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ASPECTS OF THREE-DIMENSIONAL STRUCTURAL SHAPE OPTIMIZATION

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Abstract. *In this work, shape optimization of three-dimensional structures described by NURBS surfaces is presented. The finite element method is applied to structural analysis. Continuous design sensitivity analysis is used to evaluate gradients of performance functionals. The expressions of shape sensitivity depend on the design velocity vector field which describes shape variations of the domain. Aspects of boundary velocity field evaluation and geometric data structures are discussed. Finally, a shape optimization of a 3D linear elastic example is presented.*

Keywords: *optimization, sensitivity analysis, finite element method, NURBS, velocity fields.*

1. INTRODUCTION

The numerical solution of a physical problem follows, in general, three phases. First, a mathematical statement is defined. This statement consists of a geometric model defining the physical domain, a set of domain's attributes (material properties, boundary conditions, loadings, etc.) and the appropriate set of differential equations. In the second step, a discretization method is applied to the mathematical statement to reduce the continuous problem in a set of discrete contributions of geometry and physical behavior. At last, a set of algebraic equations is obtained whose solution approximates the solution of the original problem (Beall and Shephard, 1998).

In cases where only an isolated solution is required, the data of a previous phase is quite independent of information manipulated at the following steps. In shape optimization problems, however, information of the three phases has to be used at the same time. The design variables are defined on the geometric model. Performance functional evaluation may require geometric model data, discrete model data, the algebraic system itself and its solution. Gradient evaluation requires information about how geometric changes affect the characteristics of the discrete model (Haug, Choi and Komkov, 1986; Choi and Chang, 1994). Geometric and discrete models have to be updated in each iteration.

Indeed, the optimization process consists in solving an orderly sequence of problems where geometric and discrete models are repeatedly updated using information obtained from the three steps discussed previously. During this process, the parameterization of the design (design variables defined on the geometry) controls which specific actions have to be executed in each module of the optimization environment. The exchange of geometric information requires an unified geometric description of the domain, specially when some modules are independent software working co-operatively. The adopted geometric description has also to be a standard among commercial CAD systems and mesh generators. A simple mathematical expression is desirable to simplify the definition of design variables that lead to a sequence of differentiable geometric models. Flexibility is also required in order to describe real-world components.

NURBS (Non Uniform Rational B-Splines) curves and surfaces (Rogers and Adams, 1990) meet those requirements. NURBS provide a single and precise mathematical formulation to represent common analytical shapes such as conic curves, circles, quadratic and sculptured surfaces. NURBS consists in interpolations of control point coordinates with highly flexible parametric basis functions which can be easily implemented by simple recursive procedures. The smoothness and continuity can be controlled very well and several types of shape design variables may be defined. Besides, NURBS are an IGES standard since 1983.

In this work, NURBS entities are applied to define three-dimensional structures for shape optimization using finite elements and continuum-based design sensitivity analysis. Aspects of the geometric data structures used in this work and boundary velocity field construction are also discussed. Finally, a numerical example is presented to illustrate the application of the concepts discussed.

2. NURBS

2.1. NURBS Curves

The coordinates of a point on a rational B-spline curve of $q+1$ control points with coordinates $\mathbf{X}^i = \{X_1^i, X_2^i, X_3^i\}^T$ are given by

$$\mathbf{x}\left(r\right) = \sum_{i=1}^{q+1} \mathbf{X}^i R_{i,\chi}\left(r\right), \qquad R_{i,\chi}\left(r\right) = \frac{\beta_i N_{i,\chi}\left(r\right)}{\sum_{j=1}^{q+1} \beta_j N_{j,\chi}\left(r\right)}. \tag{1}$$

The set $\{R_{i,\chi}\left(r\right)\}$ is the rational B-spline basis and the β_i's are weights associated to the control points. In fact, the weight and the physical coordinates of a point belong to a 4D homogeneous coordinate space. The expression (1) defines a ratio of polynomials of order χ (degree $\chi - 1$). The non-dimensional quantity r is the internal curve parameter.

The nonrational B-spline basis $\{N_{i,\chi}\left(r\right)\}$ can be evaluated by the recursive Cox-de Boor's formula (Rogers and Adams, 1990),

$$\begin{cases} N_{i,1}\left(r\right) = 1, & \kappa_i \leq r \leq \kappa_{i+1}, \\ N_{i,1}\left(r\right) = 0, & \text{other cases,} \end{cases}$$

and

$$N_{i,k}(r) = \frac{(r - \kappa_i) N_{i,k-1}(r)}{\kappa_{i+k-1} - \kappa_i} + \frac{(\kappa_{i+k} - r) N_{i+1,k-1}(r)}{\kappa_{i+k} - \kappa_{i+1}}, \qquad k = 2, \ldots, \chi,$$

where the κ_i's are elements of a *knot vector* $\boldsymbol{\kappa}$ with dimension $q + \chi + 1$.

Three types of knot vectors are generally used: periodic (uniform), open uniform and nonuniform. The only requirement is that a knot vector has to be a monotonically increasing series of real numbers. A periodic knot vector has values which are spaced evenly and distributed between 0 and some maximum value with increments of 1. They are commonly used to generate closed curves. Open uniform knot vectors are given by

$$\begin{cases} \kappa_i = 0, & 1 \le i \le \chi, \\ \kappa_i = i - \chi, & \chi + 1 \le i \le q + 1, \\ \kappa_i = q - \chi + 2, & q + 2 \le i \le q + \chi + 1. \end{cases}$$

Nonuniform knot vectors can have either unequally spaced and/or multiple internal values.

Rigorously, a NURBS curve is a rational B-spline with basis functions generated with a nonuniform knot vector. However, since NURBS is the most general form of rational B-splines, all rational or nonrational B-splines can be called NURBS.

2.2. NURBS Surfaces

NURBS surfaces are the generalization of the previous concepts for bi-parametric coordinates (r, s). A rational B-spline surface is given by

$$\mathbf{x}(r, s) = \sum_{i=1}^{q+1} \sum_{j=1}^{t+1} \mathbf{X}^{ij} S_{ij}(r, s), \qquad S_{ij}(r, s) = \frac{\beta_{ij} N_{i,\chi}(r) M_{j,\vartheta}(s)}{\sum_{k=1}^{q+1} \sum_{l=1}^{t+1} \beta_{kl} N_{k,\chi}(r) M_{l,\vartheta}(s)}, \tag{2}$$

where the $\mathbf{X}^{ij}$'s are the vertices of a three-dimensional polygon net and the $S_{ij}(r, s)$'s are rational B-spline surface basis functions. $N_{i,\chi}(r)$ and $M_{j,\vartheta}(s)$ are nonrational B-spline basis functions given by the Cox-de Boor recursive formula in each parametric direction for any kind of periodic, open or nonuniform knot vectors. It is observed that the boundaries of the surfaces are NURBS curves.

3. OPTIMIZATION AND SENSITIVITY ANALYSIS FORMULATIONS

A large number of structural optimization problems can be expressed as

$$\text{Minimize } f(u, \nabla u, \mathbf{x}, \mathbf{p}) \quad \text{subject to} \quad \begin{array}{ll} g_i(u, \nabla u, \mathbf{x}, \mathbf{p}) \le 0, & i = 1, \ldots, m, \\ h_j(u, \nabla u, \mathbf{x}, \mathbf{p}) = 0, & j = 1, \ldots, p, \end{array} \tag{3}$$

where f is the objective function, g_i and h_j are the constraint functionals, u is the solution of the structural state equation defined in $\mathcal{B} \subset \Re^3$, $\mathbf{x} \in \mathcal{B}$ is a material point, and $\mathbf{p} \in \Re^n$ is the design variable vector.

It can be observed in (3) that f, g_i, and h_j depends on the design variables implicitly, i.e., those functionals rely on the solution u and $u \equiv u(\mathbf{p})$.

An important class of optimization algorithms uses first order information of the performance functional gradients (Belegundu and Arora, 1985; Bazaraa, Sherali and

Shetty, 1993). For many important problems, the performance functionals depend on a structural problem solution and specific methods are needed for gradient evaluation (Haftka and Grandhi, 1986; Haftka and Adelman, 1989). The discrete and continuous design sensitivity analysis methods are commonly used (Haug et al., 1986; Choi and Seong, 1986).

In the discrete case, the derivatives are evaluated from the discretized form of the continuous equations. In the continuous case, sensitivity expressions are obtained analytically from the original continuous model and posteriorly discretized and evaluated. The continuous formulation is adopted in this text and has advantages such as the determination of an analytical design sensitivity form from the original equations of continuum media and independence of the structural analysis code. The sensitivity evaluation can be calculated as a post processing step from the usual finite element output.

The elastic structural problem can be stated in the following variational form: *Find the displacement field* $u \in \mathcal{V}$ *such that*

$$\int_{\mathcal{B}} \mathbf{T}(u) \cdot \mathbf{E}(v) \, dV = \int_{\Gamma^2} \mathbf{\Phi} \cdot v \, dA + \int_{\mathcal{B}} \mathbf{b} \cdot v \, dV, \qquad \forall v \in \mathcal{V}\ , \tag{4}$$

where $\mathcal{B} \subset \Re^3$ is the region of the Euclidian space occupied by the structure, $\mathbf{T}(\cdot)$ is the Cauchy's stress tensor, $\mathbf{E}(\cdot) = \nabla u^S$ is the linear strain tensor, $\mathbf{\Phi}$ is the surface force vector field, $\mathbf{b}$ is the body force vector field, Γ^2 is the boundary portion with prescribed traction $\mathbf{\Phi}$, and $\mathcal{V}$ is the kinematically admissible displacement space. For a linear isotropic material, the tensors $\mathbf{T}$ and $\mathbf{E}$ are related by the Hooke's law, $\mathbf{T} = \mathsf{C}[\mathbf{E}] = \mathsf{C}\left[\nabla u^S\right]$, where C is the elasticity tensor.

The previous expression and many other mechanical problems can be written in the following variational form

$$a(u, v) = l(v), \qquad \forall v \in \mathcal{V}, \tag{5}$$

where $a(\cdot,\cdot) : \mathcal{V} \times \mathcal{V} \to \Re$ is a bilinear limited elliptic form and $l(\cdot) : \mathcal{V} \to \Re$ is the associated linear limited form. The forms $a(\cdot,\cdot)$ and $l(\cdot)$ depend on the design variables $\mathbf{p}$.

3.1. Continuum-based design sensitivity analysis

Structural performance functionals such as volume, displacement, stress, and strain energy can be described by the following general equation (Silva and Bittencourt, 1999*a*)

$$\psi(\mathbf{p}) = \int_{\mathcal{B}} \mathcal{G}(u, \nabla u, \mathbf{p}) \, dV, \qquad u = u(\mathbf{x}, \mathbf{p}), \qquad \mathbf{x} \in \mathcal{B}. \tag{6}$$

The functional sensitivity for shape design variables is given by the following expression (Haug et al., 1986; Silva and Bittencourt, 1999*b*)

$$\begin{aligned} \dot{\psi} &= \int_{\mathcal{B}} \left[\mathcal{G}_{,u} \cdot \dot{u} - \mathcal{G}_{,u} \cdot \nabla u \mathbf{V} + \mathcal{G}_{,\nabla u} \cdot \nabla \dot{u} - \mathcal{G}_{,\nabla u} \cdot \nabla(\nabla u \mathbf{V})\right] dV \\ &\quad + \int_{\mathcal{B}} \left[\mathcal{G}_{,\mathbf{x}} + \mathcal{G} \ \mathrm{Div}\mathbf{V} + \mathbf{V} \cdot \nabla \mathcal{G}\right] dV. \end{aligned} \tag{7}$$

The smooth vector field $\mathbf{V}(\mathbf{x})$ is called the design velocity field. In fact, $\mathbf{V}(\mathbf{x}) = \mathbf{V}_s(\mathbf{x}^\tau, \tau)|_{\tau=0}$, where $\mathbf{V}_s(\mathbf{x}^\tau, \tau)$ is the vector field defined as the velocity spatial description of the deformation $\mathbf{x}^\tau = \mathsf{X}^\tau(\mathbf{x}, \tau) = \mathbf{x} + \tau \mathbf{V}(\mathbf{x})$, for $\mathbf{x} \in \mathcal{B}$ and τ is a parameter.

The application of (7) demands the evaluation of $\dot{u}$ and $\nabla\dot{u}$. The adjoint method replaces the dependency of (7) on $\dot{u}$ and $\nabla\dot{u}$ using an adjoint variational problem with the same bilinear form of (5): *Find the displacement field* ς *such that*

$$a(\varsigma, v) = \int_{\mathcal{B}} \left(\mathcal{G}_{,u} \cdot v + \mathcal{G}_{,\nabla u} \cdot \nabla v\right)\, dV, \qquad \forall v \in \mathcal{V}. \tag{8}$$

¿From the differentiability properties assumed for $\mathcal{G}$, the right hand side term in (8) is a continuous linear functional. Therefore, the Lax-Milgram's theorem guarantees the existence and uniqueness of the adjoint state ς.

Using the material derivative (Gurtin, 1981) of (5) and the adjoint state ς, we have for variables related to the shape of $\mathcal{B}$ that (Haug et al., 1986; Silva and Bittencourt, 1999*b*)

$$\begin{aligned}\dot{\psi} \;=\; & \int_{\mathcal{B}} \left[(\nabla \mathbf{b})\,\mathbf{V} \cdot \varsigma + \mathbf{b} \cdot \varsigma \;\mathrm{Div}\mathbf{V}\right]\, dV + \int_{\Gamma^2} \left[\mathbf{\Phi}' + (\nabla \mathbf{\Phi})\,\mathbf{V} \cdot \varsigma + \mathbf{\Phi} \cdot \varsigma\;\; (\mathbf{V} \cdot \mathbf{n})\,\mathrm{Div}\;\mathbf{n}\right]\, dA \\ & - \int_{\mathcal{B}} \left\{\mathsf{C}\left[\nabla u^S\right] \cdot \nabla \varsigma^S \;\mathrm{Div}\mathbf{V} - \mathsf{C}\left[(\nabla u \nabla \mathbf{V})^S\right] \cdot \nabla \varsigma^S - \mathsf{C}\left[\nabla u^S\right] \cdot (\nabla \varsigma \nabla \mathbf{V})^S\right\} dV \\ & + \int_{\mathcal{B}} \left[\mathcal{G}_{,\mathbf{x}} + \mathcal{G}\;\mathrm{Div}\mathbf{V} - \mathcal{G}_{,\nabla u} \cdot \nabla u \nabla \mathbf{V}\right]\, dV.\end{aligned} \tag{9}$$

After solving the adjoint problem (8), the expression (9) determines the sensitivity of the functional ψ for a variation of the domain described by the velocity field $\mathbf{V}$.

Suppose that the points on the boundary $\partial\mathcal{B}$ are specified by a position vector $\mathbf{x}(\mathbf{p}) \in \Re^3$, where $\mathbf{p} \in \Re^n$ is the vector of design variables. The sensitivity expressions can be stated in terms of a variation $\delta\mathbf{p}$. For this purpose let

$$\mathbf{p}^\tau = \mathbf{p} + \tau\delta\mathbf{p}, \tag{10}$$

where $\mathbf{p}^\tau$ defines the boundary $\partial\mathcal{B}_\tau$ of $\mathcal{B}_\tau$ through an expression $\mathbf{x}^\tau(\mathbf{p}^\tau)$. The components of the vector $\mathbf{p}^\tau$ are chosen as characteristics of the NURBS curves and surfaces, such as control point coordinates or weights. Indeed, the velocity field on the boundary is defined by

$$\mathbf{V}(\mathbf{x}) = \left.\frac{d}{d\tau}\left[\mathbf{x}^\tau(\mathbf{p}^\tau)\right]\right|_{\tau=0} = \nabla_{\mathbf{p}}\mathbf{x}(\mathbf{p}) \left.\frac{d\mathbf{p}^\tau}{d\tau}\right|_{\tau=0} = \nabla_{\mathbf{p}}\mathbf{x}(\mathbf{p})\,\delta\mathbf{p}. \tag{11}$$

Each column of $\nabla_{\mathbf{p}}\mathbf{x}(\mathbf{p})$ gives the contribution of a design variable on the velocity field such that

$$\left.\mathbf{V}(\mathbf{x})\right|_{p_k} = \frac{\partial \mathbf{x}(\mathbf{p})}{\partial p_k}\delta p_k, \qquad k = 1, \ldots, n. \tag{12}$$

If the velocity field is defined inside the domain $\mathcal{B}$, according to the smoothness assumptions given in (Choi and Chang, 1994), the functional sensitivity can be written as a function of design variables such as

$$\dot{\psi} = \nabla_{\mathbf{p}}\psi \cdot \delta\mathbf{p}.$$

The $k-$th component of the functional gradient in terms of the design variables is obtained replacing (12) in (9),

$$
\begin{aligned}
(\nabla_{\mathbf{p}}\psi)_k \;=\; & \int_{\mathcal{B}} \left[(\nabla \mathbf{b}) \frac{\partial \mathbf{x}}{\partial p_k} \cdot \varsigma + \mathbf{b} \cdot \varsigma \ \mathrm{Div} \frac{\partial \mathbf{x}}{\partial p_k} \right] dV \\
& + \int_{\Gamma^2} \left[\frac{\partial \mathbf{\Phi}}{\partial p_k} \cdot \varsigma + \frac{\partial \mathbf{x}}{\partial p_k} (\nabla \mathbf{\Phi}) \cdot \varsigma + \mathbf{\Phi} \cdot \varsigma \ \left(\frac{\partial \mathbf{x}}{\partial p_k} \cdot \mathbf{n} \right) \mathrm{Div}\ \mathbf{n} \right] dA \\
& - \int_{\mathcal{B}} \mathsf{C} \left[\nabla u^S \right] \cdot \nabla \varsigma^S \ \mathrm{Div} \frac{\partial \mathbf{x}}{\partial p_k}\ dV \\
& + \int_{\mathcal{B}} \mathsf{C} \left[\left(\nabla u \nabla \frac{\partial \mathbf{x}}{\partial p_k} \right)^S \right] \cdot \nabla \varsigma^S\ dV + \int_{\mathcal{B}} \mathsf{C} \left[\nabla u^S \right] \cdot \left(\nabla \varsigma \nabla \frac{\partial \mathbf{x}}{\partial p_k} \right)^S dV \\
& + \int_{\mathcal{B}} \left(\frac{\partial \mathcal{G}}{\partial p_k} + \mathcal{G}\ \mathrm{Div} \frac{\partial \mathbf{x}}{\partial p_k} - \mathcal{G}_{,\nabla u} \cdot \nabla u \nabla \frac{\partial \mathbf{x}}{\partial p_k} \right) dV.
\end{aligned} \tag{13}
$$

The discrete form of (13) using the finite element method is

$$
\begin{aligned}
(\nabla_{\mathbf{p}}\psi)_k \;=\; & \sum_{e=1}^{Nels} \{ \int_{\mathcal{B}e} \left[(\nabla \mathbf{b})\, \tilde{V}_e \cdot \varsigma_e + \mathbf{b} \cdot \varsigma_e \, \mathrm{Div}\, \tilde{V}_e \right] dV \\
& + \int_{\partial \mathcal{B}e \cap \Gamma^2} \left[\frac{\partial \mathbf{\Phi}}{\partial p_k} \cdot \varsigma_e + \frac{\partial \mathbf{x}}{\partial p_k} (\nabla \mathbf{\Phi}) \cdot \varsigma_e + \mathbf{\Phi} \cdot \varsigma_e \ \left(\frac{\partial \mathbf{x}}{\partial p_k} \cdot \mathbf{n} \right) \mathrm{Div}\ \mathbf{n} \right] dA \\
& - \int_{\mathcal{B}e} \mathbf{D}\mathbf{B}\mathbf{u}_e \cdot \mathbf{B}\boldsymbol{\varsigma}_e \, \mathrm{Div}\, \tilde{V}_e \ dV \\
& + \int_{\mathcal{B}e} \mathbf{D} \left[\mathbf{G}\mathbf{u}_e \mathbf{G} \tilde{\mathbf{V}}_e \right]^S \cdot \mathbf{B}\boldsymbol{\varsigma}_e dV + \int_{\mathcal{B}e} \mathbf{D}\mathbf{B}\mathbf{u}_e \cdot \left[\mathbf{G}\boldsymbol{\varsigma}_e \mathbf{G} \tilde{\mathbf{V}}_e \right]^S dV \\
& + \int_{\mathcal{B}e} \frac{\partial \mathcal{G}}{\partial p_k} + \mathcal{G}\ \ \mathrm{Div}\, \tilde{V}_e - \mathcal{G}_{,\nabla u} \cdot \mathbf{G}\mathbf{u}_e \mathbf{G} \tilde{\mathbf{V}}_e \ dV \}
\end{aligned}
$$

where $\mathbf{D}$ is the elasticity matrix. $\mathbf{B}$ is the deformation matrix and $\mathbf{G}$ is the gradient matrix of shape functions. $\mathbf{u}_e$, $\boldsymbol{\varsigma}_\mathbf{e}$ and $\tilde{\mathbf{V}}_e$ are, respectively, the vectors of nodal displacements, adjoint state and design velocity restricted to the element e. ς_e and $\tilde{V}_e$ are the interpolations of nodal values using the shape functions.

The adjoint method gives a precise mathematical formulation of the functional sensitivity. The solution of ajoint structural problems is generally required by functionals of strain, stress and displacements. But there are also functionals, such as the volume and strain energy functionals, whose expression do not require an adjoint problem for gradient evaluation (Silva and Bittencourt, 1999*a*).

4. VELOCITY FIELDS

Assume that the shape variable is the coordinate l ($l = 1, 2, 3$) of an *internal* control point ij of a NURBS surface (2), i. e., $p_k = X_l^{ij}$. Therefore,

$$
\frac{\partial \mathbf{x}}{\partial p_k} = \frac{\partial \mathbf{x}}{\partial X_l^{ij}}, \tag{14}
$$

where

$$\frac{\partial \mathbf{x}}{\partial X_1^{ij}} = \left\{ \begin{array}{c} S_{ij}(r,s) \\ 0 \\ 0 \end{array} \right\}, \qquad \frac{\partial \mathbf{x}}{\partial X_2^{ij}} = \left\{ \begin{array}{c} 0 \\ S_{ij}(r,s) \\ 0 \end{array} \right\}, \qquad \frac{\partial \mathbf{x}}{\partial X_3^{ij}} = \left\{ \begin{array}{c} 0 \\ 0 \\ S_{ij}(r,s) \end{array} \right\}.$$

If the variable is the weight ij, i.e., $p_k = \beta_{ij}$, then

$$\frac{\partial \mathbf{x}}{\partial p_k} = \frac{\partial \mathbf{x}}{\partial \beta_{ij}} = \frac{1}{\beta_{ij}} \left[\mathbf{X}_{ij} - \mathbf{x}(r,s) \right] S_{ij}(r,s). \tag{15}$$

In both cases, the boundary velocity field on the discrete model is obtained evaluating (14) and (15) at the parametric coordinates (r,s) of each node on the surface.

In the case of a control point variable associated to a NURBS curve on the intersection of two surfaces, any change in the characteristics of the curve leads to modifications on both surfaces. Those modifications are described by partial velocity fields $\mathbf{V}_1$ and $\mathbf{V}_2$. The resultant velocity field is the union $\mathbf{V} = \mathbf{V}_1 \cup \mathbf{V}_2$. In the same way, if the variable is associated to a point that is the vertex of n_{surf} surfaces, the resultant velocity field is the union of partial velocity fields $\mathbf{V} = \mathbf{V}_1 \cup \mathbf{V}_2 \cup \ldots \cup \mathbf{V}_{n_{surf}}$.

Mechanical parts may also present certain features (e.g. symmetry or equality) that have to be maintained during optimization. To handle these features, a set of *geometric variables* is introduced in addition to the design variables of the optimization problem. A geometric variable can be any characteristic of the geometric model while a design variable represents an association of geometric variables. For example, if the symmetry relative to the XY-plane of a component has to be maintained and the z-coordinate of a control point is selected as a design variable, this design variable will actually represent two linked geometric variables since the z-coordinate of the symmetric control point will also change. Such associations of geometric variables are introduced in the optimization algorithms through proper design updating and construction of boundary velocity fields. For a symmetry relationship between two variables, the velocity field is $\mathbf{V} = \mathbf{V}_1 + (-\mathbf{V}_2)$. For equality relationships of n_{var} geometric variables, the velocity field of the representative design variable is $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \ldots + \mathbf{V}_{n_{var}}$.

5. DATA STRUCTURES

As observed in previous expressions, the design variables associated to points, curves or surfaces need to access the characteristics of the NURBS entities of the geometric model and the nodes placed on the boundary of the model to evaluate velocity fields.

The structural optimization environment that was implemented (Silva and Bittencourt, 1999*a*) connects design variables, performance functionals, velocity fields, discrete model and a reduced geometric model. This reduced geometric model receives information from the CAD software for geometric modeling, selects and stores only the information related to the parameterized geometric entities. Indeed, all the relevant geometric data can be manipulated by the optimization software kernel. This fact reduces data exchange between different software.

In the reduced geometric model all numerical information about weights and coordinates of control points is stored in surface objects. Curve objects do not store geometric data but just the numbers of the surfaces that share the curve. Curves access methods and data of surface objects.

6. NUMERICAL EXAMPLE

Figure 1 shows a mechanical component to be optimized. The objective function is to minimize the volume with restrictions on the maximum strain energy (0.2 kNcm) and upper and lower limits on the design variables.

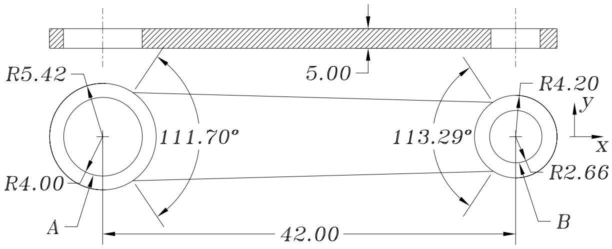

Figure 1: Dimensions [cm] of the initial design ($E = 21.0 \times 10^3$ kN/cm^2, $\nu = 0.3$, $\rho = 7.81 \times 10^{-3}$kg/cm^3). Nodes on surface A are fixed. A distributed force ($F_x = 0.12$ kN/cm^2, $F_y = 0.06$ kN/cm^2, $F_z = 0.04$ kN/cm^2) is applied on surface B.

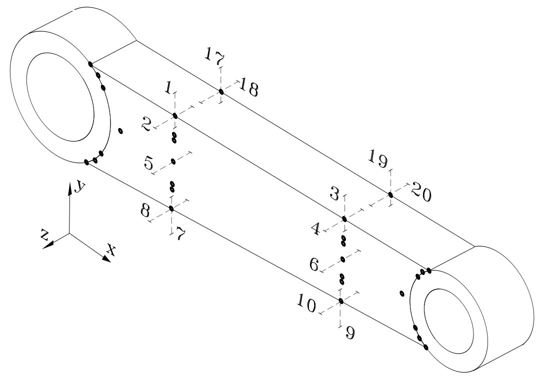

Figure 2: Design variables. The dashed lines indicate the direction and range of variation (upper and lower limits) for each shape variable.

Figure 2 illustrates the 20 geometric design variables which are control point coordinates of NURBS curves and surfaces. Meshes of linear tetrahedric elements were used.

The optimization procedure converged in 14 iterations within a precision of 10^{-5}. Solution of 19 finite element structural problems was required and an interior point algorithm used (Herskovits and Coelho, 1989). The final volume was 1295.3720 cm^3 with a decrease of 26.57% over the initial volume of 1763.9958 cm^3. Figure 3 shows the shape evolution of the domain during the optimization process. Figures 4 and 5 illustrate, respectively, the objective function and strain energy along 14 iterations. The final strain energy was 0.199925 kNcm.

7. CONCLUSIONS

The techniques presented here were successfully applied to structural shape optimization. The use of NURBS parameters as shape design variables provides great flexibility for the control of the changes in the geometry. Smoothness and continuity are kept even for large shape variation. It also provides simple expressions for boundary velocity fields. This kind of variables, however, cannot control shape requirements such as symmetry or equality. For that purpose design variables that represent associations of geometric variables are used. Furthermore, those associations decrease the number of variables, simplify mathematical programming evaluations and reduce the costs to extend velocity fields into the domain.

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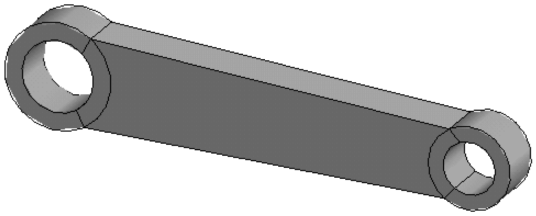
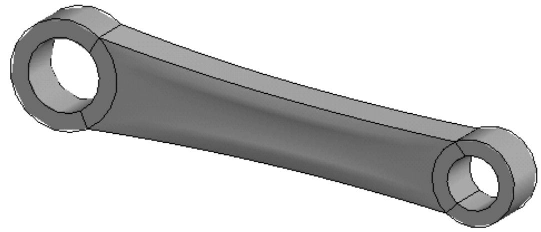
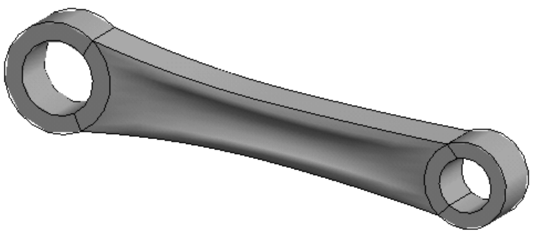


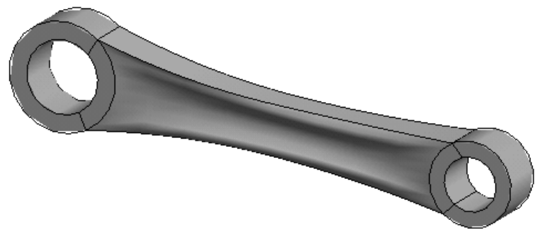
(a) Initial design.



(b) One iteration.



(c) Four iterations.



(d) Optimal shape.

Figure 3: Shape evolution.

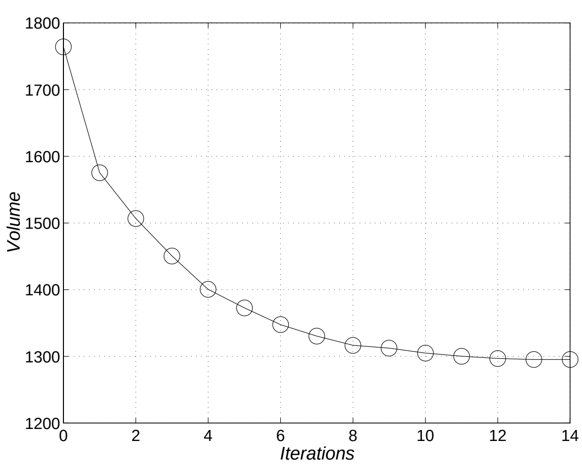


Figure 4: Volume evolution.

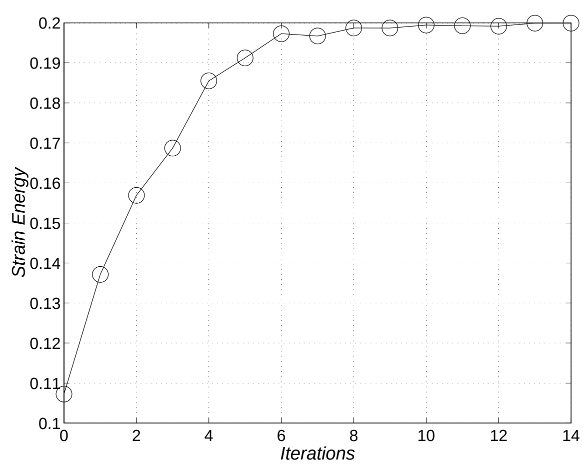


Figure 5: Strain energy evolution.

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