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A GENERAL METHOD FOR THE FLUID-DYNAMIC CALCULATION OF GAS AND STEAM PIPES

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Abstract. *The general problem of the three-dimensional non-adiabatic compressible turbulent gas flow through constant circular pipes is analyzed. The integral forms of the continuity, momentum and energy equations are employed near the state equation. The dynamic framework is rounded with three cross section mediation theorems. We apply these theorems to the flow equations and derive a system of three integro-differential equations.. The system derived is then transformed into an implicit system of four ordinary differential equations, which involves only the cross section mean parameters of the turbulent-average motion. We prove that the matrix inversion is always possible for subsonic flows. Finally, a numeric example is performed*.

***Keywords*:** Gas pipes, Turbulent compressible gas-pipe flow.

1. INTRODUCTION

The study of the gas flow through uniform section pipes has a great importance both from practical and a theoretical viewpoint. The specialized literature in the domain deals with the applied or theoretical aspects according to the aim taken into account. The applied studies deal with the problem of the fluid-dynamic calculus, while the theoretical ones are centered on the flow analysis. This paper belongs to the former classification.

The calculation methods developed up to now have a common denominator: two simplifying hypotheses which reduce the problem of the fluid-dynamic calculus to the known problem of the hydraulic calculation (Gluck, 1988).

a) The motion is assumed to be unidimensional.

b) The density is considered constant on sections of pipeline.

In this paper we shall leave these restrictive hypotheses aside. We shall deduce the equations of the mean motion by taking into account both the radial and longitudinal variations of the velocity and the continuous changing of the density along the pipe.

2. BASIC HYPOTHESES

Let us consider an infinite horizontal pipe of constant circular section. We shall refer to the cylindrical coordinates (x_1, r, θ), in which the axis Ox_1 coincides with the pipe axis.

The motion hypotheses are the following:

(**H1**) *The turbulent-average motion is stationary*.

(**H2**) *The motion is axis-symmetrical.*
(**H3**) *The pressure and density fluctuations* p' and ρ' *are negligible.*
(**H4**) *The velocity field has the form*:

$$u_1 = \tilde{u}_1 + u_1', \qquad u_2 = u_2', \qquad u_3 = u_3' \tag{1}$$

where $\tilde{u}_1$ -or $\langle u_1 \rangle$- denotes the turbulent-average value.
Under these hypotheses, the continuity equation applied to a control volume D (Fig.1) is

$$\int_{\partial D} \tilde{\rho}\, \tilde{u}_j n_j d\sigma = 0 \tag{2}$$

We shall use further down the Einstein's convention. The momentum equation on the control volume D has the following form

$$\int_{\partial D} \tilde{\rho}\tilde{u}_i\tilde{u}_j n_j d\sigma = \int_D \tilde{\rho} g_i dx - \int_{\partial D} \tilde{p} n_i d\sigma + \int_{\partial D} (\tilde{T}_{ij}^V + T_{ij}^R) n_j d\sigma \quad i = 1,2,3 \tag{3}$$

where g is the acceleration of gravity, $\tilde{T}_{ij}^V = \mu_2 \mathrm{div}\, \tilde{u} \delta_{ij} + 2\mu_1 \tilde{D}_{ij}$, $D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$, and $T_{ij}^R = -\tilde{\rho}\left\langle u_i' u_j' \right\rangle$.

Under hypothesis (H3), there results T'=0 and from here h'=0, where h is the specific mass-enthalpy. On noting with q the unitary heat flux vector and neglecting radiation heat transfer, the energy equation can be written as

$$\int_{\partial D} \tilde{h}\, \tilde{\rho}\, \tilde{u}_j n_j d\sigma = \int_D \tilde{\rho}\, \tilde{u}_j\, \tilde{v} \frac{\partial \tilde{p}}{\partial x_j} dx + \int_D \tilde{\Phi}_d dx - \int_{\partial D} q_j n_j d\sigma \tag{4}$$

where Φ_d is the Rayleigh dissipative function (Germain, 1973): $\Phi_d = T_{ij}^V \cdot D_{ij}$, and v is the specific mass-volume

The aim of the dynamic-gas calculation is the determination of the mean-cross section parameters variation along the pipe. With this end in view, we shall reduce the tridimensional motion to a quasi-unidimensional one. The equations (2), (3), and (4) will be brought to a proper integro-differential form. For this, we consider the control volume D (Fig.1) parametrized as $D = \{(x_1, r, \theta) : x_1 \in [x_1^0, x_1^0 + \Delta x_1], r \in [0, R], \theta \in [0, 2\pi]\}$. The boundary of the control volume will be $\partial D = S(x_1^0) \cup S_w \cup S(x_1^0 + \Delta x_1)$. $S(x_1^0)$ and $S(x_1^0 + \Delta x_1)$ are the influx and efflux surfaces, respectively. The rigid boundary-surface of the pipe wall is denoted with S_w.

The equations transformation method consists in their particularization for the control volume D parametrized above and, in a second stage, on using the below mediation theorems, bringing them to an integro-differential form from which will result a system of ordinary differential equations in which only the cross-section mean flow parameters intervene.

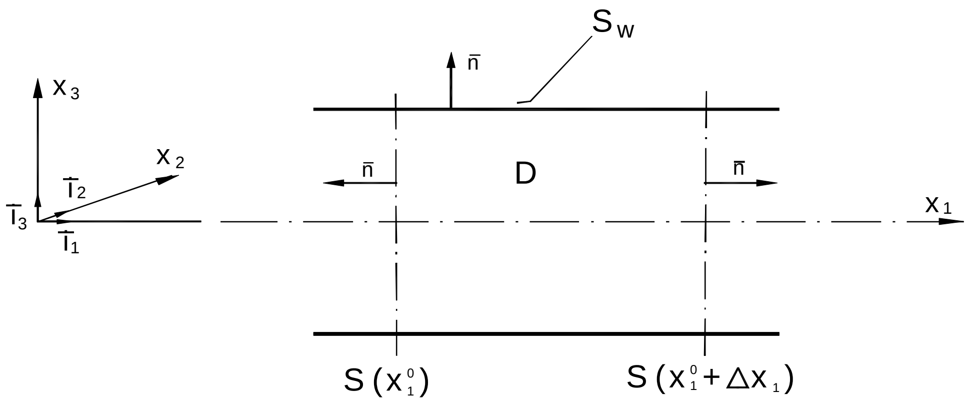


Figure 1 - The control volume D

3. MEDIATION THEOREMS

Theorem 1. *If f is a function of C^1 class, then*

$$\lim_{\Delta x_1 \to 0} \frac{1}{\Delta x_1} \int_D f\, dx = \int_{S(x_1^0)} f d\sigma \qquad (5)$$

Proof: Because f is a function of C^1 class, we have

$$f(x_1, x_2, x_3) = f(x_1^0, x_2, x_3) + \frac{\partial f}{\partial x_1}(x_1^0, x_2, x_3)(x_1 - x_1^0) + \alpha(x_1, x_2, x_3)(x_1 - x_1^0) \qquad (6)$$

We integrate on D; for the last integral we apply the integral-mediation theorem, the rest α being continuous. We arrive at

$$\int_D f dx = \Delta x_1 \int_0^R f\left(x_1^0, r\right) 2\pi r dr + \frac{\Delta x_1^2}{2} \int_0^R \frac{\partial f}{\partial x_1}\left(x_1^0, r\right) 2\pi r dr + \frac{\Delta x_1^2}{2} \int_0^R \alpha(\bar{x}_1, r) \cdot 2\pi r dr$$

We divide by Δx_1. When $\Delta x_1 \to 0$ we obtain what had to be demonstrated.

Theorem II. *If f is a function of C^1 class, then*

$$\lim_{\Delta x_1 \to 0} \frac{1}{\Delta x_1} \left(\int_{S(x_1^0 + \Delta x_1)} f d\sigma - \int_{S(x_1^0)} f d\sigma \right) = \int_{S(x_1^0)} \frac{\partial f}{\partial x_1} d\sigma \qquad (7)$$

Proof: On particularizing (6), we can write

$$f\left(x_1^0 + \Delta x_1, x_2, x_3\right) = f\left(x_1^0, x_2, x_3\right) + \frac{\partial f}{\partial x_1}\left(x_1^0, x_2, x_3\right) \cdot \Delta x_1 + \alpha\left(x_1^0 + \Delta x_1, x_2, x_3\right) \cdot \Delta x_1$$

For $\Delta x_1 \to 0$ the left term becomes $\int_0^R \left(\frac{\partial f}{\partial x_1}(x_1^0, r) + \alpha(x_1^0, r) \right) 2\pi r dr$. Because $\alpha(x_1^0, r) = 0$ it results what had to be demonstrated.

Theorem III. *If f is a function of C^1 class, then*

$$\lim_{\Delta x_1 \to 0} \frac{1}{\Delta x_1} \int_{S_w} f d\sigma = \int_0^{2\pi} f(x_1^0, R, \theta) R d\theta \tag{8}$$

Proof: On parametrizing on S_w, we obtain

$$\frac{1}{\Delta x_1} \int_{S_w} f d\sigma = \frac{1}{\Delta x_1} \int_{x_1^0}^{x_1^0 + \Delta x_1} \int_0^{2\pi} f(x_1, R, \theta) R d\theta dx_1$$

On using again (6); for $\Delta x_1 \to 0$ it results what had to be demonstrated. If f is axis-symmetric, then the integral from (8) becomes $2\pi \cdot R \cdot f(x_1^0, R)$.

Some methodological comments are useful. The integral-form flow equations lose information. Because x_1^0 and Δx_1 are arbitrary, equations (2), (3), and (4), particularized on the control volume above parametrized, will lose only the information from the cross section. Therefore, as a conclusion, we can state that: although they imply partial derivatives, the mentioned equations have a unidimensional character, a character which, as we shall see, will be kept and after the transformations undergone applying the mediation theorems.

4. THE INTEGRO-DIFFERENTIAL EQUATIONS OF MOTION

Let us particularize the continuity equation (2) to the control volume D defined in **2.** On using the result of Theorem 2, it results

$$\int_{S(x_1^0)} \frac{\partial}{\partial x_1} (\tilde{\rho} \tilde{u}_1) d\sigma = 0 \tag{9}$$

The projection according to axis Ox_1 of the momentum equation (3) is

$$\int_{S(x_1^0 + \Delta x_1)} \tilde{\rho} \tilde{u}_1^2 d\sigma - \int_{S(x_1^0)} \tilde{\rho} \tilde{u}_1^2 d\sigma = - \int_{S(x_1^0 + \Delta x_1)} \tilde{p} d\sigma + \int_{S(x_1^0)} \tilde{p} d\sigma +$$
$$+ \int_{S(x_1^0 + \Delta x_1)} \left(\tilde{T}_{11}^V + T_{11}^R \right) d\sigma - \int_{S(x_1^0)} \left(\tilde{T}_{11}^V + T_{11}^R \right) d\sigma + \int_{S_p} \left(\tilde{T}_{1r}^V + T_{1r}^R \right) d\sigma$$

On taking into account that the fluctuation is cancelled at the wall, the application of Theorems 2 and 3 leads to the following

$$\int_{S(x_1^0)} \frac{\partial}{\partial x_1} \left(\tilde{\rho} \cdot \tilde{u}_1^2 \right) d\sigma = - \int_{S(x_1^0)} \frac{\partial \tilde{p}}{\partial x_1} d\sigma + \int_{S(x_1^0)} \frac{\partial}{\partial x_1} \left(\tilde{T}_{11}^V + T_{11}^R \right) d\sigma + 2\pi R \tilde{T}_{1r}^V (x_1, R) \tag{10}$$

where $\tilde{T}_{11}^{V} = (\mu_2 + 2\mu_1)\dfrac{\partial \tilde{u}_1}{\partial x_1}$, and $T_{11}^{R} = -\tilde{\rho}\cdot\langle u_1' u_1' \rangle$

On denote with λ_c the gas thermal conductivity in the well-known relation $q(x,\bar{n},t) = -(\bar{q}(x),\bar{n}) = \lambda_c(\mathrm{grad}T,\bar{n})$. The equation (4) becomes

$$\int_{S(x_1^0+\Delta x_1)} \tilde{i}\tilde{\rho}\tilde{u}_1 d\sigma - \int_{S(x_1^0)} \tilde{i}\tilde{\rho}\tilde{u}_1 d\sigma = \int_D \tilde{\rho}\cdot\tilde{u}_1\cdot\tilde{v}\frac{\partial\tilde{p}}{\partial x_1}dx + \int_D \tilde{\Phi}_d dx +$$

$$+ \int_{S(x_1^0+\Delta x_1)} \lambda_c \frac{\partial\tilde{T}}{\partial x_1} d\sigma - \int_{S(x_1^0)} \lambda_c \cdot \frac{\partial\tilde{T}}{\partial x_1} d\sigma + \int_{S_W} q(\bar{i}_r) d\sigma$$

On applying the three mediation theorems and neglecting the conductivity λ_c variation, in the hypothesis (H2), we obtain

$$\int_{S(x_1^0)} \frac{\partial}{\partial x_1}\left(\tilde{i}\tilde{\rho}\tilde{u}_1\right)d\sigma = \int_{S(x_1^0)} (\tilde{\rho}\tilde{u}_1)\left(\tilde{v}\frac{\partial\tilde{p}}{\partial x_1}\right)d\sigma + \int_{S(x_1^0)} \tilde{\Phi}_d d\sigma + \int_{S(x_1^0)} \lambda_c \frac{\partial^2\tilde{T}}{\partial x_1^2} d\sigma + 2\pi R q(\bar{i}_r) \quad (11)$$

5. THE MEAN MOTION EQUATIONS.

If we take into account the parametrization $S(x_1^0) = \{(x_1, r, \theta) : x_1 = x_1^0, r \in [0,R], \theta \in [0,2\pi]\}$ as well as the fact that x_1^0 is a certain value, it results that for any cross section $S(x_1)$

$$\frac{d}{dx_1}\int_{S(x_1)} \tilde{\rho}\tilde{u}_1 d\sigma = 0 \quad (9')$$

The interpretation of equation (9') is obvious, i.e., the mass flow rate $\dot{m}$ is constant along the pipe.

In order to obtain an operational equation system, characteristic for engineering applications, we introduce a supplementary hypothesis, justified by experimental researches (Hinze, 1975; Reynolds, 1982):

(**H5**) *The variations of the pressure* $\tilde{p}$ *and of the density* $\tilde{\rho}$ *in the cross section S are negligible.*

We introduce the mass mean velocity by the integral mediation of the 1-st order

$$\bar{u}(x_1) := \frac{1}{\sigma(S(x_1))}\int_{S(x_1)} \tilde{u}_1 d\sigma \quad (12)$$

On applying the same mediation operation, it results $\tilde{\rho}(x_1) = \bar{\rho}(x_1)$ and $\tilde{p}(x_1) = \bar{p}(x_1)$.

We replace in eq.(9) and take into account that $\sigma(S(x_1)) = \pi\cdot R^2$ is constant along the pipe. We arrive at the following differential equation

$$\frac{d}{dx_1}(\bar{\rho}\cdot\bar{u}) = 0 \quad (13)$$

On using the same procedure, eq. (10) becomes

$$\frac{d}{dx_1}\int_S \left(\tilde{\rho}\tilde{u}_1^2\right)d\sigma = -\frac{d}{dx_1}\int_S \tilde{p}\,d\sigma + \frac{d}{dx_1}\int_S \left(\tilde{T}_{11}^V + T_{11}^R\right)d\sigma + 2\pi R\tilde{T}_{1r}^V(x_1, R)$$

On taking into account hypothesis (H5), as well as the expression of the mass flow rate $\dot{m} = \int_S \tilde{\rho}\tilde{u}_1 d\sigma = \bar{\rho}\,\bar{u}\,\sigma(S)$, it results $\frac{d}{dx_1}\int_S \tilde{\rho}\tilde{u}_1^2 d\sigma = \bar{\rho}\,\bar{u}\,\sigma(S)\frac{d}{dx_1}\left(\beta\cdot\bar{u}\right)$, where β is the Boussinesq coefficient.

We shall assume the usual interpretation (Hinze, 1975) $\tilde{T}_{1r}^V(x_1, R) = \tau_0 = -\lambda\frac{1}{D}\frac{\bar{u}^2}{2}\cdot\frac{R}{2}$ where λ is the Darcy coefficient and D is the inner diameter of the pipe. On using the mediation operator introduced in a particularized way through (12) and taking again into account (H5), we arrive at

$$\bar{\rho}\cdot\bar{u}\cdot\frac{d}{dx_1}\left(\beta\cdot\bar{u}\right) = -\frac{d\bar{p}}{dx_1} + \left[\left(\mu_2 + 2\mu_1\right)\frac{d^2\bar{u}}{dx_1^2} - \frac{d}{dx_1}\left(\overline{\rho\langle u_1' u_1'\rangle}\right)\right] - \bar{\rho}\cdot\lambda\cdot\frac{1}{D}\frac{\bar{u}^2}{2} \qquad (14)$$

In a first approximation, we shall neglect the term in square brackets. On neglecting as well the term that contains the variation with x_1 of the Boussinesq coefficient, we obtain the following equation

$$\bar{u}\cdot\beta\frac{d\bar{u}}{dx_1} = -\bar{v}\frac{d\bar{p}}{dx_1} - \lambda\frac{1}{D}\frac{\bar{u}^2}{2} \qquad (15)$$

whose interpretation is as follows: the pressure variation along the pipe is due both to the pressure losses and to gas acceleration (there is a cumulated effect of the pressure losses and the gas acceleration). We have to note that these two processes are interdependent.

We shall introduce the cross-section mean enthalpy and, implicitly, the cross-section mean temperature of the 2-nd order by an integral mediation of the 2-nd order

$$\bar{\bar{h}} = \frac{\int_S \tilde{\rho}\cdot\tilde{u}_1\cdot\tilde{h}\,d\sigma}{\int_S \tilde{\rho}\cdot\tilde{u}_1 d\sigma} \qquad (16)$$

On noting with T_e the outer temperature and with K_L the global linear heat transfer coefficient, we can write $2\pi R\, q(\bar{i}_r) = K_L(Te - \bar{\bar{T}})$.

We shall neglect in (11) the term containing λ_c. This is equivalent with neglecting the conductive transfer through the flux surfaces as against the heat transfer through the rigid boundary (S_w). According to all these considerations and using again (H5) - as in the case of (13) - eq. (11) becomes

$$\dot{m}\frac{d\bar{\bar{h}}}{dx_1} = \dot{m}\,\bar{v}\frac{d\bar{p}}{dx_1} + \int_{S(x_1)} \tilde{\Phi}_d d\sigma + K_L\left(T_e - \bar{\bar{T}}\right) \qquad (17)$$

The significance of this equation is also obvious. The variation of the mean temperature along the pipe is generated by three reasons: the gas expansion, the inner losses and the heat transfer through the pipe wall.

Unlike the incompressible case, in the case of gas flow through the pipe there appears a new dissipative factor, produced by the gas acceleration along the pipe. It is difficult to estimate the accurate rate of this factor in the total dissipation. Because the velocity longitudinal gradient is much smaller compared to the radial gradient, we can state that this rate is insignificant. Therefore, we shall assume that $\int_{S(x_1)} \tilde{\Phi}_d d\sigma = \dot{m} \cdot \lambda \frac{1}{D} \frac{\bar{u}^2}{2}$. On neglecting the longitudinal variation of the specific mass-heat capacity c_p, eq. (17) will be written as:

$$c_p \frac{d\bar{\bar{T}}}{dx_1} = \bar{v} \frac{d\bar{p}}{dx_1} + \lambda \frac{1}{D} \frac{\bar{u}^2}{2} + \frac{K_L}{\dot{m}} \left(T_e - \bar{\bar{T}} \right) \tag{18}$$

Equations (13), (15) and (18) make up a sub-abundant differential system which will be necessarily completed with two equations. The first one is the state equation mediated

$$f\left(\bar{p}, \bar{v}, \bar{T}\right) = 0 \tag{19}$$

The second relation will have to make the connection between the 1-st order mean temperature $\bar{T}$ resulted from (19) and the 2-nd order mean temperature $\bar{\bar{T}}$. According to the implications of hypothesis (H5) as well as the aim given at the beginning of the paper, we shall consider that $\bar{\bar{T}} = \bar{T}$.

6. THE CASE OF PERFECT GAS

We propose to study the system (13)+(15)+(18)+(19) for the case of perfect gases. We shall choose as unknown values: the mean velocity $\bar{u}$, the mean specific volume $\bar{v}$ and the mean pressure $\bar{p}$.

According to (H5), $\tilde{p} = \bar{p}$ and $\tilde{v} = \bar{v}$. On applying the 1-st order mediation operation, we obtain the mediate state equation $\bar{p} \cdot \bar{v} = R \cdot \bar{T}$. There results

$$\bar{v} \frac{d\bar{p}}{dx_1} + \bar{p} \frac{d\bar{v}}{dx_1} - R \frac{d\bar{T}}{dx_1} = 0 \tag{20}$$

From (13), (15), (18), and (20), we obtain the implicit differential system

$$A \cdot \frac{d}{dx_1} \begin{bmatrix} \bar{u} \\ \bar{p} \\ \bar{v} \\ \bar{T} \end{bmatrix} = b + c \tag{21}$$

where

$$A=\begin{bmatrix}\bar{v} & 0 & -\bar{u} & 0\\ \beta\bar{u} & \bar{v} & 0 & 0\\ \beta\bar{u} & 0 & 0 & c_p\\ 0 & \bar{v} & \bar{p} & -R\end{bmatrix},\quad b=\lambda\frac{1}{D}\frac{\bar{u}^2}{2}\cdot\begin{bmatrix}0\\-1\\0\\0\end{bmatrix},\quad c=\frac{K_L}{\dot{m}}\cdot\left(T_e-\bar{T}\right)\cdot\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$$

To explicit the system, it is necessary that matrix A be non-singular. We have $\det(A)=\frac{R}{k-1}\cdot\bar{v}\cdot(\beta\cdot\bar{u}^2-k\cdot\bar{p}\cdot\bar{v})$. We introduce the Mach number associated to the mean motion velocity $\overline{M}=\frac{\bar{u}}{\sqrt{k\cdot\bar{p}\cdot\bar{v}}}$. It can be shown that A is singular if and only if

$$\overline{M}=\frac{1}{\sqrt{\beta}} \tag{22}$$

Equation (22) can be interpreted as a condition for sonic blocking. The sub-unitary value is explained by the non-uniformity of the velocity field: the central areas get to the sonic regime before the peripheral regions.

The motion regimes currently met in practice are many inferiors to the sonic regime. In this case system (21) can be explicited, by multiplying (21) with A^{-1}, as follows

$$\frac{d}{dx_1}\begin{bmatrix}\bar{u}\\ \bar{p}\\ \bar{v}\\ \bar{T}\end{bmatrix}=\frac{\lambda\frac{1}{D}\frac{\bar{u}^2}{2}}{\bar{v}\left(k\bar{p}\bar{v}-\beta\bar{u}^2\right)}\cdot\begin{bmatrix}k\bar{u}\bar{v}\\ -\left(k\bar{p}\bar{v}+\beta(k-1)\bar{u}^2\right)\\ k\bar{v}^2\\ -\frac{\beta(k-1)}{R}\bar{u}^2\bar{v}\end{bmatrix}+\frac{\frac{K_L}{\dot{m}}\left(T_e-\bar{T}\right)}{\bar{v}\left(k\bar{p}\bar{v}-\beta\bar{u}^2\right)}\cdot\begin{bmatrix}(k-1)\bar{u}\bar{v}\\ -\beta(k-1)\bar{u}^2\\ (k-1)\bar{v}^2\\ \frac{k-1}{R}\bar{v}\left(\bar{p}\bar{v}-\beta\bar{u}^2\right)\end{bmatrix} \tag{23}$$

On adding the following

$$\bar{u}(x_1^0)=\bar{u}_0,\quad \bar{p}(x_1^0)=\bar{p}_0,\quad \bar{v}(x_1^0)=\bar{v}_0,\quad \bar{T}\left(x_1^0\right)=\bar{T}_0 \tag{24}$$

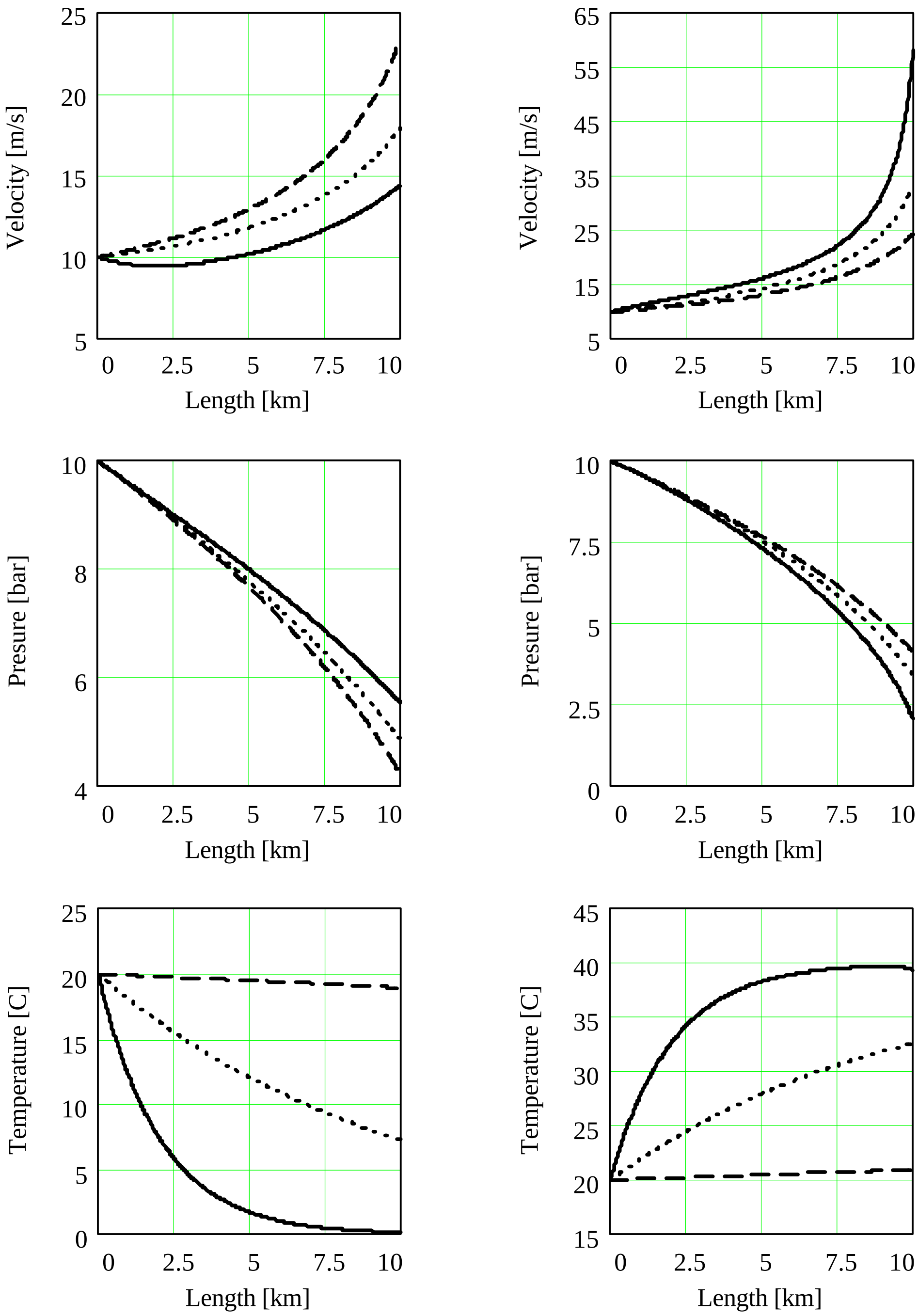
we obtain an initial-value problem.

Because the implied functions are supposed to be of class C^1, the local existence and uniqueness of the solution for the problem (23)+ (24) are ensured.

Let us notice that the last eq. from (23) is not independent. If we rewrite it as $\left[\bar{u}',\bar{p}',\bar{v}',\bar{T}'\right]^t=f\left(\bar{u},\bar{p},\bar{v},\bar{T}\right)$, where $f=\left[f_{\bar{u}}\ f_{\bar{p}}\ f_{\bar{v}}\ f_{\bar{T}}\right]^t$, then $f_{\bar{T}}=\left(\bar{v}f_{\bar{p}}+\bar{p}f_{\bar{v}}\right)/R$, as a result of the state equation.

For $\overline{M}<\sqrt{\beta^{-1}}$, system (23) can be integrated and studied numerically.

Let us perform a numerical example for a 0.2 m.-diameter non-isolated pipe of methane gas (CH_4). The initial values are $\bar{u}_0=10\,m/s$, $\bar{p}_0=10\,bar$, $\bar{T}_0=293.15\,K$. Dealing with the equation system numerically is facilitated by the introduction of the dimensionless quantities: $x=\frac{x_1}{L}$, $u=\frac{\bar{u}}{U}$, $p=\frac{\bar{p}}{P}$, $v=\frac{\bar{v}}{V}$, $T=\frac{\bar{T}}{\Theta}$, $Eu=\frac{P\cdot V}{U^2}$, $Ec=\frac{U^2}{c_p\Theta}$, where Eu and Ec are the Euler and Eckert numbers respectively. The solution was obtained by using the fourth-order classic multi-step method of the predictor-corrector type (Adams-Bashforth, Adams-Moulton). The reference values adopted are $U = \bar{u}_0, V = \bar{v}_0, \Theta = \bar{T}_0, Eu = 1$. The coefficients β, λ, K_L were considered as varying with x_1 , and their values were computed separately for each sub-interval of the integral division. The dimensionless step is h=0.2. The results are presented graphically in Fig.2. α_e is the external convection efficiency.



T_e=273.15 K T_e=313.15 K

Figure 2 - Numerical results

α_e=0.1 W/m^2K - - - - α_e=2 W/m^2K .... α_e=10 W/m^2K____

7. OBSERVATIONS AND CONCLUSIONS.

1) It is often supposed that the gas motion is isothermal. We have to notice that this is a supplementary hypothesis and there appears a compatibility problem between the energy equation and this hypothesis. The numerical results are proving that this compatibility does not exist. For the adiabatic motion (K_L=0) the temperature decreases. For a non-adiabatic motion, the temperature evolution will be strongly determined by the magnitude of the external convection efficiency α_e.

2) The system (23) has been obtained as a result of some approximations, by neglecting the very small terms. Although for engineering computations the accuracy is more than satisfactory, we can be tempted to increase the computation precision, thus getting closer to the exact solution. This problem may be considered as being of successive approximations. Thus, after solving the problem (23)+ (24) we shall determine in a first approximation the values of some of the neglected terms, such as, for instance $(\mu_2 + 2\mu_1)\frac{d^2\overline{u}}{dx_1^2}$, $\lambda_c \frac{d^2\overline{T}}{dx_1^2}$ (for $T_e = 273.15, \alpha_e = 0.1$, the Tchebishev norms of those terms are $\sim 6 \cdot 10^{-11}, 1 \cdot 10^{-9}$ respectively). Others, such as, for instance $\frac{d}{dx_1}\left(\overline{\rho} \cdot \left\langle u_1' u_1' \right\rangle\right)$, will be estimated on the basis of the experimental data found in literature. Thus, we obtain the system in the second approximation (Popescu, 1996), where the supplementary vector is due to the presence of the terms neglected in the first approximation.

3) The theoretical study of saturated solutions is a difficult task for the general case. For the adiabatic motions, or for $T_e > \overline{T}_0$, is obvious that $f_{\overline{u}} > 0, f_{\overline{p}} < 0, f_{\overline{v}} > 0$. One can prove that the solution will develop a singularity after a finite length of pipe ("blow-up"). The singularity appears when the condition of sonic blocking (22) is accomplished and it was observed in all cases numerically investigated.

4) The method can be extended as well to the case of real gases and steam. In the case of the perfect gas, the state equation can be solved algebraically. System (23) can be replaced by a 3 equation system (Popescu, 1996). The state equation for real gases and steam, especially near the saturation curve, has a very complicated form and it can not be solved algebraically in an explicit form. Especially for this case, a qualitative analysis of the small parameters influence on the solution is required. We will focus on this subject in a future paper.

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