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A CONTINUOUS MECHANIC APPROACH TO TEACH SOLID MECHANICS

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Abstract. *Generally, engineering students have been taught Mechanics of Solids and Strength of Materials through a technical theory based on one-dimensional models of bars and beams. Basic concepts such as strain and stress are defined for specific problems with no relation to the general case. Therefore, students do not learn how to apply those concepts to models that are common in real problems such as plane stress, elastic solids, plates, and shells. Students are not also able to link related concepts in solid and fluid mechanics.*

It can be said that students have missed a good background in basic concepts of mechanics. Consequently, they will not be prepared to use engineering software confidently. In order to overcome this limitation, students should be taught starting from the basics concepts of mechanics to acquire a strong background. Moreover, it is not possible to teach specific concepts while the applications of mechanical engineering have demanded background in multidisciplinary areas.

Some educational projects have been developed to achieve an interactive course in formulation and numerical solution of mechanical problems. For this purpose two books have been prepared based on a continuum mechanics and variational formulation approach (Bittencourt and Feijóo, 1999; Feijóo and Bittencourt, 1999). Internet homepages, Java programs, and multimedia applications have also been developed (Bittencourt, Gomes, Driemeier and Feijóo, 1998; Bittencourt, Driemeier and Feijóo, 1998). The general objective is to present engineering students the basic mechanical concepts using multimedia and software resources to simplify understanding and assimilation of knowledge. This paper presents a review of the status of those educational projects.

Keywords: *Continuous mechanics, variational formulation, java, multimedia, internet.*

1. INTRODUCTION

Computers have changed the way of life in the modern society. The presence of computers are notorious in areas such as education, banking, shopping, and many others. In the educational area, the use of multimedia and Internet resources have allowed a larger expansion of teaching activities, information, and knowledge around the world. In the near future it will be possible to reduce the traditional learning cycle in all teaching levels. At the same time students will acquire a huge amount of a higher level of knowledge. For this purpose, it is essential to apply software, multimedia, and Internet resources.

Some educational projects have been developed related to an interactive introductory course in solid mechanics for undergraduate and beginning graduate engineering students (Bittencourt, Gomes, Driemeier and Feijóo, 1998). This course is based on a book (Bittencourt and Feijóo, 1999), Internet homepages (www.fem.unicamp.br/ ˜em421 and www.fem.unicamp.br/ ˜em505) (Bittencourt, Gomes, Driemeier and Feijóo, 1998), some Java and C++ programs (Bittencourt, Galvão, Silva and Feijóo, 1998), and multimedia applications developed using the software Toolbook (Bittencourt, Gomes and Feijóo, 1998).

Continuum mechanics, convexity, and variational formulation (duality and the Principle of Virtual Power) have been used to present concepts such as kinematics, admissible configurations, deformation, internal (stress) and external forces, constitutive relations, and equilibrium equations for beams, plane stress, plane strain, solids of revolution, potential, and fluids problems (see (Feijóo, Pereira and Taroco, 1980) and references therein). The traditional books of Solid Mechanics and Strength of Materials address the classical theory based on one-dimensional models of bars and beams. In spite of the relative success in the treatment of these problems, this approach does not allow students to recognize clearly the primal and dual variables, the relations between them, the hypotheses associated to the model and how to manage the hypotheses in order to deal with commonly practical problems such as plane stress, plates, shells, and solid components.

The technical theory of Strength of Materials is insufficient to achieve an overall view of mechanics. The variational formulation (see (Feijóo et al., 1980) and references therein) is a natural and a general approach to formulate and solve many types of mechanical problems. Initially, the kinematic hypotheses are defined and students can verify the simplifications imposed on the general three-dimensional case. Based on the hypothesis of continuous media and concepts of tensors analysis, it is possible to introduce general definitions of deformation and stress valid for any solid and fluid continuous media. Stresses and strains are related by material constitutive laws such as the Hooke's law. The (weak/strong) equilibrium equations are obtained from the Principle of the Virtual Power. Another advantage of the variational formulation is to induce naturally the variational solution methods of mechanical problems.

This paper is organized as follows. Initially, aspects of the textbooks are discussed. After that multimedia applications, a Java program, and internet homepages are presented. Finally, some conclusions are addressed.

2. TEXTBOOKS

Two textbooks have been prepared with an introduction to continuum mechanics, variational formulation, and numerical solution of mechanical problems (Bittencourt and Feijóo, 1999; Feijóo and Bittencourt, 1999). The first volume presents the basic concepts and apply them to deduce the differential equations of basic problems in solid and fluid mechanics. It is organized in chapters for equilibrium of rigid bodies (Newtonian and variational approaches), concepts of continuum mechanics (tensors analysis, differentiation, and eigenvalue problem), deformation, stress, constitutive relations, variational formulation, bars and shafts, beams, plane strain and stress problems, axy-symmetric problems, elastic solid, heat transfer, and fluids.

The second volume presents the variational numerical methods for the solution of mechanical problems with emphasis on the Finite Element Method (FEM). Chapters for basic mathematical definitions, variational numerical methods (collocation, weighted residue, Galerkin, least squares), finite element method, shape functions for h and p finite elements, numerical integration, application of the FEM to the problems presented in the first volume, linear sensitivity analysis, and structural optimization have been included. All chapters in the two volumes include theoretical concepts, examples, solved problems, and proposed exercises.

3. MULTIMEDIA APPLICATIONS

The multimedia software Toolbook has been used to develop interactive applications for teaching solid mechanics. The objective is to use multimedia resources such as pictures, sounds, and animations to review, illustrate, and practice concepts presented in the textbooks. A review of each textbook chapter will be provided. By using interactive exercises, it is possible to check the student's performance and advice about points to be improved. Further, some Java and C++ programs are accessible from the multimedia applications. Figure 1 shows the beginning page for the interactive course developed with Toolbook.

Figure 1: Interactive course.

At present, interactive applications to variational formulation and solution of beam differential equations were developed. The objective of these applications were to define some requisites and the layout for the interactive course. The initial interactive prototypes overcame the expectations. This prototype motivates the student's learning through a pleasant interface with sounds, texts, and animations.

Toolbook software offers many resources to develop highly interactive applications. It is possible to develop some simple animations and manage sounds and pictures in an easy way. Many animation sequences have been developed with the 3D Studio software.

The following section discusses the multimedia application for the solution of beam differential equations.

4. SOLUTION OF BEAM EQUILIBRIUM EQUATIONS

The beam (strong) equilibrium equations for the problems of axial loading, torsion of circular sections, and bending are given, respectively, by (Bittencourt and Feijóo, 1999)

$$\frac{d}{dx}\left(E(x)A(x)\frac{du(x)}{dx}\right) = p(x),$$
$$\frac{d}{dx}\left(G(x)I_p(x)\frac{d\theta(x)}{dx}\right) = t(x),$$
$$\frac{d^2}{dx^2}\left(E(x)I_z(x)\frac{d^2v(x)}{dx^2}\right) = q(x),$$

where $E(x)$ is the Young elastic module, $G(x)$ is the shear elastic module, $A(x)$ is the cross section area, $I_p(x)$ is the polar inertia moment, $I_z(x)$ is the inertia moment in the z-axis, $u(x)$ is the axial displacement, $\theta(x)$ is the torsion angle, $v(x)$ is the transversal displacement, and $p(x)$, $t(x)$, and $q(x)$ are the load equations. Assuming that the material and geometrical properties are constant ($E(x) = E$, $G(x) = G$, $A(x) = A$, $I_p(x) = I_p$, $I_z(x) = I_z$) the previous equations are simplified to

$$EA\frac{d^2u(x)}{dx^2} = p(x), \qquad GI_p\frac{d^2\theta(x)}{dx^2} = t(x), \qquad EI_z\frac{d^4v(x)}{dx^4} = q(x). \tag{1}$$

Two integrations are necessary to solve the traction and torsion equations resulting in two unknown integration constants. The bending equation demands four integrations and the determination of four constants. The constants are determined from the beam boundary conditions.

Taking the axial loading equation, the integrations give, respectively, the normal force $N(x) = EA\frac{du(x)}{dx}$ and the axial displacement $u(x)$. The two integrations for the torsion problem result in the torsion moment $M_x(x) = GI_p\frac{d\theta(x)}{dx}$ and the torsion angle $\theta(x)$. Finally, the four integrations in the bending equation result, respectively, in the shear force $V_y(x) = EI_z\frac{d^3v(x)}{dx^3}$, the bending moment $M_z(x) = EI_z\frac{d^2v(x)}{dx^2}$, the rotation $\alpha(x) = EI_z\frac{dv(x)}{dx}$, and the transversal displacement $v(x)$.

The load terms in (1) are represented by a singular function notation which allows to represent concentrated and distributed loads in the same equation (Bittencourt and Feijóo, 1999; Bittencourt, Driemeier and Feijóo, 1998).

Based on that, the following general procedure can be applied to the solution of problems (including statically indeterminate examples) governed by expressions (1) (Bittencourt and Feijóo, 1999):

1. Write the load term expression using a singular function notation. If there is any support between the beam endpoints (in addition to the boundary conditions) the

unknown reaction forces and rotation discontinuities on hinges must be included in the load term.

2. Integrate the differential equations (twice for the axial loading and torsion problems and four times for beams).

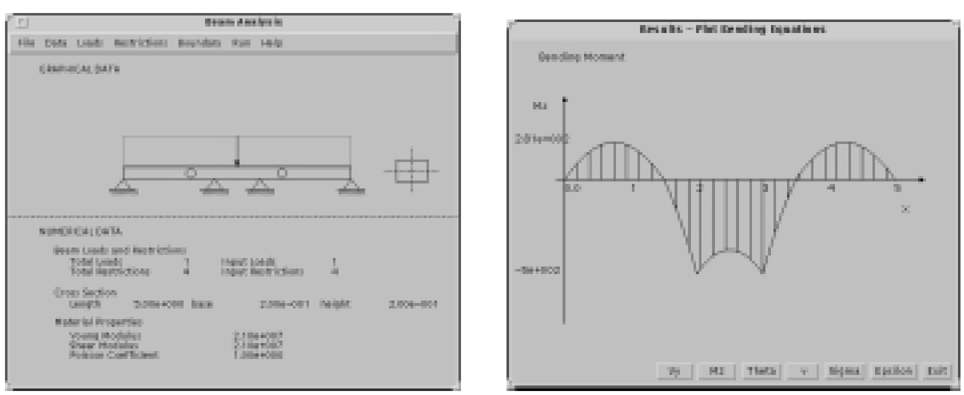
3. Apply the boundary conditions (given at the beam endpoints) and additional restrictions (given between the beam endpoints) to calculate the integration constants, reaction forces, and rotation discontinuities.

4. Write the equations for the internal loads (normal force, shear force, flexure moment, torsion moment), rotations, and displacements.

5. Calculate the reaction forces and moments at the beam endpoints.

6. Plot the diagrams.

A Java program was implemented based on the previous procedure (Bittencourt, Driemeier and Feijóo, 1998). Several boundary conditions types (e.g., support, cantilevered, elastic support) and additional restrictions (e.g., supports and hinges) are available. Similarly, several types of cross sections can be manipulated (e.g., circular, rectangular, profiles, general). The internal loads, deformation, and stress distributions, as well as, support reaction forces and moments are obtained through the integration of the differential equations and application of the boundary conditions and restrictions on the beam. All attributes and results are specified and presented graphically by using a graphic interface. It is possible to verify a beam or even design it by calculating the cross section dimensions. Figures 2a) and 2b) present respectively the user interface with an example and a bending moment diagram.



(a) Main program. **(b)** Bending moment diagram.

Figure 2: Java program.

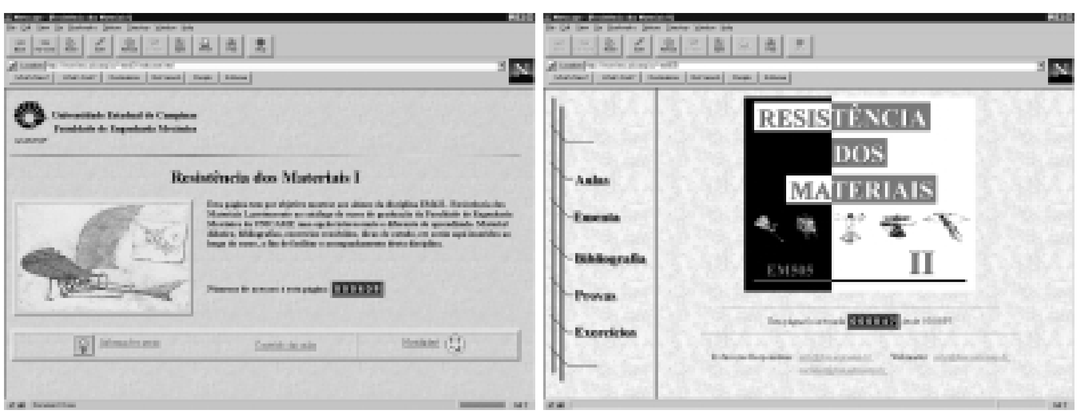
The beginning pages of the multimedia application are illustrated in Figure 3. Concepts related to bars such as basic definitions, differential equations in terms of normal forces, boundary conditions, and an animation of the kinematic hyphoteses are also illustrated. A glossary of terms was also included to review definitions of related concepts. The students can also check the assimilation of concepts by solving simple interactive exercises. Based on that, the application suggests points to be improved. In addition, exercise solutions are available in pdf format files. Finally, students can run the Java program from the multimedia application to solve more complex problems.

Figure 3: Multimedia application for the solution of beam equations.

5. HOMEPAGES

Two homepages for undergraduate courses in Mechanical Engineering at State University of Campinas were developed. The pages are available in the website addresses http://www.fem.unicamp.br/ em421 and http://www.fem.unicamp.br/ em505 by using browsers such as Netscape and Internet Explorer.

All information in the homepages are presented in a clear and organized form. The students have access to the main topics related to the disciplines, from general information to summaries of each class. In addition, it is possible to send e-mails to the teachers. Therefore, the students can have a new interactive learning resource.



(a) EM421. **(b)** EM505.

Figure 4: Homepages for the undergraduate courses EM421 and EM505.

Figure 4 illustrates the opening pages of the homepages. Starting from the page showed in Figure 4a), the following pages are available:

General Information : in this page the students have access to general discipline information such as course subject, bibliography, how to enter in contact with teachers, exam dates, and other information;

Class Summaries : a description of each class can be consulted starting from this page;

Support Texts : the student can access and consult some previous exams, resolved exercises, course notes, and the textbook. This information is available in the format pdf (Portable Document Format).

6. CONCLUSIONS

The textbook has been used in undergraduate and graduate courses. The students have enjoyed strongly the possibility to learn mechanical concepts in a broader form using continuum mechanics and variational formulation approaches. The homepages have been accessed by many students and professors around the world. Many e-mails have been received with questions, suggestions and positive comments. The variational formulation requires tensors analysis and differentiation concepts. Multimedia resources allow to present those concepts in a clear and didactic form using pictures, sounds, and animations. In addition, the student can practice all concepts through interactive examples, exercises and Java programs.

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