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NEURAL-SLIDING CONTROL WITH CMAC IN VARIABLE STRUCTURE SYSTEMS

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Abstract. *This work proposes a new method of dynamical control systems implementation using the Cerebellar Model Articulation Controller (CMAC) neural network as part of a variable structure system. CMAC is a neural network easily implemented in hardware, quite efficient in the reproduction of multivariable functions and extremely fast in its training process. These features yield CMAC applications in robust multivariable on line control in the presence of uncertainties in plant model. Variable Structure Systems (VSS) theory yields the concatenation of at least two, possibly unstable, dynamical structures, in order to realize a low cost controller that guarantees robustness with respect to nonidealities in the control system. This is done by a deliberate introduction of a special behavior named sliding mode where the state of the plant is confined to a subspace determined by the eigenvalues of the desired resultant dynamics until the origin is reached. Further, if the system model equations are written in a particular regular form and complete access to states is guaranteed for any time, the CMAC network in a variable structure system collects information enough about plant dynamics for the closed loop eigenvalues be allocated on line with no previous knowledge of plant model other than its order and structure. Furthermore, if the supposed unknown plant parameters change due to environment interaction, the resultant dynamics is invariant.*

Keywords: CMAC, Variable structure systems, Sliding mode control.

1. INTRODUCTION

The actual growth of works on variable structure control systems with sliding mode (Ackermann & Utkin, 1998) is due to their quasi invariance property to parameter disturbances (Chen & Chang, 1996), to the extension to the multivariable control problems and to the attractive simplicity of the relay type control usually applied to the plant. The variable structure control systems can reach desired dynamic behavior and show stability certainty as a feature but, usually, the knowledge of the linear system model matrices A and B is a need for sliding mode controller design, (Furuta, 1990) and (Iordanou & Surgenor 1997). This article proposes a method for variable structure control of partially known and/or uncertain dynamical systems where a CMAC neural network approximates a quantity depending on the system state and on the system parameter structure. The experimental results show that the approximation obtained is accurated enough to yield the system state to reach the discontinuity surface where it evolves in sliding mode to the origin.

2. CEREBELLAR MODEL ARTICULATION CONTROLLER - CMAC

Since its conception in the mid-seventies (Albus, 1975), the Cerebellar Model Articulation Controller (CMAC) has been used in on line nonlinear control systems, including robotics and chemical processes (Brown & Harris, 1994). Also, applications in reinforcement learning control, state estimation (Wang, Brown & Harris, 1994), digital filtering and color calibration hardware realization (Ker, Kuo & Wen, 1997) are reported. CMAC approximates multivariable functions over a compact domain of interest only (Ker, Kuo & Wen, 1997), and in order to improve accuracy, higher order basis functions than the binary one can be used. Memory requirements is a critical issue in using CMAC due to its exponential growth with accuracy requirements (Bordon, 1995). Albus (1975) suggested hashing code to randomly attribute one weight to many association cells.Teixeira & Bordon(1995) used discrete in value weights to reduce the memory needs in function approximation.

The CMAC output, (Hirashima & Iiguni, 1997) and (Tolle & Ersü, 1992), is the sum of ρ, weights, where ρ is a positive integer, chosen from a set with N weights, $N >> \rho$, by quantization functions applied to the set of inputs, X, where X is a compact set of $\mathbb{R}^n$. Each quantization function determines a weight associated with one interval of the partition in X that contains the input vector to the CMAC. In all, ρ, quantizations are made and there exists an overlapping of $^1/\rho$ of an interval in each coordinate axis. In Fig.1, the set $X \subset \mathbb{R}^2$ is partitioned by $\rho = 2$ quantization functions. The first one associates one weight to each interval in the set $\{ Aa, Ab, Ba, Bb \}$. The other weight is associated with one of the intervals $\{ Cc, Cd, Dc, Dd \}$ of the second quantization function. The CMAC output is defined in Eq.(1):

$$\tilde{r} = \sum_{i=1}^{\rho} w_i \ , \qquad (1)$$

where w_i are the weights chosen by the quantization functions for some $x \in X$. For two input vectors x_1 and x_2 sharing weights determined by the quantization functions, the corresponding outputs $\tilde{r}_1$ and $\tilde{r}_2$ are similar, and, in this context, ρ is a measure of the generalization inherent to the CMAC.

We define the learning rule for CMAC in Eq.(2):

$$w(k+1) = w(k) + \Delta w(k),$$
$$\Delta w(k) = \frac{\beta}{\rho}(r - \tilde{r}(k)), \quad (2)$$

r is the desired value for the output $\tilde{r}$ associated to some $x \in X$, $w(k)$ are the weights whose summation results in $\tilde{r}(k)$ and β is a positive constant. Usually $\beta \in (0, 1]$.

3. VARIABLE STRUCTURE SYSTEMS

This section collects some aspects of Variable Structure Systems (VSS) Theory for multivariable linear systems that lead to a nice fusion between sliding mode control and CMAC based on line training control. Let S be the null space of a linear transformation $G \in \mathbb{R}^{m \times n}$, defined by

$$S = \{ x \in \mathbb{R}^n \mid s(x) = Gx = 0 \}. \quad (3)$$

S is the intersection of m surfaces in $\mathbb{R}^n$ and, in multivariable VSS control, it is called sliding subspace (Utkin, 1992).

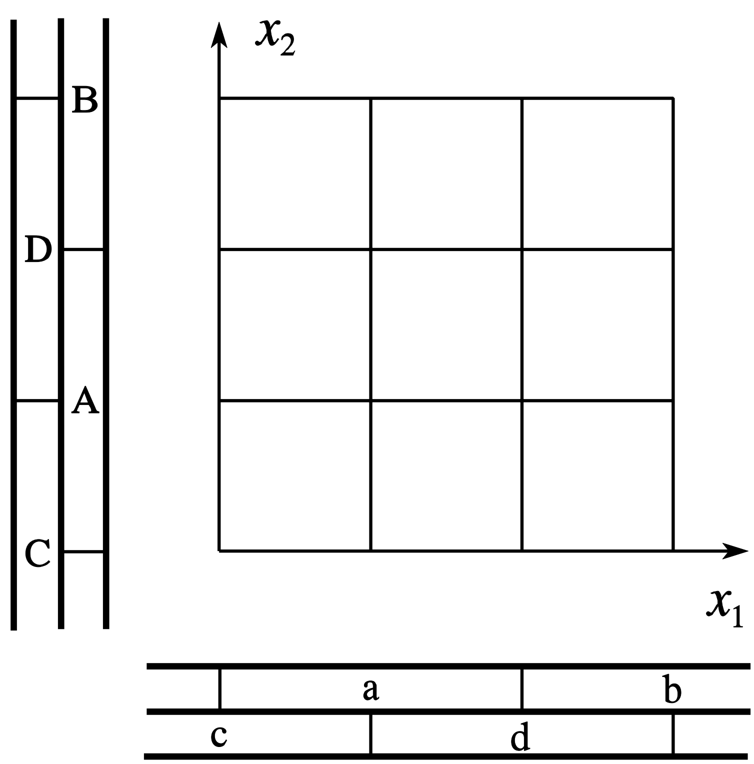

Figure 1: Structure of a two input CMAC.

Let the time invariant multivariable linear system written in a particular regular form be:

$$\dot{x} = Ax + Bu$$
$$x \in \mathbb{R}^n, u \in \mathbb{R}^m. \tag{4}$$

We suppose that the state is accessible but the matrix A is not completely known. Let's consider the following partitioning of matrix A and B:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$
$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad B_1 = 0. \tag{5}$$

where B_2 is an identity matrix and the eigenvalues of A are due only to A_{21} and A_{22} (A_{11} and A_{12} could represent a priori known relations about state variables).

Let the discontinuous control law be:

$$u_i = \begin{cases} u_i^+(x, t), & s_i(x) > 0 \\ u_i^-(x, t), & s_i(x) < 0 \end{cases}, \tag{6}$$

where $s_i(x)$ is the i th component of the vector $s(x)$ and $u^+(x,t)$ and $u^-(x,t)$ are continuous functions with respect to x and t.

The procedure for stabilizing this system may be divided in two phases (Utkin, 1978):

· to yield the state, initially equal to x_0, to a subspace $S \in \mathbb{R}^n$ in the state space called sliding subspace:

$$S = \{ x \in \mathbb{R}^n | \ s = Gx = 0 \}, \tag{7}$$

where G is a matrix of $\mathbb{R}^{m \times n}$ chosen accordingly to desired conditions for the dynamic behavior during the next phase;

· to keep the system state in a sliding domain of the discontinuity surface where a fast switching process, named sliding mode, is established due to discontinuous control law (6).

The system state remains in sliding mode in the subspace $S \in \mathbb{R}^n$ if there exists a function $E(s,x,t)$ satisfying the following conditions, (Utkin, 1978) and (Utkin, 1992):

$$
\left.\begin{array}{l} E(s,x,t) > 0 \\ \dot{E}(s,x,t) < 0 \end{array}\right\} \forall s \mid s \neq 0, \forall x, \forall t, \qquad (8)
$$

$$
E(0,x,t) = 0.
$$

Once the state reaches the sliding subspace $S \in \mathbb{R}^n$ and remains on it, with a switching frequency as high as desired, the resultant dynamics becomes independent of the plant parameters and depends only on G. So, if the sliding mode is established and the control system parameters satisfy the existence conditions for sliding, the closed loop dynamics is independent of the parameters of the plant.

Let a standard quadratic Lyapunov function be

$$
E(s,x,t) = \frac{1}{2} s^T s. \qquad (9)
$$

Consider the matrix G is partitioned as in Eq.(10)

$$
G = \left[\, G_1 \;\; G_2 \, \right], \qquad G_2 = I. \qquad (10)
$$

where I stands for the identity matrix. When the sliding regime is established, we have $\dot{s} \equiv 0$ and

$$
\dot{s} = G\dot{x} = GAx + GBu = 0, \qquad (11)
$$

in addition, the sliding mode for the system (3) in a neighborhood of the subspace $S \subset \mathbb{R}^n$ exists if:

$$
\begin{array}{l} u_i^+(x,t) < -g_i A x \\ u_i^-(x,t) > -g_i A x \end{array} \qquad (12)
$$

where g_i is the i^{th} row of matrix G.

Discrete Sliding Mode. Unlike the continuous time VSS, where changes in the structure due to switching may occur at any instant, in a computer controlled system, a quasi-sliding regime appears due to switching changes only at sampling instants. An alternative approach to approximate the sliding regime that occur in a digital control procedure to the ideal case is increasing sampling frequency; see in Furuta, (1990). In this way, the structure changing delay will be limited to the sampling period.

On an ideal sliding situation, the switching frequency tends to infinity and apparently the state remains on the subspace S. Equivalent control method

(Utkin, 1978) may then be applied to study the properties of the continuous resultant dynamics. But in a real sliding situation, the state moves in a finite neighborhood of the subspace $S \in \mathbb{R}^n$ and, so,

$$s(x) = Gx = \gamma, \qquad \|\gamma\| \in (0, \Delta] \subset \mathbb{R}. \tag{13}$$

Utkin (1978) showed that, for a certain class of systems, the real sliding occurs in a neighborhood close to the trajectory described by the ideal sliding.

4. ALGORITHM DESCRIPTION AND RESULTS

The proposed controller was applied in the stabilization of a partially unknown plant, with access to all states, and given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{14}$$

Figure 2 shows the proposed controller performance. In this work, $\rho = 4$, each partition divides the coordinate axes in 30 intervals, $\beta = 0.90$, and the CMAC is trained at each $\Delta = 10ms$. Figure 3 and 4 show the control law and the value of $s(x)$ during simulation. The simulated time was $10s$ and values of k_{ii} taken in the interval $[5, 10]$ showed to be satisfactory. The CMAC used in this application has three inputs related to the components of the vector x which represents the state of the system. The control law employed is defined by

$$u(x, t) = -\tilde{r} - k\,sign(s)\|x\| \tag{15}$$

where k is a diagonal matrix and

$$sign(s) = \begin{bmatrix} sign(s_1) \\ sign(s_2) \end{bmatrix}, \qquad \tilde{r} = \begin{bmatrix} \tilde{r}_1 \\ \tilde{r}_2 \end{bmatrix}. \tag{16}$$

CMAC output and training are given by:

$$\tilde{r} = \sum_{i=1}^{\rho} w_i \tag{17}$$

$$\Delta w = \frac{\beta}{\rho}(G\tilde{R} - u - \tilde{r}), \tag{18}$$

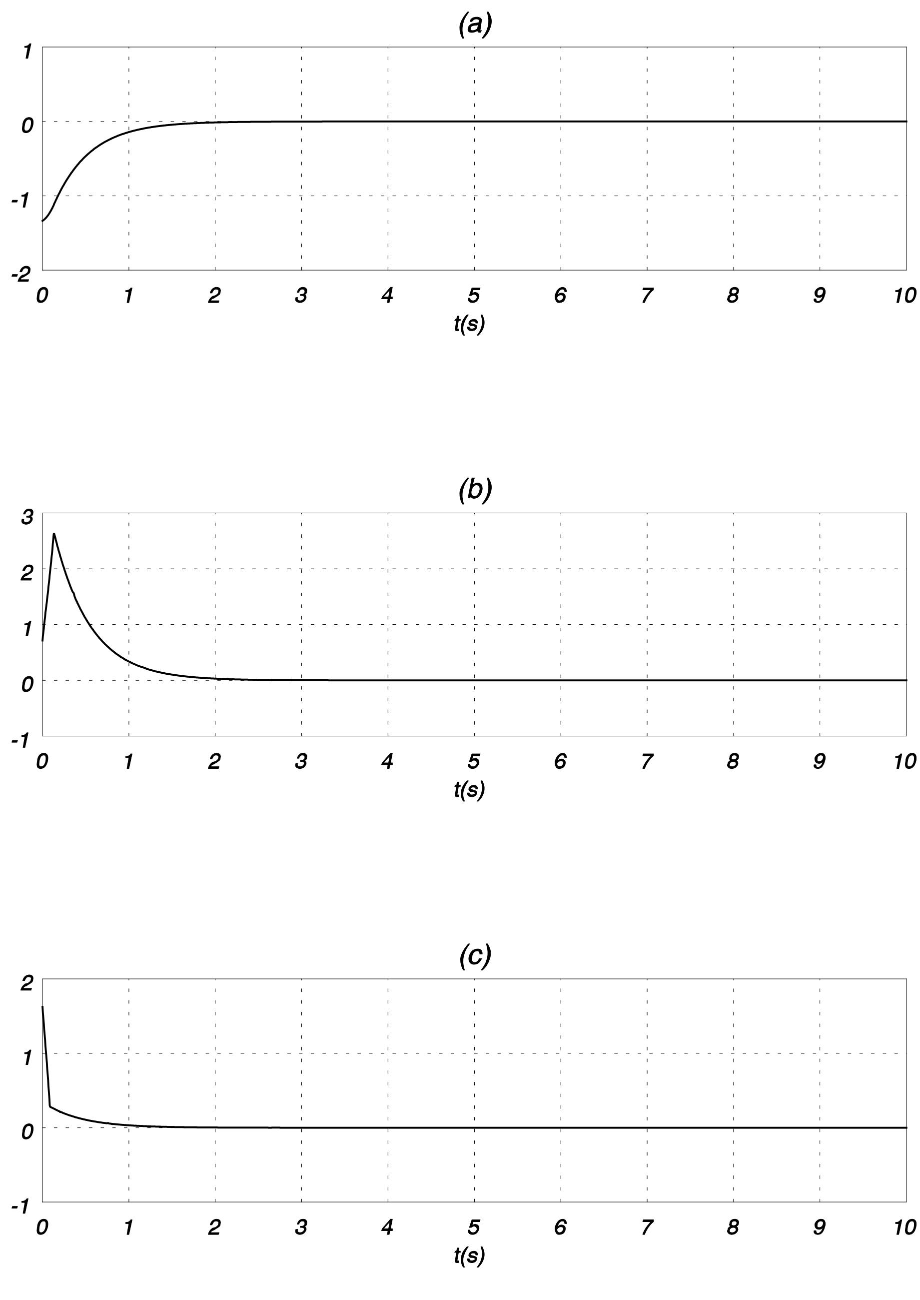

Figure 2: State paths for the plant reaching origin. (a) $x_1 \times t$, (b) $x_2 \times t$ and (c) $x_3 \times t$.

where G is a matrix chosen accordingly to desired dynamic properties for the closed loop system; $\tilde{R}$ is an approximation to $\dot{x}$ obtained by digital differentiation, for example; k is a diagonal matrix that controls the sliding robustness. Supposing the state is far away the origin, the quantity $k\|x\|$ guarantees the validity of the existence conditions (12) if $\tilde{r}$ is not close enough to $r = GAx$. Initially, the state is far away the origin, the CMAC output is not adjusted to yield the state in the sliding surface direction and the norm of $s = Gx$ may increase. At the end of some learning iterations, $\tilde{r}$ approximates GAx, the neighborhood that contains the state becomes a sliding domain, for the existence conditions are now verified, and the sliding starts at the first crossing of the discontinuity surface. According to existence conditions, to yield the sliding mode, the control law (6) must be such that

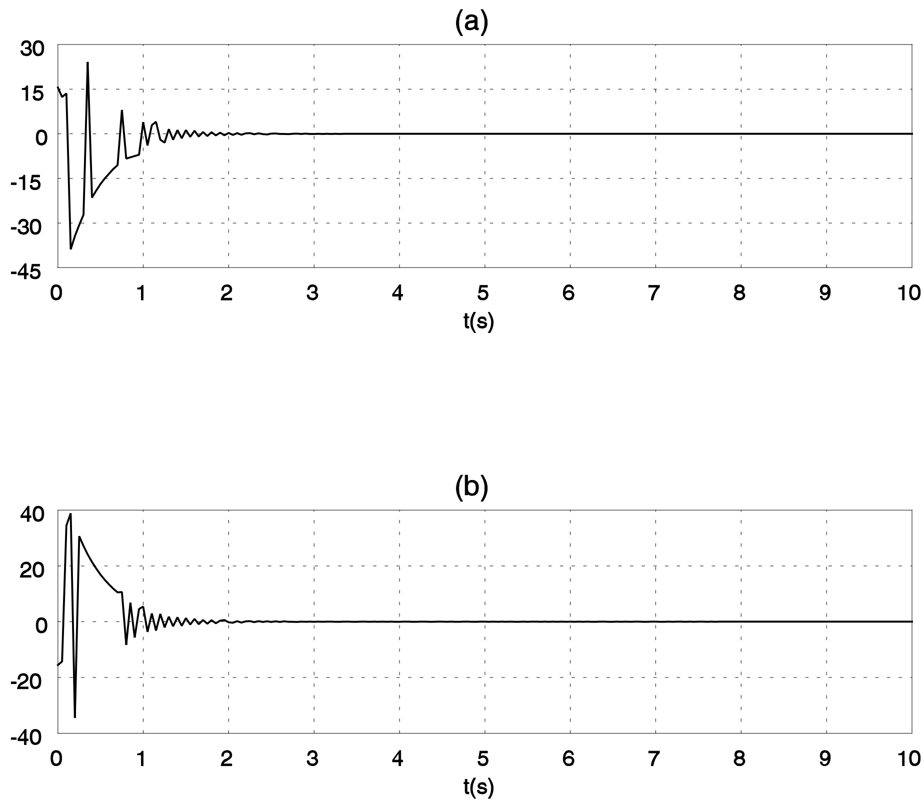

Figure 3: Control signals during simulation. (a) $u_1 \times t$ and (b) $u_2 \times t$.

$$\begin{aligned} u_i^+ &< -g_i A x \\ u_i^- &> -g_i A x \end{aligned}, \tag{19}$$

and, in this application,

$$\begin{aligned} u_i^+ &= -\tilde{r}_i - k_{ii}\|x\| \\ u_i^- &= -\tilde{r}_i + k_{ii}\|x\| \end{aligned}. \tag{20}$$

As the CMAC output, $\tilde{r}$, is intended to imitate GAx, if $\|\tilde{r} - gAx\| < k\|x\|$ the existence conditions for sliding are satisfied and the robustness is guaranteed.

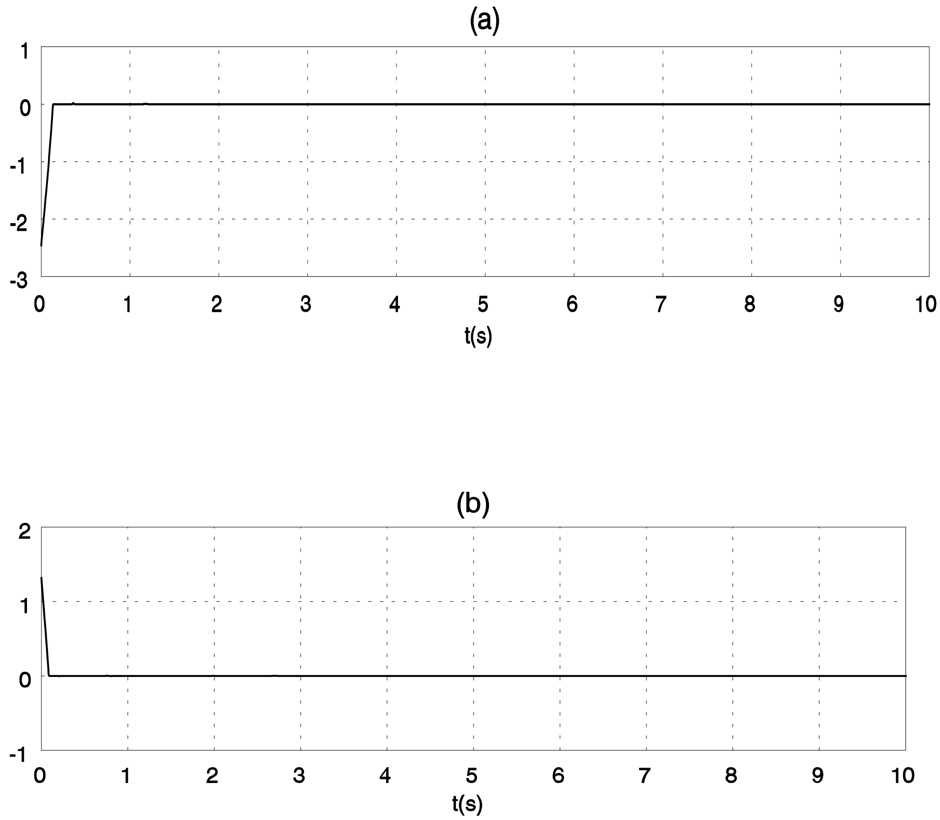

Figure 4: Evolution of the variable $s(t)$ over time. (a) $s_1 \times t$ and (b) $s_2 \times t$.

5. CONCLUSIONS

A procedure for neural-sliding controller realization with CMAC and VSS is proposed. Due to robustness properties of sliding mode behavior, this approach yields the stabilization of the controlled process to a quasi-invariant regime, and this is done with a chattering free control action in the stabilization point due to nonlinear state feedback employed. For a measure of the tolerance of the controller to variations in plant parameters it is necessary a deeper theoretical study. Further studies include discrete time systems analysis which yields lower switching/sampling frequencies.

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