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MICROMECHANICS OF PARTICULATE METAL MATRIX COMPOSITES: MODELING EFFECTIVE MODULI

Antonio F. Ávila
Universidade Federal de Minas Gerais, Department of Mechanical Engineering,
6627 Antonio Carlos Avenue – 31270-901 Belo Horizonte, MG, Brazil
Fúlvio D. Deniculi
EMBRAER, Department of Structures, 2170 Brigadeiro Faria Lima Avenue
São José dos Campos, SP, Brazil

Abstract. *Metal Matrix composites are attractive for high temperature engineering applications due to their high strength even under extreme situations. A key point on composite applications is the material effective properties. In this paper, the thermal-elastic moduli for metal matrix composites are calculated based on micromechanical models. This work is focused on particulate metal matrix composites, whose fibers are assumed to be as spherical inclusions on a matrix, and on weak interface fiber/matrix condition. This spherical inclusion is surrounded by an interface in a concentric spherical shell shape. Due to this geometric consideration the concentric spheres model is applied in such a way that the weak interface fiber/matrix condition, which is represented by a displacement and traction discontinuity across the interface, is satisfied. The material property bounds for such type of problem are calculated, and new expressions for the thermal conductivity are proposed based on the analogy between shear loading and conductivity. Moreover, on the proposed model, the interface material properties are described by the modified rule of mixtures. Numerical simulations for two sets of metal matrix composites, Ti/SiC and Al/SiC, are shown and compared against experimental and numerical simulation results available in the literature with good agreement.*

Keywords: Metal matrix composites, Micromechanics, Bounds for effective moduli, Thermal conductivity.

1. INTRODUCTION

The idea of bounds for elastic moduli was proposed in the early 60's when Hashin and Shtriknam (1963) based on the variational principle of "minimum energy" derived the effective elastic moduli for a multiphase material. However, it was only on the late 60's and early 70's that the bounds for the thermal conductivity and the thermal expansion coefficient were derived. A comprehensive study on thermoelastic properties was given by Rosen (1970) and Hashin (1972). However, none of them considered the inclusion of the fiber/matrix interface.

When dealing with metal matrix composites at elevated temperatures, it is needed to take into account the mechanism of fracture caused by chemical reactions between the matrix and fibers, which can result in debonding, crack formation and propagation. Due to these chemical reactions and the process of manufacture itself., an interface region grows between the fiber and the matrix. Various researchers have studied the nature and the effects of this interface. Needleman (1987), Hashin (1990), Jones *et al*. (1994), Aboudi (1994) are just a few of them. Aboudi (1994), for instance, concluded that for MMC the most critical mechanism of failure is debonding.

In this work we introduce some new bounds for thermomechanical properties of particulate composites where the weak interface fiber/matrix condition is applied. It is an extension of the first author previous work (Avila, 1998) where the weak interface condition was studied, and Avila and de Miranda's work (1999) where the analogy between shear deformation and thermal conductivity is applied.

2. NEW BOUNDS FOR THERMOELASTIC MODULI

To be able to model composites under weak interface fiber/matrix condition, we should first establish the interface material properties. In the present model, it is assumed that the interface properties, in special the shear and bulk moduli, are given by the modified rule of mixtures (Tsai & Hahn, 1980). The Poisson's ratio and the Young's modulus are calculated considering the equations of the elasticity theory (Jones, 1999). For the present formulation, the weak interface condition is represented by a discontinuity on the displacement and traction functional across the interface as in Avila (1999).

In the present model, it is assumed that the interface grows from the fiber diameter, which implies that the fiber volume fraction remains constant while the matrix volume fraction is decreasing, see Fig. 1. By considering the concentric spheres model and the interface growing law, the fiber, the matrix and the interface volume fractions are written as:

$$v_f = \left(\frac{a}{b}\right)^3, \quad v_i = \frac{(a+t)^3 - a^3}{b^3}, \quad v_m = \frac{b^3 - (a+t)^3}{b^3} \tag{1}$$

where a, b, and t are the fiber diameter, the matrix diameter and the interface thickness. The subscripts f, m, and i are representative of fiber, matrix, and interface properties, respectively.

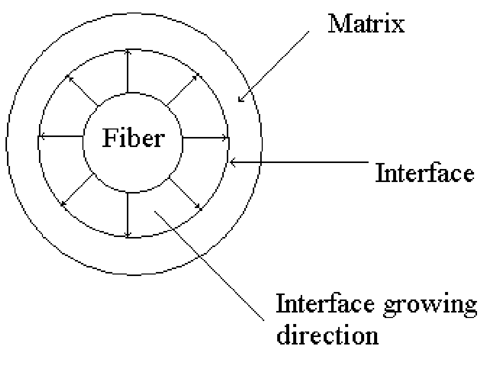

Figure 1: Modeling for interface growing

The analogy between the shear loading and the thermal conductivity was first discussed by Springer and Tsai (1967), and later on applied by Ávila and de Miranda (1999) to unidirectional composites. The interface thermal conductivity for this model is given by an analogous expression to the one used by the modified rule of mixtures to the shear modulus. Thus:

$$\frac{1}{\mu_i} = \frac{1}{v_f + \eta_i v_m}\left(v_f \frac{1}{\mu_f} + \eta_i v_m \frac{1}{\mu_m} \right) \tag{2}$$

where

$$\eta_\mu = \frac{1}{4(1 - v_m)}\left(1 + \frac{\mu_m}{\mu_f} \right) \tag{3}$$

μ_i, μ_f and μ_m are the thermal conductivity for the interface, the fibers and the matrix. For particulate composites, Hashin (1992) proposed bounds for the shear modulus, and applying the shear loading-thermal conductivity analogy we got expressions as

$$\mu^*_{(+)} = \mu_m v_m + \frac{\mu_f v_f}{1 + 5\mu_f / (2D'_n + 3D'_s + 3D'_t)a} \tag{4}$$

$$\mu^*_{(-)} = \left[\frac{v_m}{\mu_m} + \frac{v_f}{\mu_f} + \frac{2v_f}{5}\left(\frac{2}{aD'_n} + \frac{3}{aD'_s} + \frac{3}{aD'_t} \right) \right]^{-1} \tag{5}$$

For an isotropic thin compliant isotropic phase interface, the interface parameters on the orthogonal system of reference (n, s, t) can be written as

$$D_s = D_t = \frac{\mu_i}{t}, \quad D_n = \frac{2(1 - v_i)D_s}{1 - 2v_i} \tag{6}$$

where the interface Poisson's ratio is given by the classical theory of elasticity (Jones, 1999).

It should be mentioned that the interface parameters (D_s, D_t, D_n) are spring-constant-type coefficients, which relate the interface normal and tangential displacement jumps to the interface tractions.

Another important thermoelastic property is the thermal expansion coefficient (CTE) due to its influence on the thermal stress calculation. In our model, it is applied the bounds for a multiphase concentric spheres composite derived by Hashin and Rosen (1970). The interface properties are calculated as before, but for the CTE the rule of mixtures is applied. The bounds for a three isotropic phase composite (fiber, interface and matrix), where the concentric spheres model is applied are given by

$$\alpha^*_b = \frac{1}{\left(\overline{\frac{1}{K}}\right) - \frac{1}{\overline{K}}}\left\{ \frac{\overline{K\alpha}}{\overline{K}}\left[\left(\overline{\frac{1}{K}}\right) - \frac{1}{K^*} \right] + \overline{\alpha}\left[\frac{1}{K^*} - \frac{1}{\overline{K}} \right] \pm \psi\left[\left(\overline{\frac{1}{K}}\right) - \frac{1}{K^*} \right]^{1/2}\left[\frac{1}{K^*} - \frac{1}{\overline{K}} \right]^{1/2} \right\} \tag{7}$$

where

$$\psi = \left\{ \left[\left(\overline{\frac{1}{K}}\right) - \frac{1}{\overline{K}} \right]\left[\overline{K\alpha^2} - \frac{\left(\overline{K\alpha}\right)^2}{\overline{K}} \right] - \left[\overline{\alpha} - \frac{\overline{K\alpha}}{\overline{K}} \right]^2 \right\}^{1/2} \tag{8}$$

and

$$\left(\overline{\frac{1}{K}}\right)=\frac{v_f}{K_f}+\frac{v_m}{K_m}+\frac{v_i}{K_i} \tag{9}$$

$$\overline{K}=v_f K_f+v_m K_m+v_i K_i \tag{10}$$

$$\overline{K\alpha}=v_f K_f \alpha_f+v_m K_m \alpha_m+v_i K_i \alpha_i \tag{11}$$

$$\overline{K\alpha^2}=v_f K_f \alpha_f^2+v_m K_m \alpha_m^2+v_i K_i \alpha_i^2 \tag{12}$$

The remaining material properties, e.g. Young's modulus and Poisson's ratio, can be calculated based on the classical theory of elasticity as in Jones (1999).

3. NUMERICAL SIMULATIONS

On this section we will discuss two set of data: Al/SiC and Ti/SiC. Our objective is to investigate such different types of composite, and how the interface fiber/matrix affects the overall material behavior.

3. 1 Case 1: Al/SiC

It is considered an Al/SiC metal matrix particulate composite due to its importance on engineering applications, and the possibility of comparisons against experimental results (McDanels, 1985). The material properties from Lynch (1998) are summarized on Table 1.We should also mention that it was considered the fiber diameter as 0.1 μm.

Table 1. Al/SiC material properties

Properties	Al	SiC
K [GPa]	67.60	259.95
G [GPa]	26.30	202.80
E [GPa]	68.95	482.60
ν	0.33	0.19
μ [W/mK]	155.80	498.20
α [με/K]	$0.24x10^{-4}$	$0.434x10^{-4}$

To be able to investigate the interface influence on the overall composite behavior, a set of different ratio interface thickness/fiber diameter was studied. The results for the thermal conductivity, lower and upper bounds, for four different interface conditions are shown on Figs. 2 and 3.

When the ratio t/a approaches to zero, the results are closer to the perfect bonding condition. The same pattern can be observed for the coefficient of thermal expansion for both upper and lower bounds obtained by the numerical simulations, see Figs. 4 and 5.

To validate the proposed model a specific t/a ratio, 0.0001, was selected. Numerical simulations for the Young's modulus and the coefficient of thermal expansion are performed, and the results compared against data available in the literature. Those comparisons are shown in figures 6 and 7. It is observed a good agreement between the predicted results and those from literature.

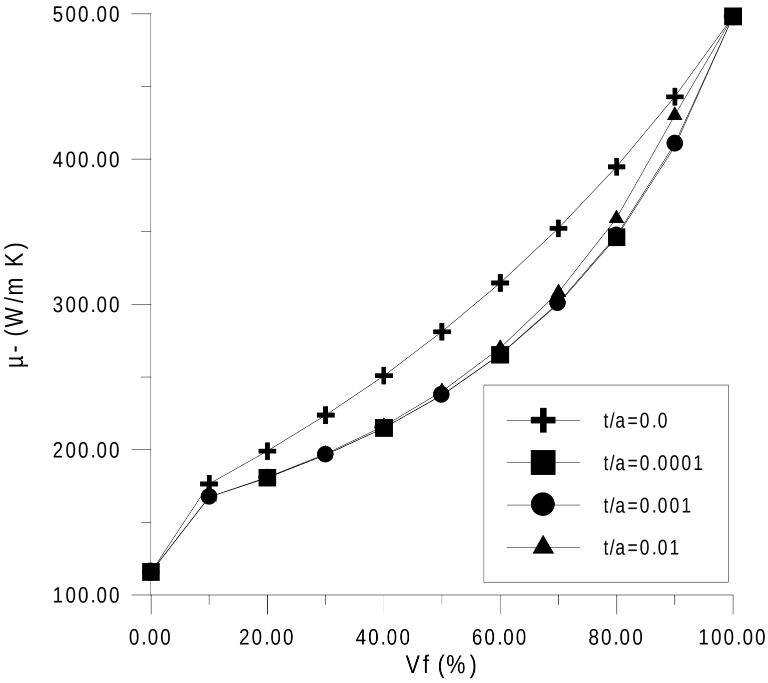

Figure 2: Thermal conductivity lower bound for four different interface conditions

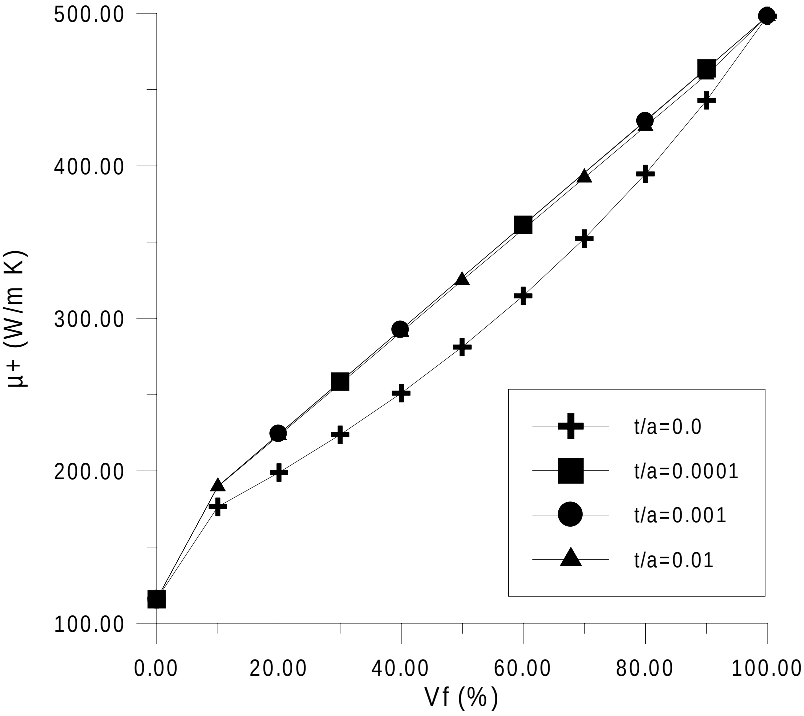

Figure 3: Thermal conductivity upper bound for four different interface conditions

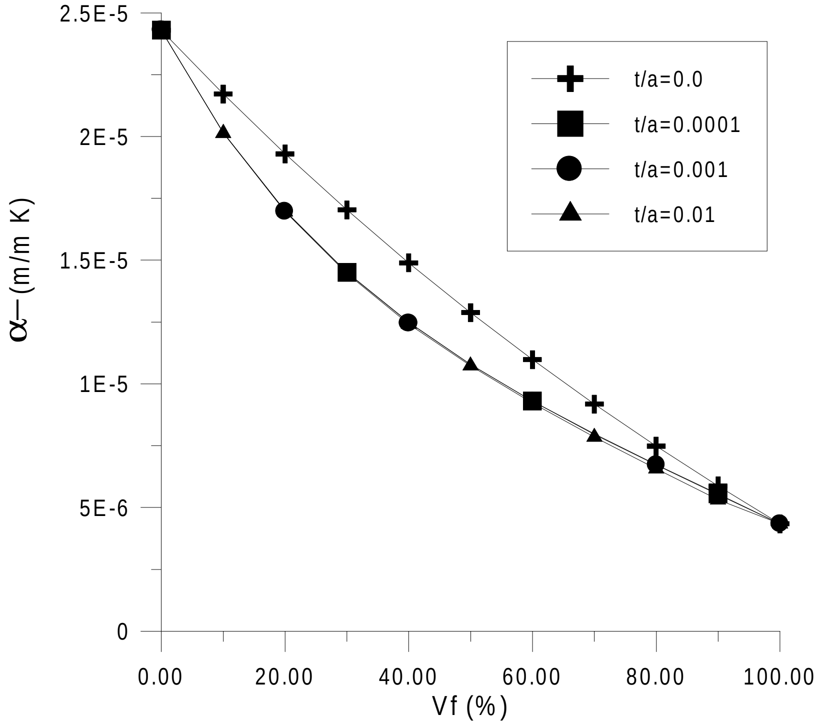

Figure 4: CTE, lower bound, for four different interface conditions

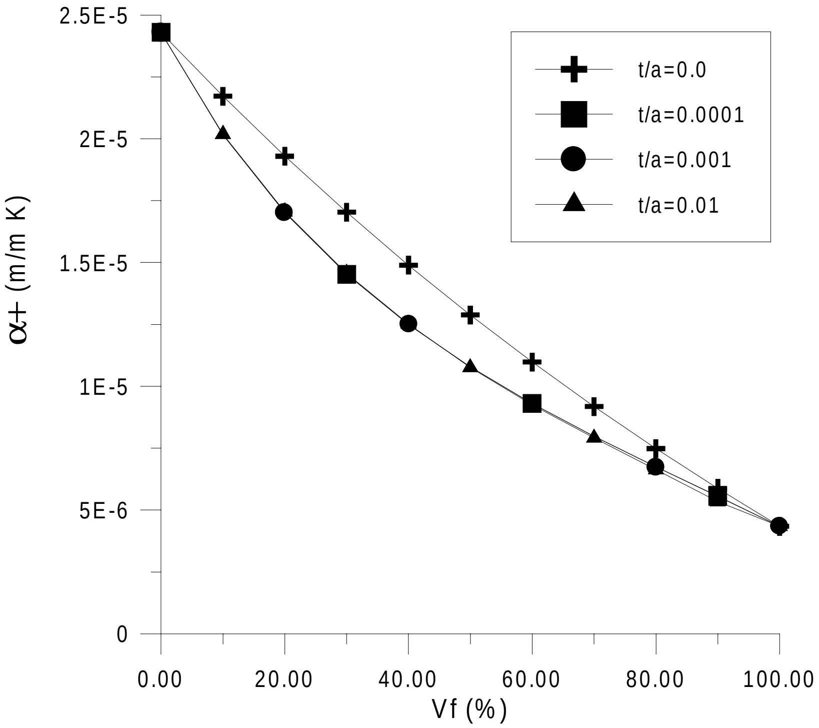

Figure 5: CTE, upper bound, for four different interface conditions

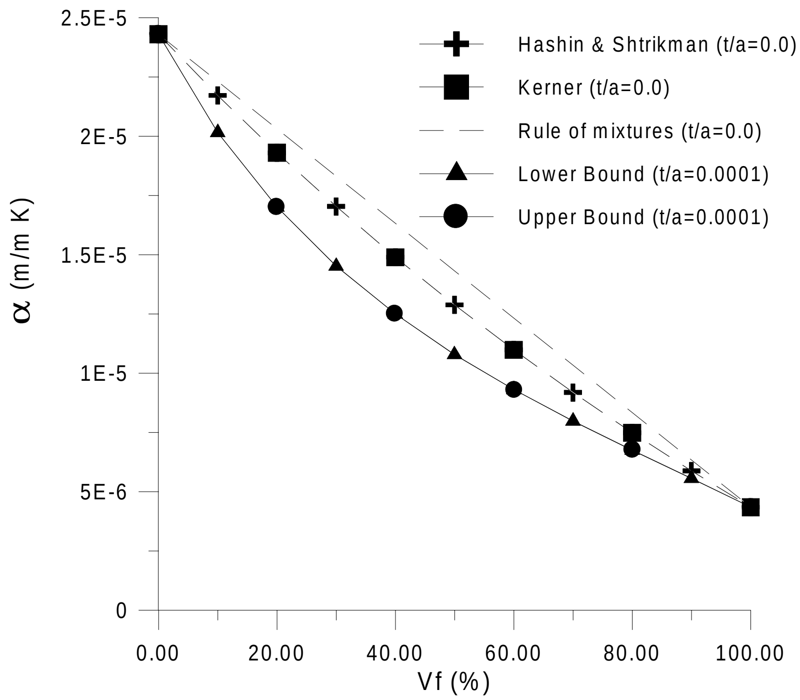

Figure 6: CTE comparison among various models

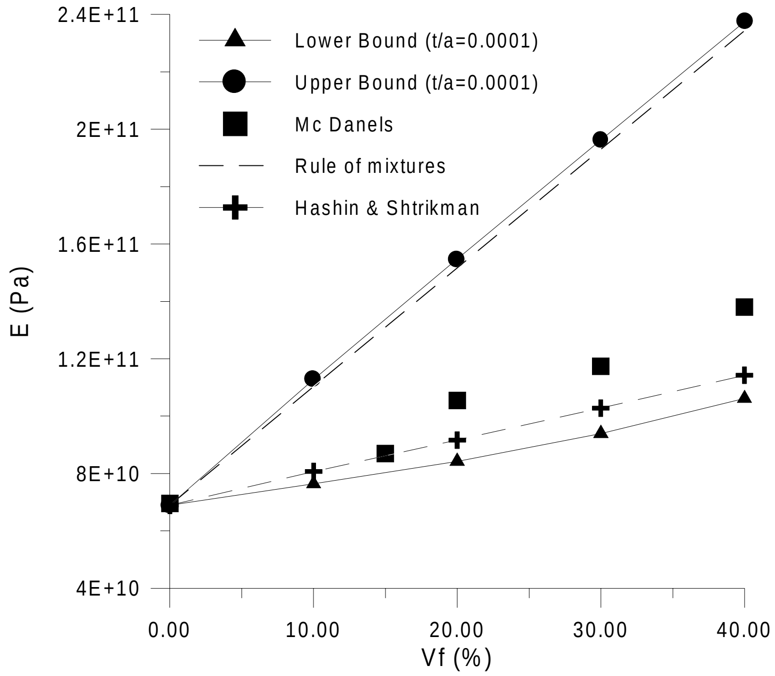

Figure 7: Young's modulus comparison among various models

3.2 Case 2: Ti/SiC

Another important set of MMC, for engineering applications, is the Ti-6Al-4V-base composite due to its large use in aerospace industry. The material properties are listed on Table 2. We focus our attention on the thermal conductivity and the CTE values. The results shown in Figs. 8 and 9 are concerned to the thermal conductivity, while Figs 10 and 11 deal with the thermal expansion coefficient. It seems that they are in good agreement with data available in the literature

Table 2: Ti/SiC material properties

Properties	Ti	SiC
K [GPa]	91.90	259.95
G [GPa]	42.43	202.80
E [GPa]	110.30	482.60
ν	0.30	0.19
μ [W/mK]	200.00	498.20
α [$\mu\varepsilon$/K]	$0.850\text{x}10^{-4}$	$0.434\text{x}10^{-4}$

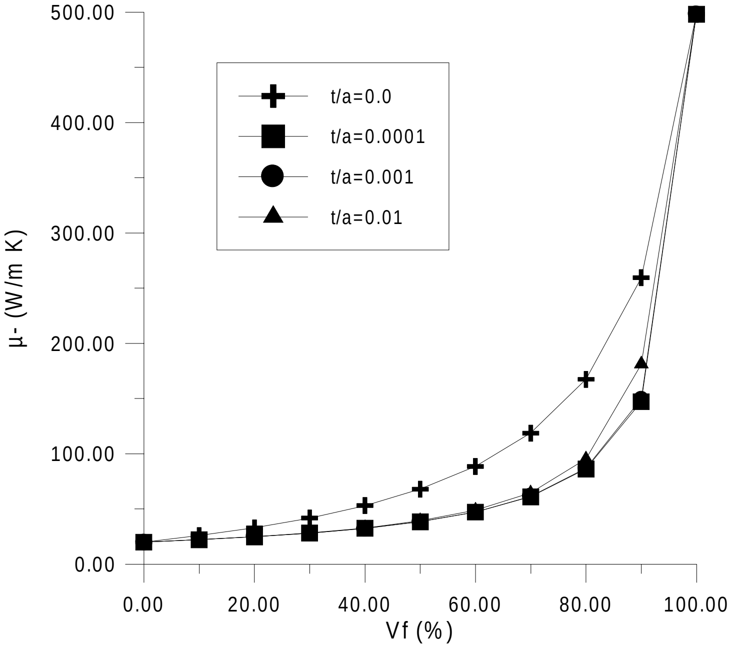

Figure 8: Thermal conductivity, lower bound for Ti/SiC particulate composite

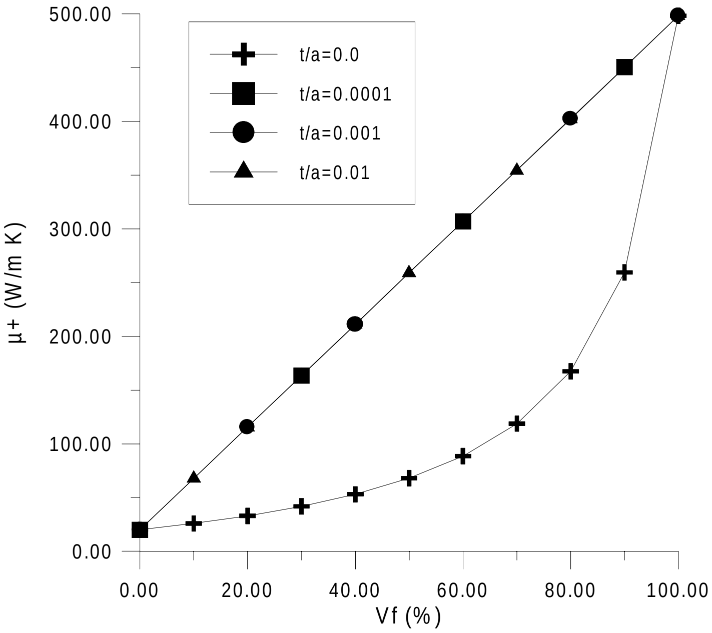

Figure 9: Thermal conductivity, upper bound, for the TiSiC particulate composite

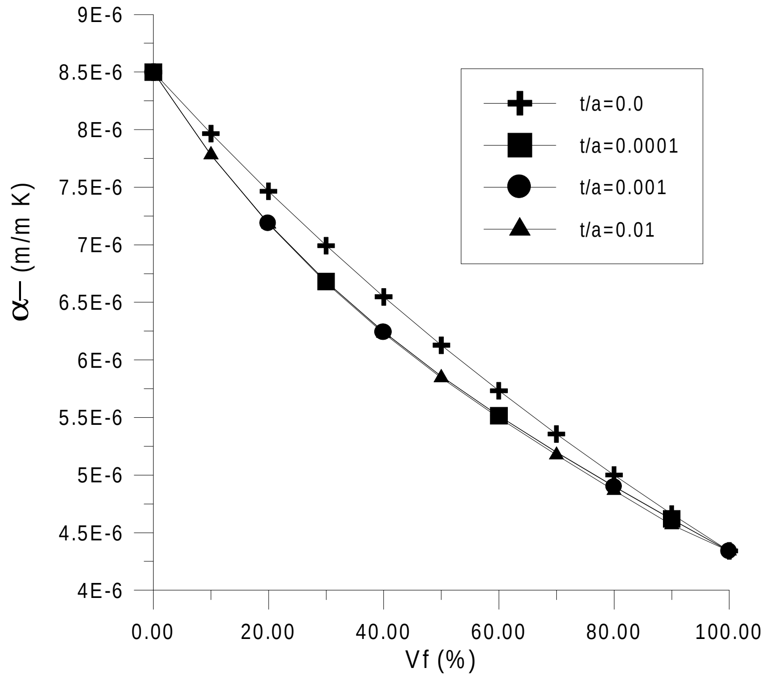

Figure 10: CTE, lower bound, for TiSiC particulate composite

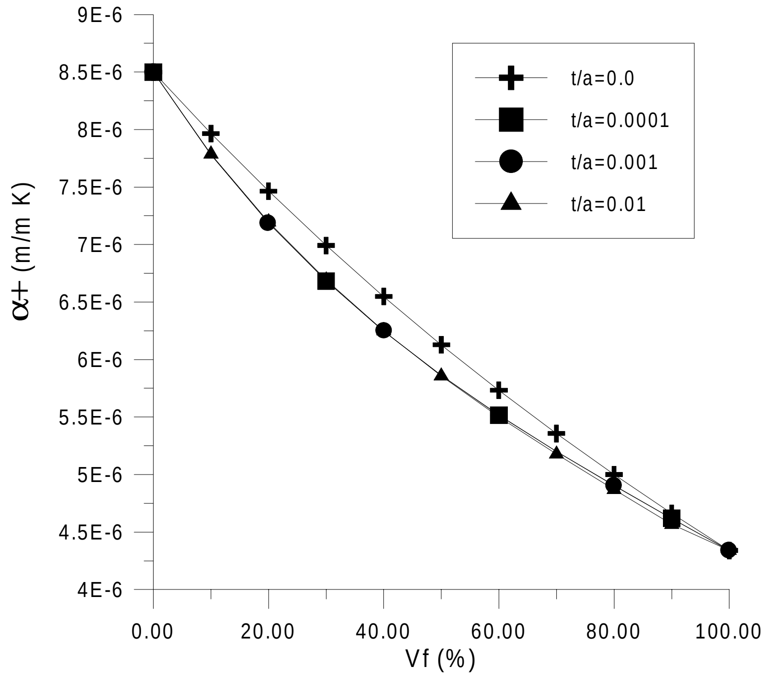

Figure 11: CTE, upper bound, for TiSiC particulate composite

4. CONCLUDING REMARKS

New bounds for thermal conductivity and thermal expansion coefficient of particulate MMC are proposed, and the results compared against results available in the literature. The results are encouraging and point out to a possible new research area. The interface fiber/matrix modeling should be more studied and new expressions for predicting interface material properties are under development.

5. ACKNOLEGMENTS

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