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A STUDY ON THE HARDENING IN A PARTICULAR MODEL FOR PSEUDOELASTIC MATERIALS

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Abstract. *In a recent paper, Souza et al. proposed a new mechanical model for pseudoelastic and shape memory alloys in the setting of three dimensional media. Qualitative agreement with experimental stress-strain data was observed when considering both proportional and nonproportional load cases. The goal of the present paper is to introduce new features into the aforementioned mechanical model in order to obtain closer quantitative agreement between model and experiments. In particular, we discuss improvements in the term associated with the hardening of the material, very often observed during stress induced phase transformation processes.*

Keywords: *Pseudoelasticity, Shape memory effect, Hardening.*

1. INTRODUCTION

Pseudoelastic or superelastic materials designate a class of materials which recovers its original shape, after the applied load is removed. It differs from a true elastic material since the stress-strain curve is history-dependent (Fig. 1). From the microscopic point of view, the behavior is associated with a crystalline stress-induced phase transformation between austenite and martensite at a constant room temperature. The stress free material is in the austenitic phase, while mechanical loading provides the energy necessary to promote the transformation into martensite. Upon unloading, the martensite transforms back into austenite.

Several studies on pseudoelastic materials have been performed in the last decade: Gurtin (1983), Ball and James (1987), Abeyaratne and Knowles (1988, 1990, 1992), Auricchio and Taylor (1997), and Pagano *et al* (1998) are representative examples of papers in the area.

Souza *et al*. (1998) proposed a model, written within the framework of Generalized Standard Materials (see, for instance, Maugin (1992)), describing some features of polycrystalline shape memory alloys, in the setting of three dimensional media. In this paper, the results obtained by Sittner *et al*. (1995) are reproduced qualitatively. Nevertheless, some difficulties were found when trying to fit model and experiments for simple traction and pure shear. Motivated by this fact, we propose an improvement associated to the transformation hardening effect observed in the experiments.

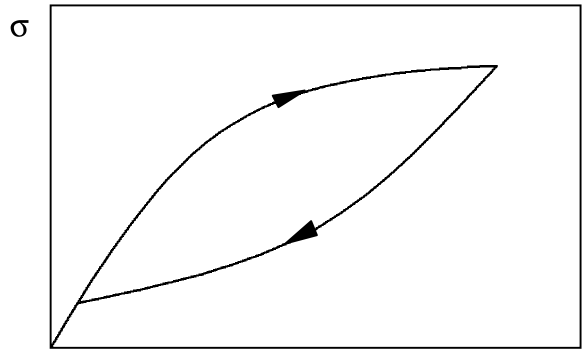

Figure 1 – Stress-strain curve for pseudoelastic materials

2. THE MECHANICAL MODEL

Let ε *and* $\boldsymbol{T}$ denote respectively the classical linear strain tensor and the Cauchy stress tensor. The corresponding deviatoric tensors are defined as:

$$\boldsymbol{e} := \varepsilon - \frac{1}{3}(tr\varepsilon)\boldsymbol{I} \qquad \text{and} \qquad \boldsymbol{S} := \boldsymbol{T} - \frac{1}{3}(tr\boldsymbol{T})\boldsymbol{I} \tag{1}$$

where $\boldsymbol{I}$ is the identity operator in R^3 and *tr* is the trace operator. Inspired on the fact that, from the kinematic point of view, stress induced phase transformations can be characterized as shear deformations, we define a symmetric and deviatoric transformation strain tensor $\boldsymbol{e_T}$ as a measure of the martensite observed in the material: when $\boldsymbol{e_T} = 0$, we say that only austenite is present in the material; a mixture of austenite and martensite is associated with $\| \boldsymbol{e_T} \| > 0$.

We present here a modified version of the mechanical model proposed by Souza *et al*. (1998). Two convex potentials are defined: a Helmholtz free energy density function and a potential of dissipation, from which the constitutive laws and the flow rules are derived. In this context, let us consider as state variables the linear strain tensor ε, its deviatoric part $\boldsymbol{e}$ and the transformation strain tensor $\boldsymbol{e}_T$. The Helmholtz free energy density function is given by:

$$\varphi(tr\varepsilon, \boldsymbol{e}, \boldsymbol{e_T}) := \left(\frac{\lambda}{2} + \frac{\mu}{3}\right)(tr\varepsilon)^2 + \mu \| \boldsymbol{e} - \boldsymbol{e_T} \|^2 + \tau_M \| \boldsymbol{e_T} \| + \frac{1}{2}\boldsymbol{e_T} \cdot \boldsymbol{A}\,\boldsymbol{e_T} \tag{2}$$

where λ and μ are the Lamé constants, τ_M is the Maxwell stress and $\boldsymbol{A}$ is the fourth order hardening tensor of the material.

Souza *et al*. (1998) describe the hardening behavior by considering a functional of the form $\frac{1}{2} h\, \boldsymbol{e}_T^2$, where *h* is a scalar material parameter. As pointed out in that paper, if the slopes in the traction test were used in order to identify the material parameters, then the slope in the stress-strain curve for pure shear would be underestimated. This fact motivated us to consider the more general form $\frac{1}{2}\boldsymbol{e}_T \cdot \boldsymbol{A}\,\boldsymbol{e}_T$ for the hardening term in the Helmholtz potential. Since the Helmholtz potential above is not differentiable at $\boldsymbol{e}_T = 0$, we make use of the concept of subdifferentials (see Ekeland and Teman (1974), for instance) to derive the following constitutive relations:

$$T_m := \frac{\partial \varphi}{\partial (tr\varepsilon)}(tr\varepsilon, \boldsymbol{e}, \boldsymbol{e}_T) = \left(\lambda + \frac{2\mu}{3}\right) tr\varepsilon \tag{3}$$

$$\boldsymbol{S} := \frac{\partial \varphi}{\partial \boldsymbol{e}}(tr\varepsilon, \boldsymbol{e}, \boldsymbol{e}_T) = 2\mu(\boldsymbol{e} - \boldsymbol{e}_T) \tag{4}$$

$$\boldsymbol{X} \in -\partial_{e_T} \varphi(tr\varepsilon, \boldsymbol{e}, \boldsymbol{e}_T) = \boldsymbol{S} - \left(\tau_M \, \partial \| \boldsymbol{e}_T \| + \boldsymbol{A} e_T\right) \tag{5}$$

Next, in order to describe the evolution of the transformation strain $\boldsymbol{e}_T$, we consider the same complementary potential of dissipation as Souza *et al*. (1998):

$$\phi^*(\boldsymbol{X}) = I_R(\boldsymbol{X}) := \begin{cases} 0 & if \ \|\boldsymbol{X}\| \le R \\ +\infty & otherwise \end{cases} \quad , \tag{6}$$

where $I_R(\boldsymbol{X})$ is the indicator function associated with the elastic domain. The material parameter R can be understood as the radius of the elastic domain.

The corresponding flow rule is then, given by:

$$\dot{\boldsymbol{e}}_T \in \partial \phi^*(\boldsymbol{X}) = \partial I_R(\boldsymbol{X}) \tag{7}$$

or, equivalently, by:

$$\dot{\boldsymbol{e}}_T = \dot{\varsigma} \frac{\boldsymbol{X}}{\|\boldsymbol{X}\|} \tag{8}$$

$$\dot{\varsigma} \ge 0 \tag{9}$$

$$f(\boldsymbol{X}) := \|\boldsymbol{X}\| - R \le 0 \tag{10}$$

$$\dot{\varsigma} \, f(\boldsymbol{X}) = 0 \tag{11}$$

3. NUMERICAL RESULTS

A backward Euler scheme, together with a return-mapping algorithm, was considered for the time discretization of the flow rule, Eqs. (8) to (11). Since the procedures are basically the same as those adopted in Souza *et al.*, we do not describe the details here.

Figures 1 and 2 compare the model proposed by Souza *et al*. (1998) with the experimental results from Sittner *et al*. (1995) for simple traction and pure torsion tensile tests. The parameters were chosen so as to fit model and experiments in the case of simple traction. Under such circumstances, the hardening slope described by the model in the case of pure shear disagrees strongly with the one observed experimentally. In order to overcome this inconsistency, we propose the following form for the hardening tensor $\boldsymbol{A}$:

$$
A_{ijkl} = \begin{cases} h & if \quad i = j = k = l, \\ h+\beta & if \quad i = k,\ j = l,\ i \neq j, \\ 0 & otherwise \end{cases} \qquad i, j, k, l = 1, 2, 3 \tag{12}
$$

Figures 2 and 3 also include the results corresponding to our updated model, where the following material parameters were considered: *E = 30.7 GPa*, ν *= 0.36*, *R = 90.0 MPa*, τ_M *= 150.0 MPa*, *h = 9.23 GPa* and β *= 4.5 GPa*. The improvement obtained due to the new description of the transformation hardening is apparent.

REMARK: A further observation of Fig. 2 and Fig.3 reveals important differences between the elastic domains described by our model and by experimental data. This question will be addressed in a future paper.

Next, we consider a nonproportional traction-torsion stress driven tensile test as performed by Sittner *et al.* (1995), using the same material parameters as in the proportional tests. The stress path sequence, Fig. 4, consists of: traction driven loading, torsion driven loading, traction unloading, and torsion unloading.

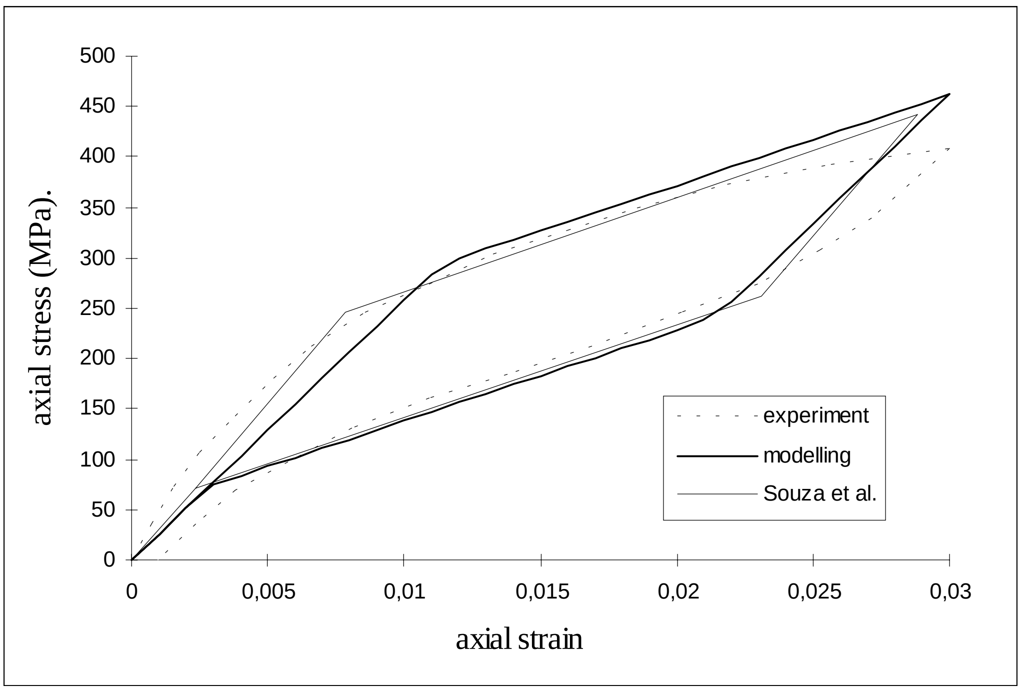

Figure 2 - Comparison between numerical simulation from Souza *et al.* and experimental results from Sittner *et al.* for the uniaxial traction test.

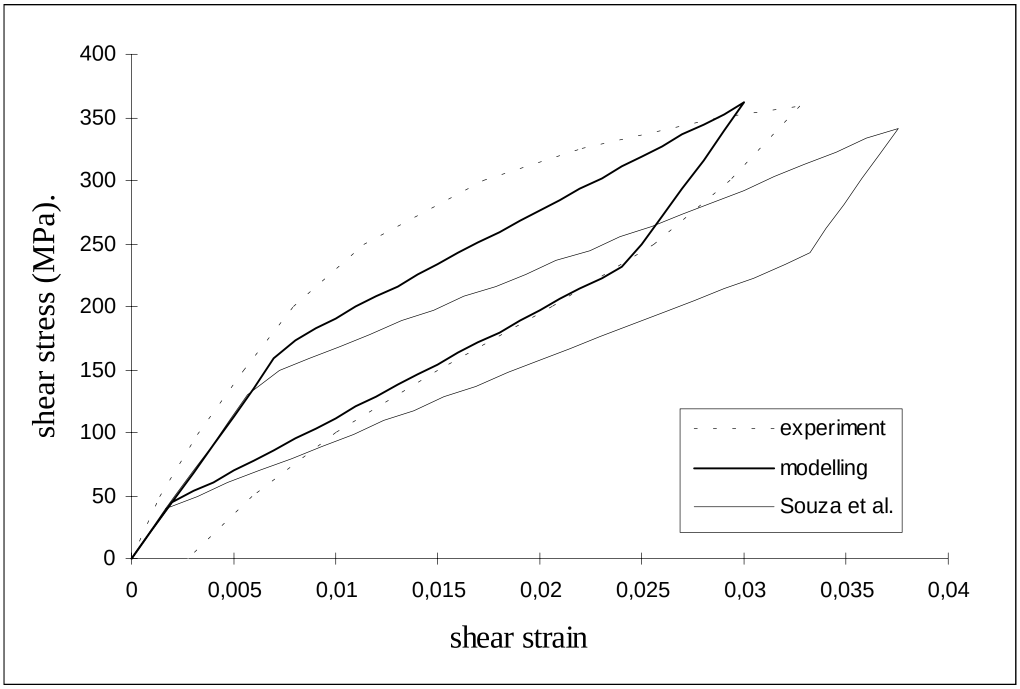

Figure 3 - Comparison between numerical simulation from Souza *et al*. and experimental results from Sittner *et al*. for the torsion test.

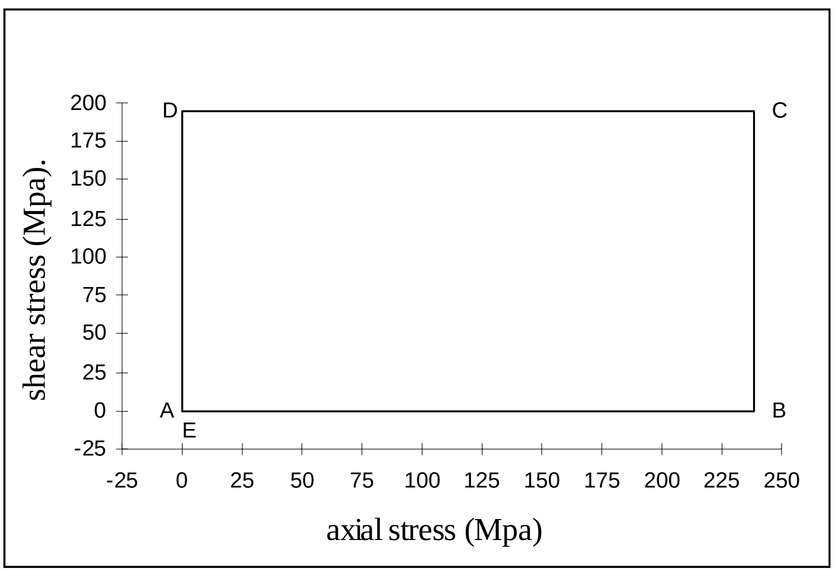

Figure 4 - Non-proportional loading path.

Figures 5 to 7 show the numerical results corresponding to the modified model when compared to the numerical results from Souza *et al*. (1998) and to the experiments by Sittner *et al*. (1995). Closer agreement with experiment is observed when considering the shear stress curve in Fig. 7, although the same trend is not observed when considering the axial stress-strain curve in Fig. 6.

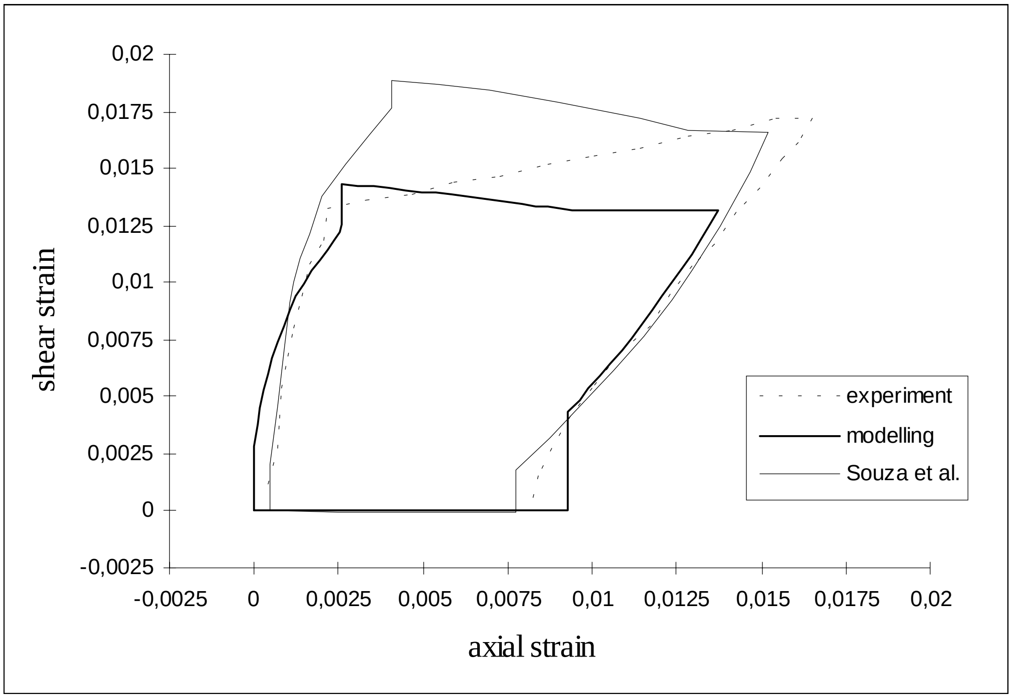

Figure 5 - Strain path curve.

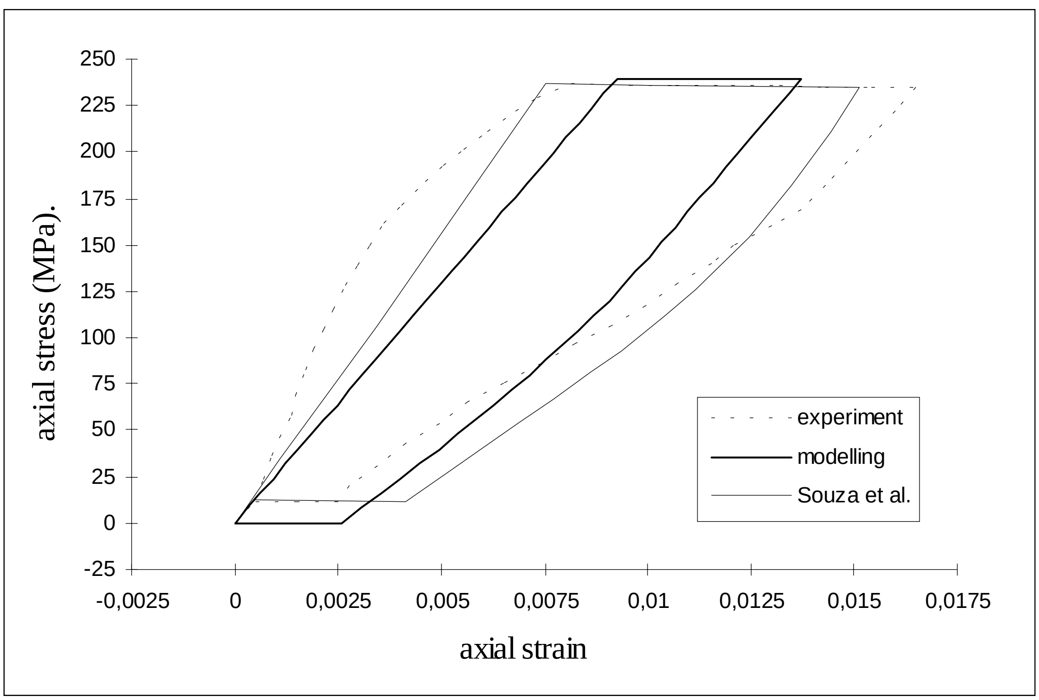

Figure 6 - Axial stress-strain curve.

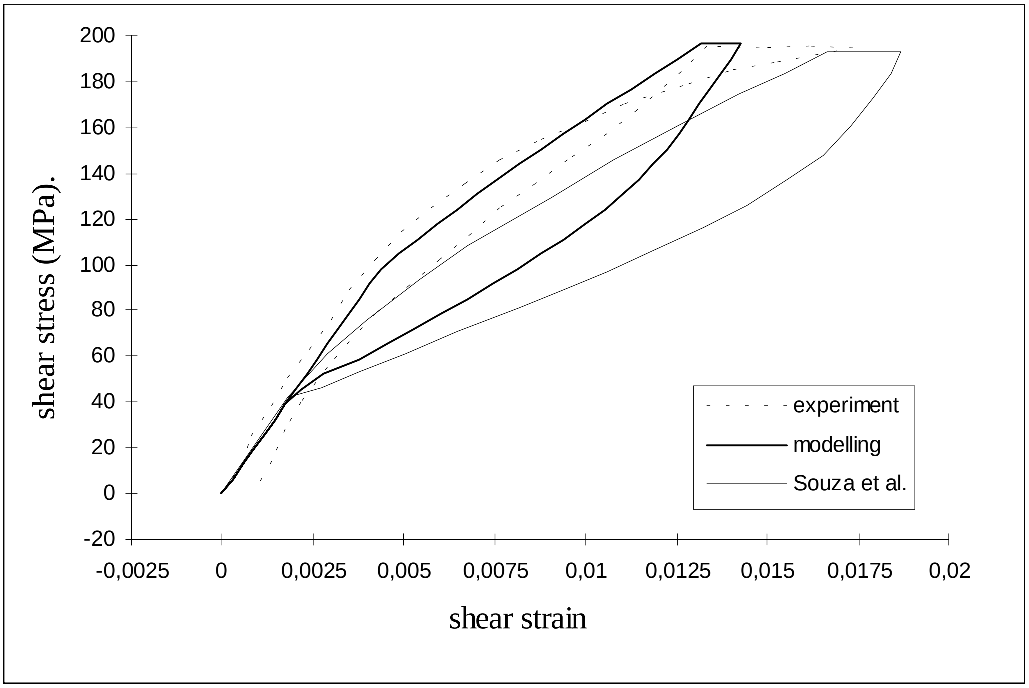

Figure 7 - Shear stress-strain curve.

4. CONCLUDING REMARKS

Although the proposed update in the hardening term led to a lack of isotropy in the mechanical response, the overall behavior of the model showed a significant improvement under distinct situations, such as proportional traction and torsion or even nonproportional load paths. Next steps toward the improvement of the model include the consideration of nonlinear hardening laws as well as geometrical nonlinearities.

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REFERENCES

Abeyratne R., Knowles J. K., 1988, On the dissipative response due discontinuous strains in bars of unstable elastic material, Int. J. Solids Struct., 24, 1021-1044.

Abeyratne R., Knowles J. K., 1990, On the driving traction acting on a surface of strain discontinuity in a continuum, J. Mech. Phys. Solids, 38, 345-360.

Abeyratne R., Knowles J. K., 1992, On the propagation of phase boundaries in solids, Q. Appl. Math.,50, 149-172

Auricchio F., Taylor R., L., 1997, Shape memory alloys: modelling and numerical simulation of the finite strain superelastic behavior, Comp. Meth. Appl. M., 143, 175-194.

Ball J. M., James R. D., 1987, Fine phase mixtures as minimizers of energy, Arch. Ration. Mech. An., 100, 13-52.

Ekeland I., Teman R., 1974, Analyse Convexe et Problèmes Variationnels, Bordas, Paris.

Gurtin M. E., 1983, Two-phase deformations of elastic solids, Arch. Ration. Mech. An., 84, 1-29.

Huang W., Pellegrino S., 1995, Shape Memory Alloy Actuators for Deployed Structures: Phase I, Tech. Report, Dept. of Engineering, University of Cambridge.

Maugin G. A., 1992, The Thermomecanics of Plasticity and Fracture, Cambridge University Press.

Pagano S., Alart P., Maisonnneuve O., 1998, Solid-solid phase transition modelling. Local and global minimizations of non-convex and relaxed potentials. Isothermal case for shape memory alloys, Int. J. Eng. Sc., 36, 1142-1172.

Rogueda C., Lexcellent C., Bocher L., 1996, Experimental study of pseudoelastic behavior of a CuZnAl polycrystaline shape memory alloy under tension-torsion proportional and non-proportional loading tests, Arch. Mech., 48, 6, 1025-1045.

Sittner P., Hara Y., Tokuda M.,1995,Experimental Study on the thermoelastic martensitic transformation in shape memory alloy polycrystal induced by combined external forces, Metall. Mater. Trans. A, 26A, 2923-2935.

Souza, Ângela C.; Mamiya, Edgar N. & Zouain, Nestor. 1998, Three-dimensional model for solids undergoing stress-induced phase transformations.