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A NEW DAMAGE IDENTIFICATION APPROACH BASED ON A CONTINUUM DAMAGE MODEL

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Abstract. *In this paper a new approach based on a continuum damage model aiming to detect and locate structural damage is presented. The algorithm takes into consideration that the available spectral information is partial, i.e., considers as entries some measured modes containing only a set of measured degrees of freedom. The problem's formulation is such that the original physical configuration of the analytical model, connectivity and sparsity, is preserved, which plays an important role in damage identification. Some examples apllied to Kabe's problem (Kabe, 1985) are presented and the results are compared with the results obtained by Zimmerman & Kaouk (1992) to the same problem.*

Keywords: *Damage Identification, Continuum Damage Model, Finite Element Model Update*

1. INTRODUCTION

It is needless to emphasize the importance of damage detection techniques and health monitoring in aeroespace, civil and mechanical engineering. It is essential to determine the safety and reliability of their systems and mechanical structures. Based on experimental modal analysis and signal processing techniques, monitoring and interpreting changes on structural dynamic measurements can be considered as a quite promissing approach for damage identification and health monitoring of mechanical structures. The main idea is to reconcile analytical modal parameters of finite element models of structures with the modal parameters identified from a dynamic test. The question applied to damage detection can be formulated as follows: is it achievable to identify the existence, to locate and to determine the severity of damage using the discrepancy between the original finite element model (FEM) parameters and post-damage test measured dynamic parameters?

Most prior work on damage detection of structures is focused on the general framework of FEM updating methods. This technique is intended to refine and validate FEM structural models before they are considered as quite accurate models of the structure (Baruch & Bar, 1978). There are several recent review articles (Ibrahim & Saafan,1987; Motterhead & Friswell,1993; Doebling *et al.*,1996) that show the great interest in model updating, model correction and health monitoring. Optimal matrix update methods which seek to determine the system property matrices, such as the stiffness matrix, using measured data have been used extensively in FEM refinement and damage detection. Baruch & Bar (1978) obtained a closed-form solution to the minumum Frobenius norm adjustment to the mass-weighted structural stiffness matrix that takes into consideration the frequencies and the mode shapes. An inherent drawback associated with these methods is the fact that the zero/nonzero sparsity pattern of the original stiffness matrix may be destroyed, what migth be followed by no physical meaning results. Aiming to preserving the original stiffness matrix pattern, some algorithms were proposed (Kabe, 1985; Kammer,1998; Smith & Beatie,1993; Smith,1992), keeping the load paths the same as the original model's. Farhat & Hemez (1993) presented a methodology based on element-by-element sensitivity for updating the finite element dynamic models. The updating algorithm is obtained through minimization of the squared norms of the modal dynamic residuals assuming that the variables to be corrected are structural parameters such as Young modulus E, Poisson ratio ν, cross-sectional area A and so on.

A new approach for damage identification based on finite element model update is presented based on a continuum damage model: Continuum Damage Identification Approach (CDIA). The approach builds on a constrained minimization of dynamic residuals and utilises a mode shape projection to match the dimension of the experimental and FEM models. The connectivity and sparsity of the original model are naturally preserved.

The remainder of the paper is organized as follows. Section 2 presents the theoretical development of the CDIA as well as the mode shape projection algorithm followed by a parameter selection procedure. Finally, Section 3 presents numerical illustrations to assess the main characteristics of CDIA and the results are compared with those obtained by the Eingenstructure Assignment Algorithm (Zimmerman & KaouK, 1992) to the same problem under the same conditions.

2. THEORETICAL BASIS

2.1. Basic equations

One can summarize the present nondestructive damage detection procedure by three main steps : modal testing, location of potential damage regions and the parameters associated with them, and assessment of severity of damage.

Consider an undamped n-DOF finite element model of an undamaged structure represented by the general equation of motion

$$[\mathbf{M}]\ddot{\mathbf{u}} + [\mathbf{K}]\mathbf{u} = \mathbf{0} \tag{1}$$

where $[\mathbf{M}]$ and $[\mathbf{K}]$ are the nxn analytical mass and stiffness matrices, respectively, $\mathbf{u}$ is an nx1 displacement vector, and $(\ddot{\ })$ denotes the second derivative with respect to time.

The equation (1) leads to the following generalized eigenproblem

$$([\mathbf{K}] - \lambda_i[\mathbf{M}])\phi^{(i)} = 0 \tag{2}$$

Introducing the experimental modal parameters into the eigenproblem (2) yields to the dynamic residue equation for each mode i, e.g.:

$$\mathbf{R}^{(i)} = ([\mathbf{K}] - \omega_{E,i}^2[\mathbf{M}])\boldsymbol{\phi}_E^{(i)} \tag{3}$$

where $\omega_{E,i}$ and $\boldsymbol{\phi}_E^{(i)}$ are the frequencies and mode shapes obtained from the experiment. The $nx1$ vector $\mathbf{R}^{(i)}$ contains information about the degrees of freedom which are most likely influenced by the damaged regions, and somehow, it contains information about the severity of damage, which, in the present work, is described by means of a scalar variable $\beta \in [0,1]$ defined over the domain occupied by the elastic body. This variable is related to the links among material points and can be interpreted as a measurement of the local cohesion state of the material. If $\beta = 1$, all the links are preserved and the initial material properties are preserved too. If $\beta = 0$ a local rupture is considered since all the links among material points have been broken. The variable β is associated with the damage variable D by the following relation: $D = 1 - \beta$. As the degradation is an irreversible phenomenon, β varies with a negative rate. A detailed presentation of the basic principles that govern the evolution of such kind of continuum damage can be found in Fermond & Nedjar (1996), Mattos & Sampaio (1995) and Mattos, Domingues & Rochinha (1997).

Roughly speaking, β appears in the stiffness matrix multiplying the elastic coefficients. For an original discrete model, like the one formed by masses and springs, this parameter will lead to $K_d = \beta\ K$, where K_d and K are, respectively, the damaged and undamaged stiffness of the spring. For a finite element model, the spatial parameter β will influence all the entries in the stiffness matrix corresponding to the neighbourhood of the damaged region. In other words, one could express the stiffness matrix as follows

$$[\mathbf{K}] = f(\beta(\mathbf{x}), K_i) \tag{4}$$

where $\mathbf{x}$ is a space coordinate vector and K_i are the stiffness parameters of the structure. As β is the interest variable now, we aim at updating the stiffness matrix by finding the most suitable set of parameters β_j that minimizes a global residue GR as follows

$$\min_{\boldsymbol{\beta}} GR \qquad \text{constrained by} \qquad \beta(\mathbf{x}) \in [0,1] \tag{5}$$

where

$$GR = \sum_{i=1}^{p} \mathbf{R}^{(i)^T}\mathbf{R}^{(i)} \tag{6}$$

where p is the number of modes, among the measured ones, chosen to be used for the damage identification process.

The constrained minimization problem presented above is solved by means of Newton Method.

2.2. Mode shape projection

Unfortunately, the number of DOF at wich the mode shape is sampled from the test is typically much smaller than the number of DOF in the FEM that defines $[\mathbf{K}]$ and $[\mathbf{M}]$. Therefore, to apply Eq. (3), either the model must be reduced to the measured DOF or the measured portion of the mode shape must be expanded to the dimension of the analytical eigenvectors. So, in order to achieve the compatibility between the dimension of the experimental and FEM models, the approach taken in the present work is to expand the measured eigenvectors to the dimension of the analytical eigenvectors, using for this the Orthogonal Procrustes Expansion (Zimmerman & Kaouk,1992) which searches for a linear relationship between these measured and analytical eigenvectors

$$[\Phi_E] = [\Phi_a]\mathbf{P} \tag{7}$$

where $\mathbf{P}$ is the Procrustes' Orthogonal Projection matrix, and the subscripts E and a mean experimental and analytical respectively. Partitioning the experimental and analytical modal matrices into their measured and unmeasured partitions, the Eq. (7) can be written as follows

$$\left[\frac{\Phi_{E,m}}{\Phi_{E,o}}\right] = \left[\frac{\Phi_{a,m}}{\Phi_{a,o}}\right]\mathbf{P} \tag{8}$$

where the subscripts m and o mean measured and omitted DOF respectively. The projection matrix can be determined by the solution of the following problem

$$\min_{\mathbf{P}} \; \|\Phi_{E,m} - \Phi_{a,m}\mathbf{P}\|_F \quad \text{subject to} \quad \mathbf{P}^T\mathbf{P} = \mathbf{I} \tag{9}$$

where the subscript F means the Frobenius' norm. One should note that to determine $\mathbf{P}$ it is required only the measured partition of the modal matrices. The Eq.(9) has well known solution (Zimmerman & Kaouk,1992) given by

$$\mathbf{P} = UV^T \tag{10}$$

where $\mathbf{U}$ and $\mathbf{V}$ are the left and right matrices of the SVD decomposition (Golub & Van Loan, 1983) of the matrix $\mathbf{S}$ defined by

$$\mathbf{S} = \Phi_{a,m}^T\Phi_{E,m} = \mathbf{U\Sigma V}^T \tag{11}$$

After obtaining the matrix $\mathbf{P}$, the omitted partition of the experimental modal matrix is given by

$$\Phi_{E,o} = \Phi_{a,o}\mathbf{P} \tag{12}$$

2.3. Parameter selection

Recalling the Eq.(3), the set of DOF which exhibits the largest values will be associated with the elements which have probably suffered the damage effect, in other words, the elements whose original elemental stiffness of the model might be in error. Hence, it is reasonable to select those parameters related to this set of DOF j, where the j-th component of the dynamic residue exceeds some threshold level. This threshold may be defined by a statistical procedure (Farhat & Hemez, 1993) or any other criterion, depending only on the designer. Another interesting approach to select the DOF was proposed by Kaouk and Zimmerman (1994), where each entry of the dynamic residue vector is defined as an inner product of vectors. After selecting the DOF, the choice of the set of parameters to be updated may be made through the conectivity matrix or by the feeling of the designer.

3. NUMERICAL ILLUSTRATION

The numerical example used in this work to assess the main characteristics of the CDIA is a widely used spring-mass example known as Kabe's problem (Kabe, 1985). This model includes 8 masses and 14 springs and a schematic representation of the structure is depicted in Fig. 1. Table 1 lists the dimensionless stiffness and mass values for the exact model.

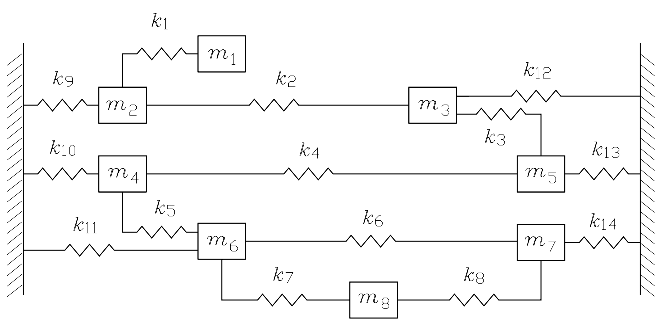

Figure 1: Kabes' problem.

Table 1: Dimensionless stiffness and mass values for the exact model.

k1	k2	k3	k4	k5	k6	k7	k8	k9	k10	k11	k12	k13	k14
1.5	10	100	100	100	10	2	1.5	1000	900	1000	1000	900	1000

m1	m2	m3	m4	m5	m6	m7	m8
0.001	1	1	1	1	1	1	0.002

This system has some particularities that make it a challenging test. It includes large relative differences in the magnitudes of some of the elemental stiffness coefficients and all the eight natural frequencies are within 27% of each other.

As the algorithm will be assessed for a damage identification case, and the damage may occur as a large local change, only one incorrect value of a connecting spring is introduced in the original model to simulate the damaged system, rather than introducing incorrect values in all of the connecting springs.

The mass and stiffness matrix of the experiment model are formed using the exact mass and stiffness values. The matrices are then used to generate the complete set of experimental frequencies and mode shapes. The data for each simulation consist of an exprimental modal set, which is taken to be a subset of the complete experimental modal set, depending only on the choice of which modes will be used to update the model and which DOF will be considered as measured. The rest of the mode shapes and frequencies are considered to be unmeasured residual modes and frequencies.

In the first case, the undamaged stiffness of the spring number 3 was 500 and it has been changed in the damaged situation to 100, what, what, according to the definition of beta, results in a damage variable D equals to 0.8. All the others springs kept the same original values. It is also assumed that only the first mode of vibration is measured, but all eigenvector components have been measured. Thus, no expansion of eigenvector is required. The Fig. 2 shows the result obtained by applying the Eigenstructure Assignement Algorithm (SEA-SR)(Zimmerman & Kaouk, 1992) and the results obtained by the CDIA. The x axis on the first 3 plots are the indices of a column vector constructed by storing the upper triangular part of the stiffness matrix in a column vector. The fourth plot is the damage plot obtained by the CDIA, where the x axis contains the stiffness parameters (springs) and the y axis their damage values (D). This plot indicates which spring is damaged as well as the extent of its damage. Both methods were capable of identifying the damage, however, the SEA-SR does not specify which element is damaged, once it only updates the global stiffness matrix.

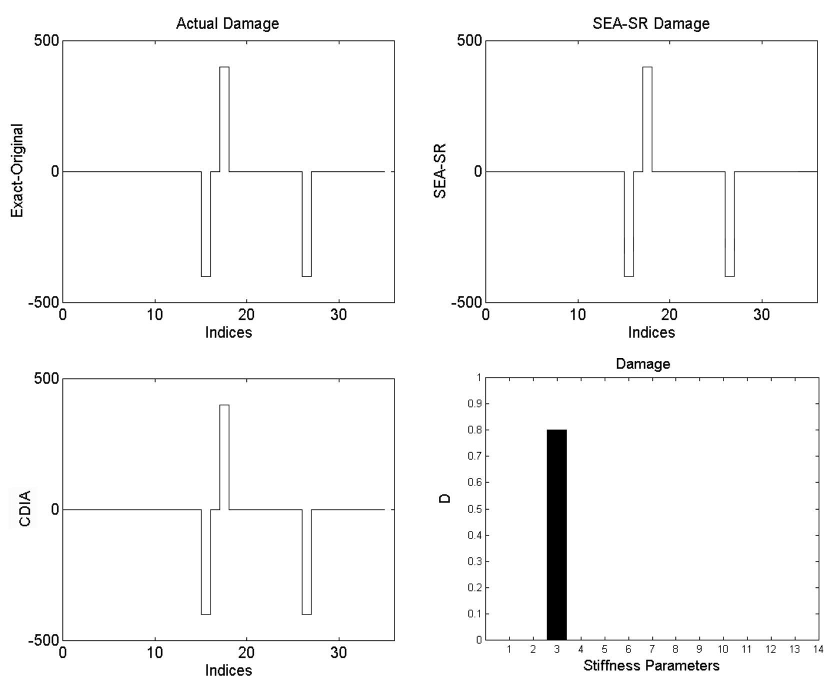

Figure 2: Results for the first case,where the first mode and all DOF are measured.

In the second case, for the same damage state, it is assumed that the first 3 modes have been measured but only the first 3 components of the eigenvector have been measured. So, an expansion process is necesssary, both SEA-SR and the CDIA were performed using the Orthogonal Procrustes Expansion. The results are shown in the Fig. 3.

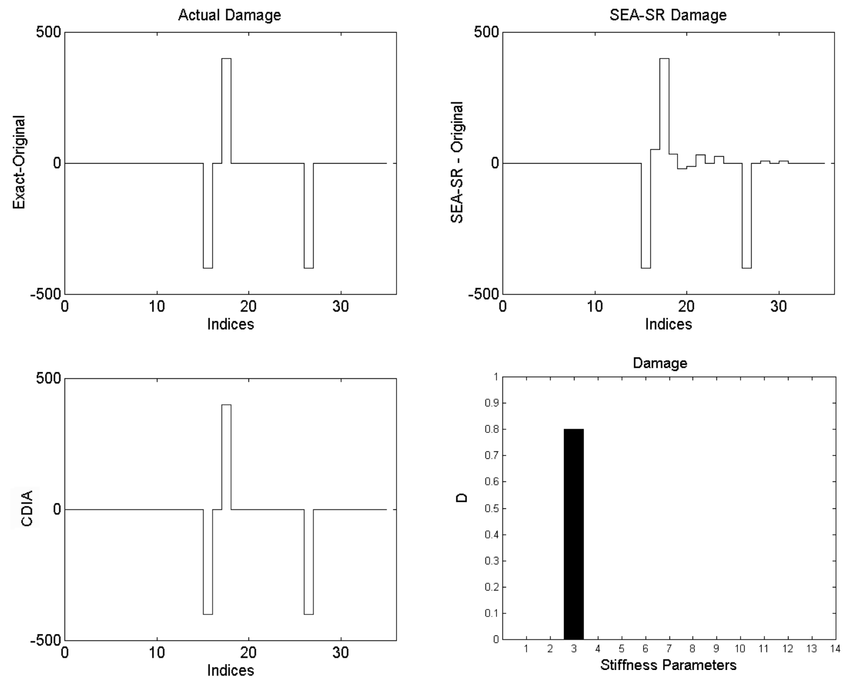

Figure 3: Results for the second case, where the first three modes and DOF are measured.

The solution achieved by the SEA-SR clearly gives an indication to both the location and extent of damage, although it has spread little errors over a few elements. The CDIA once again gives the indication of the location and the extent of the damage. The result obtained by the CDIA using the first 2 modes rather than the first 3 is the same as shown in Fig. 3 .

In the third case, the undamaged stiffness of the spring number 5 was 200 and it has been changed in the damaged to 100, what leads to a damage variable D equals to 0.5. It is assumed that only the first 3 modes of vibration and only DOFs 1,3 and 6 of the eigenvector have been measured. The SEA-SR again gives a clear indication to the location of damage, but is unable to predict its exact extent. As the foregoing cases, the CDIA gives a quite clear indication of the location and extent of the damage, as shown in Fig. 4.

Although it is not the focus of the work, it is worth verifying whether the proposed approach may be utilised to adjust the original model, what is a different task from damage detection. The system chosen to be updated is the same of the foregoing examples, but now most of the springs have had their stiffness changed. For this purpose it is required not to consider the constraint of the Eq.(5), due the fact that adjustment does not have a commitment to achieve results with physical meaning. The results are compared to the methods SER and KMA proposed by Arruda & Verçosa (1996) and by Kabe (1985) respectively. The dimensionless exact and original model stiffness for each spring are shown in Table 2, as the results obtained by the different approaches and methods when the two first modes are used in the updating process.

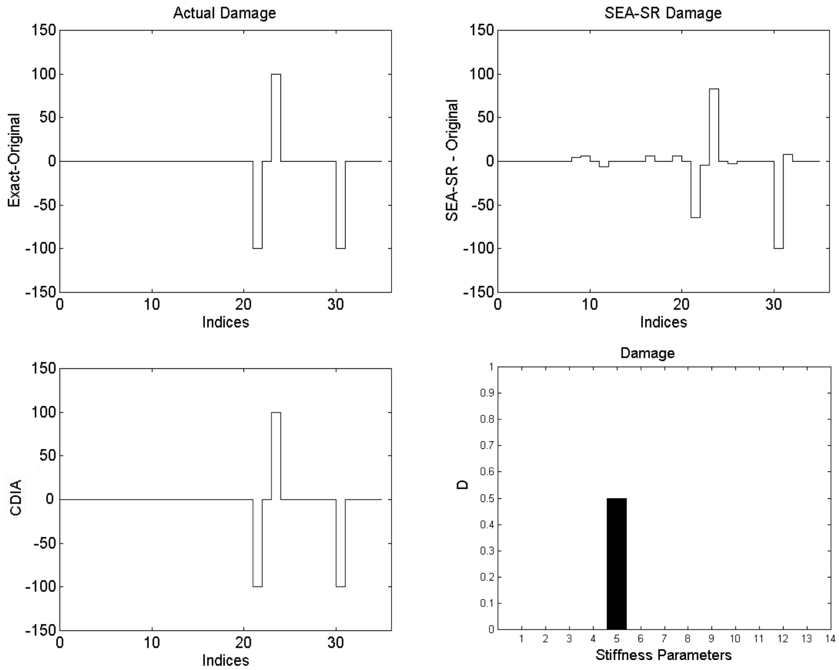

Figure 4: Results for the third case, where the first three modes and DOF 1,3 and 6 are measured.

Although the proposed approach has not been developed for model adjustment, the results depicted in Table 2 show that CDIA was capable of adjusting all the parameters exactly whereas the other 2 methods generated a few errors in some elements.

Table 2: Adjustment of the dimensionless parameters of the siffness matrix of the Kabe's problem.

			Modes 1 and 2		
	Exact	Original Model	KMA	SER	CDIA
k1	1.5	2	1.5	1.5	1.5
k2	10	10	10	10	10
k3	100	200	100	100	100
k4	100	200	100	100	100
k5	100	200	100	100	100
k6	10	10	9	8.9	10
k7	2	4	3	3	2
k8	1.5	2	2.3	2.4	1.5
k9	1000	1500	1000	1000	1000
k10	900	450	900	900	900
k11	1000	1500	1001.2	1001.4	1000
k12	1000	1500	1000	1000	1000
k13	900	450	900	900	900
k14	1000	1500	1001.1	1001.2	1000

CONCLUDING REMARKS

An approach for detecting damage based on a continuum damage model and using partial experimental modal parameters has been presented. It is built on a constrained minimization of dynamic residuals and utilises a mode shape projection to match the dimension of the experimental and FEM models. The approach has been assessed on the Kabe's problem and it has been shown to be efficient for estimating damaged parameters.

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