm starts with an initial population randomly generated over the whole search space. Each member of the population can be seen as a *chromosome* or a binary string. After codification and initial evaluation, genetic algorithms work iteration by iteration on this population of strings, in a way similar to natural population growth, where each generation is evolved into another through reproduction, making use of the three operators.

The *selection* process is responsible for the choice of which individual, and how many copies of it, will be passed to the next generation. An individual is selected if it has a high fitness value, and the choice is biased towards the fittest members. A way to do that is to use a proportional selection procedure, with the number of copies given by $n = f_i / f_{avg}$ , where $f_i$ is the fitness index of the *i*-th individual and $f_{avg}$ is the average fitness of the whole population.

*Crossover* takes two strings (parents) from the mating pool and performs a randomly exchange in some portions between them to form a new string (children). The crossover occurs with a probability $p_c$.

In a binary coding scheme, *mutation* involves switching individual bits along the string, changing a zero to one or vice-versa. This operator keeps the diversity of the population and reduces the possibility that the GA's find a local minimum or maximum instead of the global optimal solution, although this is not ever guaranteed. The mutation occurs with a probability $p_m$ .

## 3. IDENTIFICATION OF FE MODELING ERRORS

### 3.1 Formulation

Let us consider that an undamped structure is modeled by a finite element model represented by stiffness matrix $\boldsymbol{K}^m \in R^{n.n}$ and mass matrix $\boldsymbol{M}^m \in R^{n.n}$, where $n$ is the number of degrees-of-freedom of the model. The stiffness and mass matrices of the real structure are written:

$$\boldsymbol{K}^s = \boldsymbol{K}^m + \Delta\boldsymbol{K} \qquad\qquad \boldsymbol{M}^s = \boldsymbol{M}^m + \Delta\boldsymbol{M} \, , \tag{2}$$

where $\Delta\boldsymbol{K}$ and $\Delta\boldsymbol{M}$ represent the errors in the model.

If the FE model is divided into a number $p$ of subdomains, then:

$$\boldsymbol{K}^m = \sum_{j=1}^{p} \boldsymbol{K}_j^m \qquad\qquad \boldsymbol{M}^m = \sum_{j=1}^{p} \boldsymbol{M}_j^m \, , \tag{3}$$

where $\boldsymbol{K}_j^m$ and $\boldsymbol{M}_j^m$ designate the stiffness and mass matrix associated to the *j*-th subdomain. The symbol $\sum$ in Eq. (3) denotes the operation of assembling of the FE matrices.

The matrix representing the errors can be expressed as linear combinations of the matrices associated to the subdomains:

$$\Delta\boldsymbol{K} = \sum_{j=1}^{p} k_j \boldsymbol{K}_j^m \qquad\qquad \Delta\boldsymbol{M} = \sum_{j=1}^{p} m_j \boldsymbol{M}_j^m \, , \tag{4}$$

where $k_j$ and $m_j$ are scalar dimensionless coefficients.

Introducing Eq. (4) and (3) in (2), the following relations are obtained:

$$\boldsymbol{K}^s = \sum_{j=1}^{p} \bar{k}_j \boldsymbol{K}_j^m \qquad\qquad \boldsymbol{M}^s = \sum_{j=1}^{p} \bar{m}_j \boldsymbol{M}_j^m \ , \tag{5}$$

with $\bar{k}_j = 1 + k_j$ , $\bar{m}_j = 1 + m_j$ , $j = 1$ to $p$.

The coefficients $\bar{k}_j$ and $\bar{m}_j$ , $j$ = 1 to $p$ are dimensionless correction parameters associated to the mass and stiffness of each subdomain. The identification of modeling errors is achieved by determining the values of these parameters, take into account that:

- If $\bar{k}_j$ (or $\bar{m}_j$ ) $= 1$ , the $j$-th subdomain is not affected by stiffness (or mass) errors.
- If $\bar{k}_j$ (or $\bar{m}_j$ ) $\neq 1$ , the $j$-th subdomain is affected by stiffness (or mass) model errors.

The same methodology can be used for identifying structural damage, given the FE model of the undamaged structure. In this case it is generally assumed that damage takes the form of cracks or loosening of connecting elements, like bolts and rivets, leading to stiffness reduction, only. Then, the parameterization of the error is made only in terms of $\bar{k}_j$ , $j$ = 1 to $p$, with the interpretation that $\bar{k}_j \langle 1$ indicates the presence of damage in the $j$-th subdomain.

### 3.2 Experimental test

In order to assess the performance of GA's when applied to the problem of localization and evaluation of the extent of FE modeling errors, a simple uniform free-free beam, shown in Fig. 1, was tested in laboratory. During the vibration tests conducted for obtaining the experimental FRFs, the test-item was suspended by soft strings, excited in the vertical direction with a impact hammer and the acceleration responses were acquired using piezoelectric accelerometers. In Fig. 1 symbols () indicate the positions were the driving point and transfer FRFs were acquired in the band [0-2000 Hz]. Three configurations of the beam were tested: the original one, associated to the healthy structure, and two others containing a localized damaged (or modeling error), which was introduced by making a saw-cut as depicted in Fig. 2. Two degrees of severity of error were simulated by varying the depth and length of the cut, as indicated in Fig. 2.

Figure 3 shows the main characteristics of the FE model used, containing 20 bi-dimensional Euler-Bernoulli beam elements, 21 nodes and 3 degrees-of-freedom per node. In the model, nodes and elements are numbered consecutively from left to right.

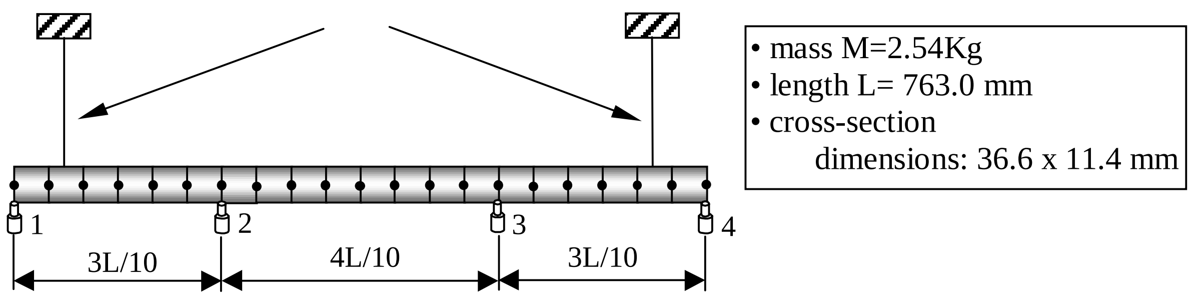
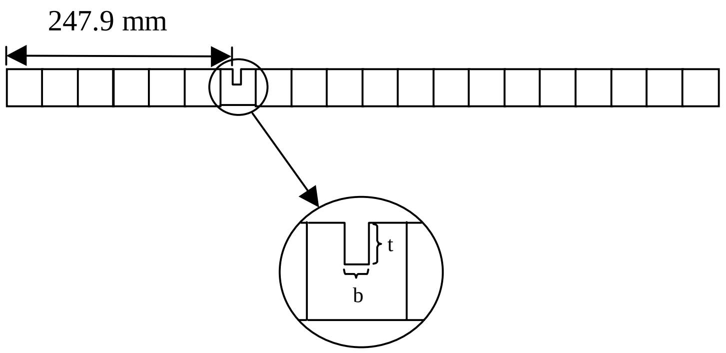
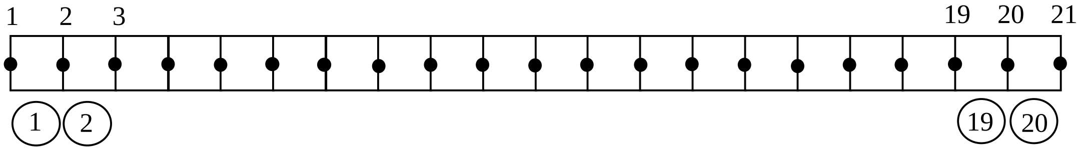



Figure 1 - Characteristics of the test structure




| Severity | b(mm) | t(mm) |
|---|---|---|
| Less severe | 5.0 | 2.5 |
| More severe | 8.0 | 4.0 |

Figure 2 - Saw-cut simulating modeling errors




$\rho$ = 7800 Kg/m$^3$ | Length L= 763.0 mm
cross-section dimension: 36.6 x 11.4 mm | E = 2.1x10$^{11}$ N/m$^2$
j: j-th element | i: i-th node

Figure 3 - Characteristics of the FE model

Figure 4 shows enables to compare the driving point FRF $H_{44}(\omega)$ of the healthy structure to that of the FE model. As can be seen, the analytical model seems to be less stiff than the real structure. In order to prevent this difference from affecting the identification results, the FE model was first adjusted by searching corrections for the bending stiffness of each of the 20 elements. The optimum correction parameters were obtained by using GA's for minimizing the differences between the natural frequencies of the structure and their analytical counterparts. Table 1 shows the results of the adjustment, whereas in Table 2 the values of the experimental eigenfrequencies for the undamaged and damaged configurations are given.

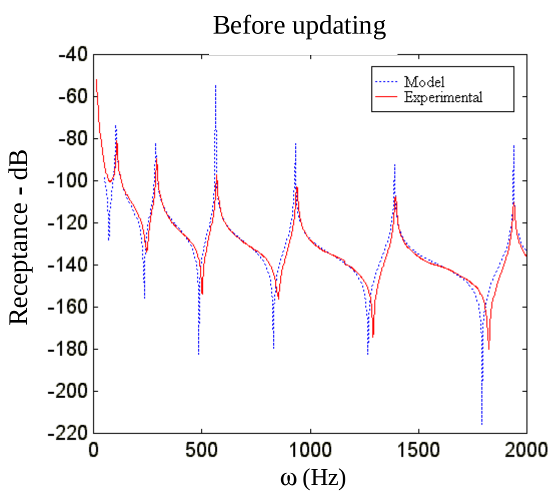



Figure 4 – FRF $H_{44}$ ($\omega$)

Table 1. Experimental and analytical eigenfrequencies of the undamaged beam (Hz)

| Elastic Modes | Experim. (Hz) | Model Before updating | Model After updating |
|---|---|---|---|
| 1 | 107.5 | 104.4 | 107.5 |
| 2 | 292.5 | 287.6 | 292.5 |
| 3 | 570.0 | 563.5 | 570.0 |
| 4 | 937.5 | 930.7 | 937.5 |
| 5 | 1393.0 | 1389.1 | 1393.0 |
| 6 | 1940.0 | 1938.4 | 1940.0 |

Table 2. Eigenfrequencies of the undamaged and damaged configurations (Hz)

| Elastic Modes | Undamaged | Less severe damage | More severe damage |
|---|---|---|---|
| 1 | 107.5 | 107.5 | 105.0 |
| 2 | 292.5 | 287.5 | 282.5 |
| 3 | 570.0 | 567.5 | 567.5 |
| 4 | 937.5 | 935.0 | 927.5 |
| 5 | 1393.0 | 1380.0 | 1355.0 |
| 6 | 1940.0 | 1935.0 | 1930.0 |

Given the "calibrated" FE model and the eigenfrequencies of the damaged structure, the problem was to identify the position and intensity of the stiffness perturbation introduced by the saw-cut. According to Section 3.1, error indicators are associated to the bending stiffness of each of the 20 elements of the FE model. The values of these indicators are the unknowns of the identification problem. The GA population size was chosen to be 100 individuals. Other features of the GA computations are: mutation probability=1%; crossover probability=90%; maximum number of generations=100; the fitness index attributed to each individual of the population is given by the minus-signed valued of the objective function, defined as:

$$J = \sum_{j=1}^{q} \frac{\left|\lambda_j^m - \lambda_j^e\right|}{\max\left(\lambda_j^m, \lambda_j^e\right)}, \tag{6}$$

where $\lambda_j^m, \lambda_j^e$, $j$=1 to $q$ are the FE and experimental (of the damaged structure) natural frequencies, respectively.

The most time consuming operation of GA's is the evaluation of the objective function (or fitness index) since it requires the calculation of the eigensolutions of the FE models corresponding to each individual of the population. To reduce the computational burden it is necessary to use efficient solvers for the eigenvalue problem. The Lanczos algorithm (Bathe, 1989), implemented as a MATLAB® code, was used in these applications.

The parameter identification results are shown in Fig. 5 for both the damage scenarios. As can be seen, the GA's were able to indicate approximately the position of the stiffness reduction, ascribing it to two neighboring elements, numbered 6 and 7 (recall that the saw-cut was introduced in a position corresponding to the element number 7). More accurate identification results were obtained for the case of more severe damage, when a more strong stiffness reduction was indicated for element 7.

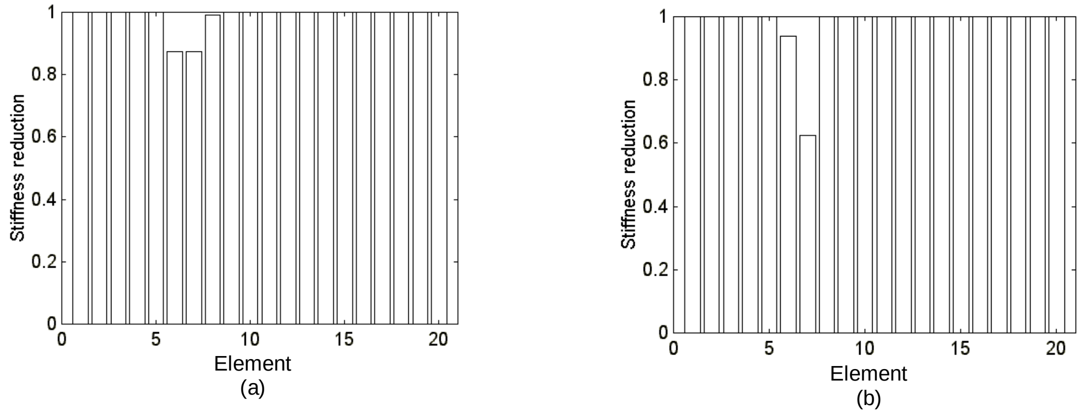



Figure 5 -Identified damage indicators (a) less severe damage ; (b) more severe damage.

By comparing the results obtained for the two cases, it can be seen that the algorithm was capable of distinguishing between to different levels of damage severity, based on the values of the errors indicators provided.

It should be also pointed out that the physical and geometrical symmetry of the FE model makes it impossible to distinguish between the effect of parameter variations affecting different symmetrically positioned elements, based on the observation of eigenfrequencies, only. Thus, although the error location was correctly identified in the numerical tests, the errors could well have been attributed to element number 14. To reduce the effect of symmetry and provide unique error localization results, the experimental data must be enriched by spatial-dependent dynamic responses, such as eigenvector components and/or anti-resonance frequencies (Silva, 1996). Other facts that render more difficult the identification are: $1^{st}$) the physical stiffness perturbation introduced (the saw-cut) is not coherent with the model error adopted (a bending stiffness reduction applied to a whole element of the model); $2^{nd}$) the presence of experimental noise in the data. As can be seen in Table 2, the effect of the saw-cut is to provoke slight reduction of the eigenfrequency values, which are likely to be masked by experimental uncertainties. This can explain why the identification of the less severe stiffness reduction was less accurate than the identification of the more severe one. In spite of these difficulties, the identification results are considered to be satisfactory and demonstrate that the GA's are reasonably robust with respect to noise and model inaccuracies.

## 4. IDENTIFICATION OF PARAMETERS OF NONLINEAR MOUNTS

The problem considered in this section is the identification of parameters of nonlinear supporting elements (SE) of vibrating elastic systems from the time-domain responses. Clearly, supports exhibiting linear behavior can be dealt with as a particular case.

The supports are modeled as nonlinear single-degree-of-freedom systems of negligible mass (as shown in Fig. 6), whose equations of motion are given by:

$$c[\dot{x}_S(t)]^a + k[x_S(t)]^b = F_S(t) \tag{7}$$

where $x_S(t)$ and $\dot{x}_S(t)$ are the displacement and velocity time histories of the coordinate through which the support is connected to the main structure, $F_S(t)$ is the total force impressed by the support to the main structure and *a*, *b*, *c* and *k* designate the parameters to be identified.

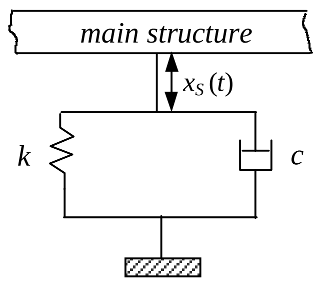



Figure 6 - Single-degree-of-freedom model for a support

The basics of the identification procedure adopted are the following: assuming that the experimental time responses of the real mounted system (in terms of either accelerations, velocities or displacements), as well as an analytical model of the system are available, the parameters of the mounts are estimated by minimizing the following objective function which defines the normalized mean-square deviations between the measured and analytical time responses:

$$J=\sum_{j=1}^{S}\left\{\frac{100}{N\cdot\sigma_{yj}^{2}}\sum_{i=1}^{N}\left[y_j^m(t_i)-y_j^s(t_i)\right]^2\right\}, \tag{8}$$

In Eq. (8), S is the number of measurement coordinates where the responses are acquired, $N$ is the number of points in the time-series, superscripts $^s$ and $^m$ indicate the experimental and analytical time responses, respectively, and $\sigma_{yj}^2$ designate the variance of the experimental time responses. It should be emphasized that, according to the procedure adopted, the analytical model of the main structure is considered to be accurate, so that the only parameters to be identified are those pertaining to the mounts.

To account for experimental noise, random perturbations were added to the simulated time responses using the following model:

$$\tilde{y}_i(t_j)=y_i(t_j)\cdot(1+r_j e_j) \qquad i=1,2,\ldots S \quad e \quad j=1,2,\ldots,N \tag{9}$$

with:

$$e_j=\frac{e_{max}-e_{min}}{|y_i|_{max}}\left(|y_i|_{max}-|y_i(t_j)|\right)+e_{min} \tag{10}$$

where $S$ is the number of instrumented coordinates, $N$ is the number of points in the time-series; $y_i(t_j)$ and $\tilde{y}_i(t_j)$ are the simulated response pertaining to the $i$-th coordinate, without noise and with noise, respectively; $r_j$ designates a sequence of uniformly distributed random numbers in the range [-1; 1]; $|y_i|_{max}$ is the maximum amplitude of the response; $e_{max}$ and $e_{min}$ are respectively the maximum and minimum values of the random errors.

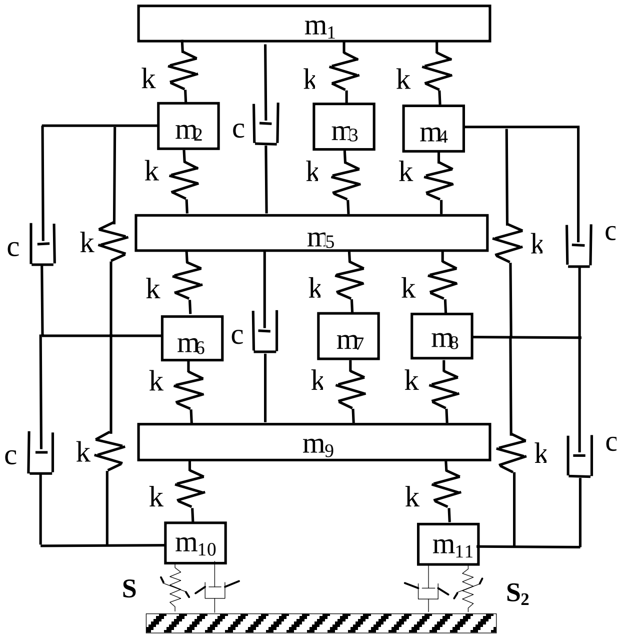
Figure 7 depicts the numerically simulated 11 degree-of-freedom system to which the identification procedure was applied. Since there are two non linear mounts $\mathbf{S_1}$ and $\mathbf{S_2}$, there is a total number of eight parameters to be identified. The primary interest in using this numerically simulated test-system is that, since the exact values of the parameters to be identified are known, conclusions can be drawn concerning the accuracy of the estimates. Moreover, important information concerning the robustness with respect to noise can be obtained by comparing the results obtained from noise-free and noise-contaminated responses.

For obtaining the time-domain responses, a short duration transient force, simulating an impact load, was applied to coordinate 1. The responses were computed by using a Newmark-type step-by-step integration scheme, with a time-step of $2.5\text{x}10^{-4}$s. The responses were assumed to have been observed at coordinates 1, 3, 5, 9, 10 and 11.

For the GA computations a population size of 150 individuals was chosen. The mutation and crossover probabilities were selected as 1% and 90%, respectively. The fitness index of the individuals of the population was defined as the minus-signed value of the objective function, given by Eq. (8). The maximum number of generations allowed for convergence was 100.

For the two cases considered: noise-free responses and noise-contaminated responses, the estimated values of the support parameters are given in Table 3, together with the deviations with respect to the exact values. As expected, parameters obtained from noise-free responses have shown to be more accurate than those obtained from the noise-contaminated responses, though no significant effect of noise amplification is observed. This fact demonstrates the GA's are quite robust with respect to noise.

The higher estimation errors obtained for some parameters can be at least partially ascribed to the fact that the time domain responses are less sensitive to these parameters, as compared to the others. In fact, by comparing the responses of the model with the identified parameters to their counterparts of the exact model (these responses are not shown in this paper), no noticeable difference is observed.




$k = 1.0\text{x}10^6$ N/m
$c = 100.0$ Ns/m
$m = 2.0$ kg
$m_1 = 4m$ ; $m_5 = 4m$ ; $m_9 = 4m$
$m_2 = m_3 = m_4 = m_6 = m$
$m_7 = m_8 = m_{10} = m_{11} = m$

Support parameters

| $S_1$ | $S_2$ |
|---|---|
| $a_1 = 2$ | $a_2 = 2$ |
| $b_1 = 3$ | $b_2 = 3$ |
| $c_1 = 2\times10^3$ Ns/m | $c_2 = 2\times10^3$ Ns/m |
| $k_1 = 7\times10^{11}$ N/m | $k_2 = 7\times10^{11}$ N/m |

Figure 7 – 11 degree-of-freedom system used for identification tests.

Table 3. Estimated nonlinear parameters

| | **Case 1 (without noise)** | | | | | |
|---|---|---|---|---|---|---|
| | **$k_1$ [N/m]** | **$k_2$ [N/m]** | **$c_1$ [N.s/m]** | **$c_2$ [N.s/m]** | **$a_1=a_2$** | **$b_1=b_2$** |
| | $7.33\times10^{11}$ | $7.33\times10^{11}$ | 2088.89 | 2079.54 | 2.02 | 3.01 |
| Error (%) | 4.74 | 4.78 | 4.44 | 3.98 | 0.99 | 0.22 |
| | **Case 2 (with noise: $e_{min}$ =0,5 % , $e_{max}$ =2,0 %)** | | | | | |
| | **$k_1$ [N/m]** | **$k_2$ [N/m]** | **$c_1$ [N.s/m]** | **$c_2$ [N.s/m]** | **$a_1=a_2$** | **$b_1=b_2$** |
| | $6.63\times10^{11}$ | $6.25\times10^{11}$ | 2303.86 | 2246.88 | 2.06 | 2.99 |
| Error (%) | 5.36 | 10.72 | 15.19 | 12.34 | 3.02 | 0.38 |

## 5. CONCLUSIONS

It has been evaluated, in this paper, the performance of genetic algorithms when applied to two kinds of inverse problems: the identification of modeling errors (or structural damage) from modal data and the identification of nonlinear parameters of support elements from time domain responses. The evaluation was made based on either numerically simulated or experimentally tested mechanical systems. Different types of complicating effects were taken into account, such as the incompleteness of experimental data, the presence of experimental noise and inconsistency between model and actual errors. The GA’s computations have shown to be very stable, providing fairly accurate results, even in the presence of noise. Although no attempt was made in this paper to compare the GA's to traditional gradient-based optimization methods, others simple numerical applications performed by Silva (1999), seem to indicate that GA's are indeed more robust with respect to noise and local minima than the traditional algorithms, as claimed by other authors.

Regarding the computational burden, GA’s have shown to be highly time consuming. Nevertheless, it is believed that computation time could be reduced to some extent by an appropriate choice of approximate reanalysis procedures, necessary for evaluating the fitness indexes, and the creation of new operators that could accelerate the convergence by enabling to work with smaller population sizes.

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