XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

H_∞ CONTROL OF A ROBOTIC MANIPULATOR WITH A FLEXIBLE LINK

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SUMMARY. *This paper focuses a H_∞ robust controller design for a one-link flexible robotic manipulator. The goal is to move the manipulator suppressing the vibration the motion produces as well as those produced for external disturbances. This particular control problem presents some characteristics which are critical for the closed-loop stability. The residual dynamics, which is not represented in the plant model for controller design, is taken in account in order to avoid stability (spillover) problems. The H_∞ method is utilized in the design to cope with these problems. Robustness is guaranteed including in the design a non-structured uncertainty model, an upper bound to the residual part of the system. In this way, the system is partitioned in the modal form and is represented as a model in the additive form. The controller design specifications require the system to track a reference as well as to suppress the bending due to the first mode, beside presenting robustness characteristics in respect to the higher, non-modeled modes. The controller so obtained was simulated in order to check for performance and robustness characteristics.*

Key words: *H_∞ controller design; Flexible robots; Robust control; Vibration suppression*

1. INTRODUCTION

The dynamic modeling and control of compliant mechanical structures is an important research area, and is specially related to aeronautical and aerospace applications, and modern robotics. In robotic research, the use of optimized structures made of light materials, and actuated by low weight, small torque motors is a very attractive goal. On the other hand, such a robotic structural design generally result in robot links with low mechanical rigidity, with a large number of vibration modes and low structural damping. These dynamic characteristics may degrade accuracy and adversely affect the actuation control system resulting in unstable behavior.

This paper focuses a robust controller design for a one-link flexible robotic manipulator. The goal is to move the manipulator suppressing the vibration the motion produces, as well as those produced for external disturbances. This particular control problem presents some characteristics which are critical for the closed-loop stability:

- the plant is a distributed-parameter, infinite-dimensional system;
- the design of the controller is based on a finite-dimensional model;
- the system is multivariable, with non-collocated sensors and actuators, i.e. they are not collocated at the same points on the structure;
- the actuator energy is limited.

Using a finite-dimensional model implies the truncation of the system dynamics. In addition, for reducing the problem complexity and the controller order, a plant model reduction is necessary. Thus, the residual dynamics (which is not represented in the plant model for controller design) should be taken in account, in some way, in order to avoid stability (spillover) problems (Inman, 1989; Berkman and Karnop, 1969). Non-collocated sensors and actuators produces a non-minimum phase system (zeros in the right complex semi-plane), which is a difficult aspect to deal, having a performance limitation impact (Middleton, 1991).

The H_∞ method is utilized in the design to cope with these problems. This method can provide suitable robustness characteristics to the controller. The specific case of H_∞ vibration control is approached in Moreira e Arruda (1998), Moreira (1998), Moreira e Arruda (1999), showing good results. The main aspect treated in these works is the inclusion of robustness characteristics with respect to the residual dynamics of the structure, eliminating spillover. Robustness is guaranteed including in the design a non-structured uncertainty model, an upper bound for the residual part of the system. In this way the structure is partitioned in the modal form and is represented as a model in the additive form.

This paper presents the dynamic modeling identification and the robust control system design for a flexible slewing structure composed of a single flexible link, as shown in Figure 1. The controller design specifications requires the system to track a reference as well as to suppress the bending due to the first mode, besides having robustness characteristics in respect to the higher, non-modeled modes. The resulting controlled system is simulated to check for performance and robustness characteristics.

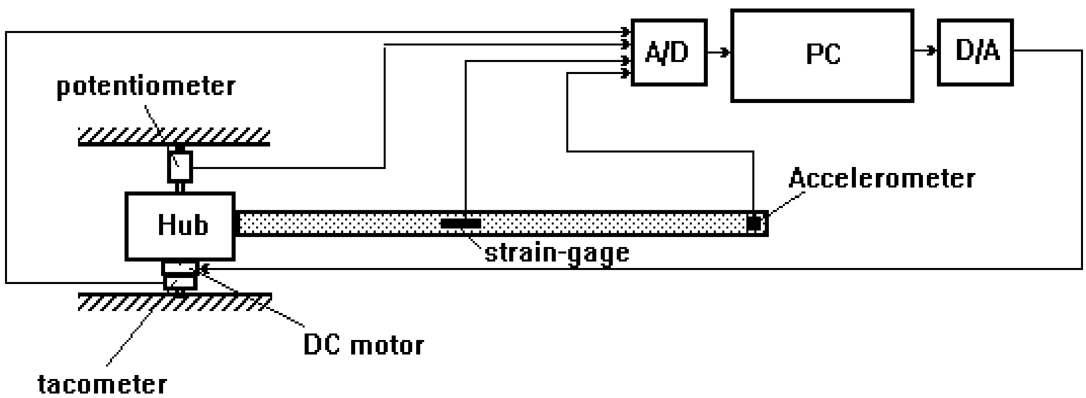

Figure 1 - One link flexible manipulator.

2. DYNAMIC MODEL OF THE PLANT

As illustrated in Figure 1, the system under consideration can be considered as a flexible beam attached to a rotating motor shaft. A set of integro-differential equations describes the coupled rigid-body and bending modes motion (Soares and Góes, 1993; Góes and Adade, 1999) of this system. Since the differential eigenvalue problem for this set of equations is difficult to solve, an approximate spatial discrete model is used. The discretization is performed by the assumed modes method, which leads to a linear, finite-dimensional, continuous-time equations. In the assumed modes method, the deflection of the flexible structure is modeled by a finite (n terms) series of a space-dependent function multiplied by a time-dependent function.

Proceeding this way, the dynamic equations of the system can be rewritten in a matrix form as follows:

$$M\,\ddot{q} + K\,q = \Gamma\tau \tag{1}$$

where

$$M = \begin{bmatrix} \mathrm{I_B} + \mathrm{I_H} & 0 \\ \left[-2\dfrac{(\mathrm{I_B}+\mathrm{I_H})\theta_\mathrm{i}}{\mathrm{I_B} - (\mathrm{I_B}+\mathrm{I_H})\theta_\mathrm{i}^2} \right]_{1xn} & [\mathrm{I}]_{nxn} \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & 0 \\ [0]_{1xn} & [\Omega^2]_{nxn} \end{bmatrix} \tag{2}$$

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}^T$$

with

$$[\Omega^2]_{nxn} = \begin{bmatrix} \omega_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \omega_n^2 \end{bmatrix}, \qquad \omega_\mathrm{i}^2 = \sqrt{\frac{\mathrm{EI}}{\rho}}\beta_\mathrm{i}^2, \qquad \mathrm{i=1,...,n} \tag{3}$$

In Eq. (1)-(3); θ_i is the angular displacement of the i^{th}-mode, ρ is the mass/unit length, EI is the constant bending stiffness of the beam, I_H is the inertia of the hub and I_B is the beam inertia about an end; β_i is a separation constant.

In the state-space form,

$$\dot{x} = A_0\,x + B_0\,\tau \tag{4}$$

$$A_0 = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix} \tag{5}$$

$$B_0 = \begin{bmatrix} 0 \\ M^{-1}\,\Gamma \end{bmatrix} \tag{6}$$

$$x = \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix}^T, \qquad q = \begin{bmatrix} \theta & q_1 & q_2 & \cdots & q_n \end{bmatrix}^T \tag{7}$$

$q_i(t)$ (i=1,...,n) are the generalized coordinates (modal amplitudes) and $\theta(t)$ is the rigid-body angular displacement.

3. MODEL ESTIMATION

The system model is identified using an identification algorithm (Moreira and Arruda, 1997) based on the eigensystem realization algorithm (ERA) approach (Juang and Pappa, 1988). The frequency response of the estimated transfer function are compared with the experimental frequency response in Fig. 2. The functions plots in Fig. 2 refer to the transfer function from the input signal (the brushless motor command) to the potentiometer, G11, the tachometer, G21, and the strain-gage, G31, *i. e.*, the position, velocity and deformation signals are the measured outputs.

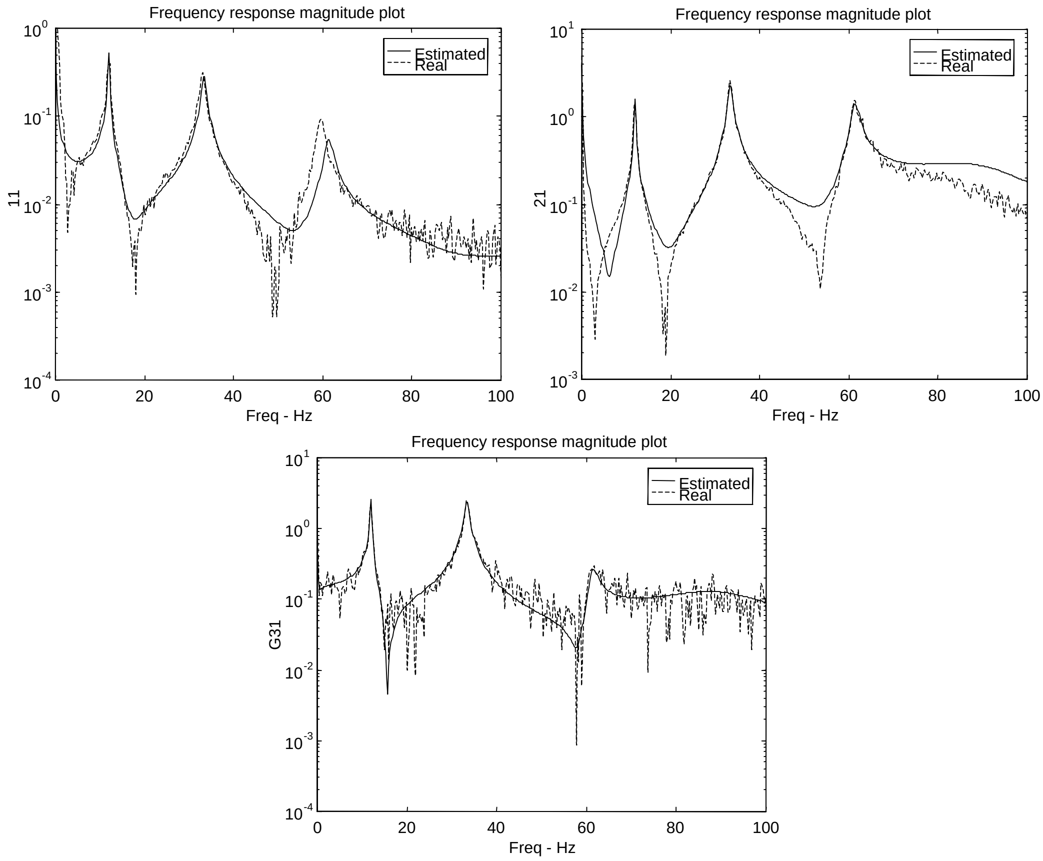

Figure 2 - Frequency response of the real system and the estimated model.

The estimated plant model, in the canonical Jordan form, yields

$$\mathbf{A} = \begin{bmatrix} \begin{bmatrix} \sigma_i & \omega_i \\ -\omega_i & \sigma_i \end{bmatrix} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} , \quad i = 1,2,3 \qquad (8)$$

$\mathbf{B}$=[-2.0957 18.5893 -2.2461 12.6562 -4.3363 -3.5241 2.9510 -1.0298]^T

$$\mathbf{C}=\begin{bmatrix} -0.0088 & 0.0310 & -0.1327 & 0.0577 & 0.1038 & -0.1667 & -0.0105 & 0.4125 \\ -0.8312 & 0.1012 & -0.6247 & -0.9779 & 0.5027 & 0.3189 & 0.1071 & 1.9062 \\ -0.0303 & 0.1828 & -0.9958 & 0.7336 & 0.3700 & -0.9409 & 0.1023 & 0.2571 \end{bmatrix}$$

where $\sigma_1 = -5.85$, $\omega_1 = 385.25$, $\sigma_2 = -3.23$, $\omega_2 = 209.60$, $\sigma_3 = -1.04$, $\omega_3 = 75.19$.

4. H_∞ ROBUST CONTROL DESIGN

Framework for controller design

The torque motor is the unique actuator in the flexible manipulator. Thus, it has to rotate the shaft through a desired angle in a way to maintain minimum vibration in the beam. So, the control target is to execute a maneuver in a short time interval, with small amplitude of the 1st bending mode, to reduce the manipulator oscillation. In addition, the system model is truncated since control implementation demands a controller order restriction. Then, the system must be robust to the residual modes, considered as model uncertainties.

The H_∞ robust control approach applies to systems with uncertainties in plant model, either in order or in the parameters. For our system, these uncertainties are considered being due to the residual sub-system which resulted from the model reduction (unstructured uncertainty). In the H_∞ control methods these uncertainties can be represented and included explicitly in the design requirements (Maciejowski, 1989).

The H_∞ problem can be summarized as a problem to find a controller **K(s)** that stabilizes an augmented system,

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{9}$$

$$u = K\, y \tag{10}$$

and satisfies the infinite-norm inequality

$$\left\| F_{z,w} \right\|_\infty = \left\| G_{11} - G_{12} K (I - G_{22} K)^{-1} G_{21} \right\|_\infty < 1 \tag{11}$$

where $F_{z,w}$ is the transfer function from the exogenous inputs w, to the output to be controlled, z. In other words, to limit the energy transfer from the input w to the output z. The general form of the problem is shown in Fig. 3. The optimal H_∞ problem minimizes the infinite-norm.

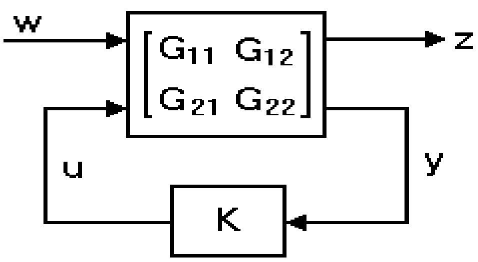

Figure 3 - General problem formulation

Model partition

To determine the nominal plant dynamics, which is used in the controller design, the system is modeled as a reduced model, G_m, and a residual model, G_r. Partitioning the system in a modal form, the function G_r in Fig. 4, can be considered an additive uncertainty to the model. This residual model, which includes all the unmodeled modes, can be substituted by a low order upper bound function, G_{up}, such that

$$\left\| \mathbf{G}_{up} \right\|_\infty \geq \left\| \mathbf{G}_r \right\|_\infty \tag{12}$$

which allows to include in the design, some elements to attain robustness to the residual dynamics.

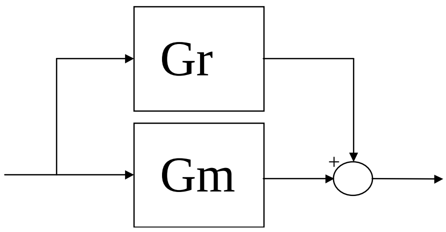

Figure 4 - Additive uncertainty model.

The resultant partition of the system is shown in Fig. 5, with G11, G21 and G31 as defined in section 3.

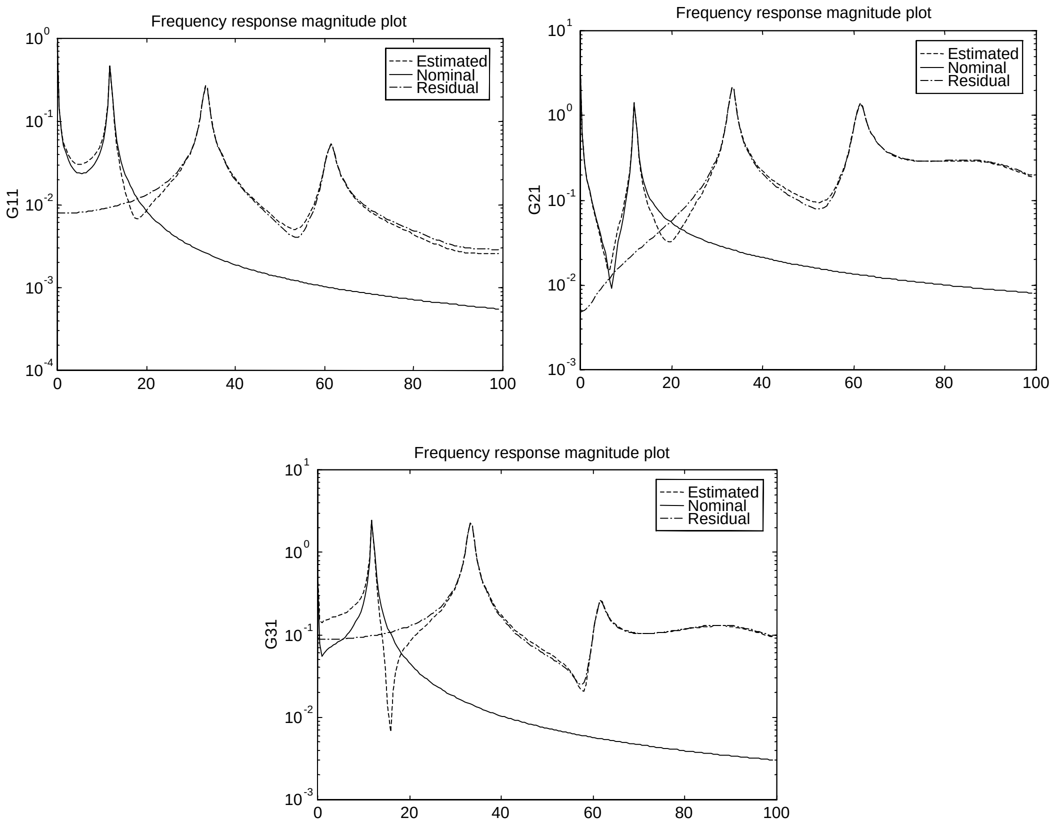

Figure 5 - System partition.

Design model

The H_∞ design model is shown in Fig. 6 as a Simulink® simulation model. The performance requirements are associated with the regulated outputs, *d* and *e*, respectively the strain and the error signals. These outputs are weighted by the functions W1 and W2, respectively. The weighting functions are the design parameters: W1 weights the error-command relation (sensitivity function) and specifies the response time and steady-state error; W2 weights the bending rates-command relation and specifies required bending modes attenuation. The uncertainty model Gup, with the respective uncertainty inputs p1, p2 and p3, and output *q*, takes the robustness to the dynamics of the residual modes in the design.

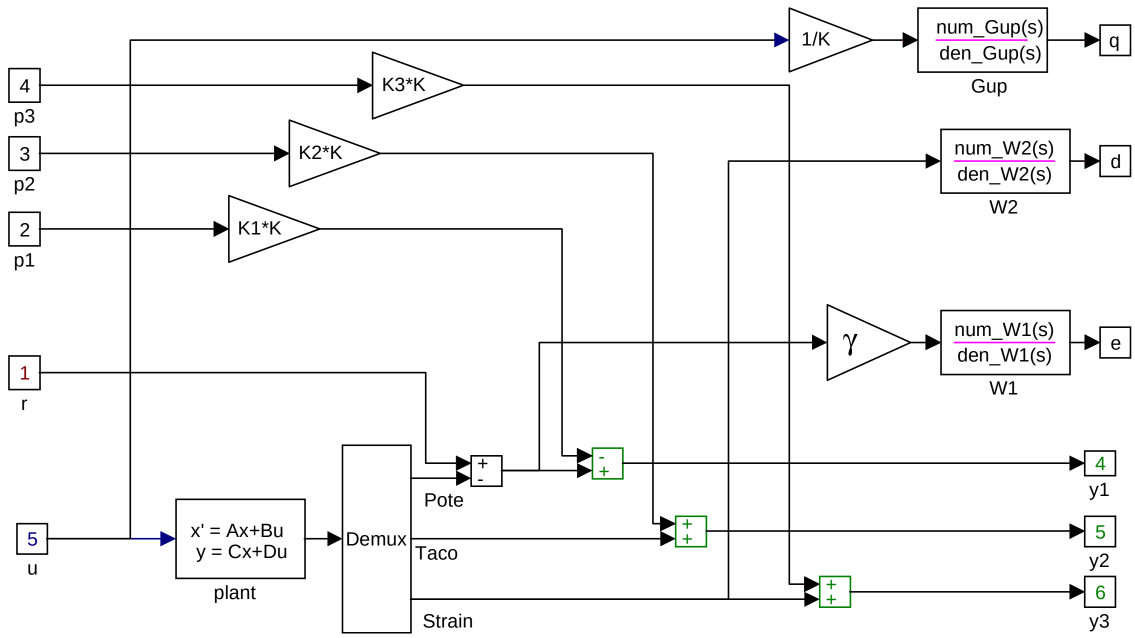

Figure 6 - Design model as a Simulink simulation model.

The specified W1 and W2 are shown in Fig. 7. The function W1 is a first-order function with high gain in the frequency region that includes the rigid-body dynamics. The function W2 is a second-order function with high gain in the frequency region of the first mode.

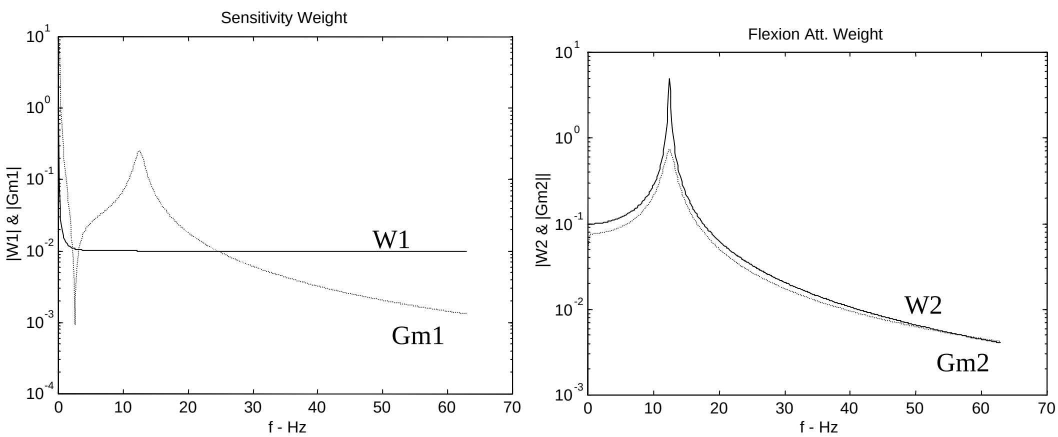

Figure 7 - Weighting functions W1 and W2: the error specification and 1st mode attenuation specification, respectively.

The selected upper bound function for the residual uncertainty is shown in Fig. 9. Only one function is used to bound the uncertainty Gr. A set of gain factors, K1, K2 and K3, is used to adjust the gain to the three input-output relationship between the motor command signal and the measured position, velocity and deformation. In addition, it is utilized a scale factor K to trade off performance and robustness.

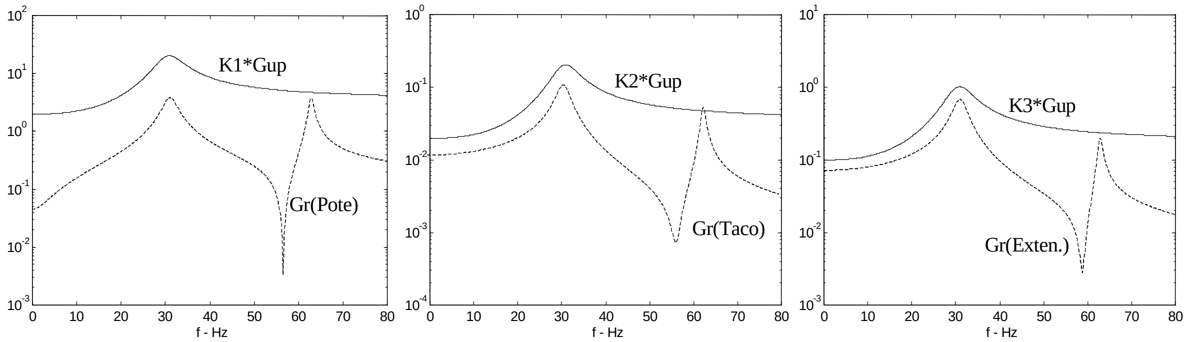

Figure 9 - Residual uncertainty specification.

Therefore, the selected weighting functions are:

$$W_1 = \frac{1}{s + 0.001}\ ,\ W_2 = \frac{6050}{s^2 + 1509\,s + 5694}\ ,\ G_{up} = \frac{0.9383\,s^2 + 89.59s + 4364}{s^2 + 41.5s + 43060}$$

H_∞ controller

The final parameters of the design are K = 0.1, K1= 0.1, K2 = 1, K3 = 1, γ = 100. This parameter γ is utilized to adjust the error specification.

The H_∞ controller (A_K, B_K, C_K, D_K) for this problem was obtained using the Matlab® LMI Toolbox yields

$$\mathbf{A}_K = \begin{bmatrix} \begin{bmatrix} \sigma_i & \omega_i \\ -\omega_i & \sigma_i \end{bmatrix} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \begin{bmatrix} \sigma_7 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma_8 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sigma_9 \end{bmatrix} \end{bmatrix} \quad , \quad \text{with i = 1,…,6} \tag{13}$$

where
σ_1 = -83.21, ω_1 = 559.23,
σ_2 = -5.86, ω_2 = 385.25,
σ_3 = -3.23, ω_3 = 209.60,
σ_4 = -48.79, ω_4 = 49.22,
σ_5 = -4.77, ω_5 = 79.23,
σ_6 = -9.33, ω_6 = 71.79,
σ_7 = -8.12,
σ_8 = 0,
σ_9 = 0.0015.

and

$$B_K = \begin{bmatrix} -1.0102e{+}001 & 1.5746e{-}005 & -2.2534e{-}005 \\ 6.1998e{+}000 & -9.6636e{-}006 & 1.3830e{-}005 \\ 2.4382e{+}000 & -3.8004e{-}006 & 5.4389e{-}006 \\ 8.0536e{+}000 & -1.2553e{-}005 & 1.7965e{-}005 \\ 1.4391e{+}000 & -2.2431e{-}006 & 3.2102e{-}006 \\ -2.0551e{+}000 & 3.2033e{-}006 & -4.5843e{-}006 \\ -2.1153e{+}002 & 3.2971e{-}004 & -4.7186e{-}004 \\ 2.5639e{+}002 & -3.9962e{-}004 & 5.7191e{-}004 \\ -2.9508e{+}000 & 4.5984e{-}006 & -6.5809e{-}006 \\ -4.9828e{+}001 & 7.7668e{-}005 & -1.1115e{-}004 \\ -7.4510e{+}000 & 1.1613e{-}005 & -1.6619e{-}005 \\ -5.3772e{+}001 & 8.3815e{-}005 & -1.1995e{-}004 \\ -7.1627e{+}002 & 1.1163e{-}003 & -1.5976e{-}003 \\ 1.1096e{-}005 & 6.8190e{-}005 & 2.9002e{-}003 \\ -1.4627e{+}003 & 7.8320e{-}004 & -1.1209e{-}003 \end{bmatrix} \quad C_K = \begin{bmatrix} 8.7552e{-}007 \\ 4.3855e{-}006 \\ 4.9774e{-}006 \\ -2.7905e{-}005 \\ 1.0646e{-}005 \\ 5.4243e{-}005 \\ 3.9896e{-}002 \\ 4.6044e{-}001 \\ 7.4096e{-}002 \\ 4.2969e{-}002 \\ -2.2328e{-}002 \\ 1.3405e{-}001 \\ 1.5480e{-}001 \\ 8.6104e{-}011 \\ -1.1703e{-}005 \end{bmatrix}^T$$

5. RESULTS

To verify the performance and the robustness of the control system, a closed-loop simulation is performed taking in account an extended order model (rigid-body and 3 bending modes). The step response of the closed-loop system is shown in Fig. 10. The simulation results are the error signal y1, the strain of the flexible link y3, and the control signal u.

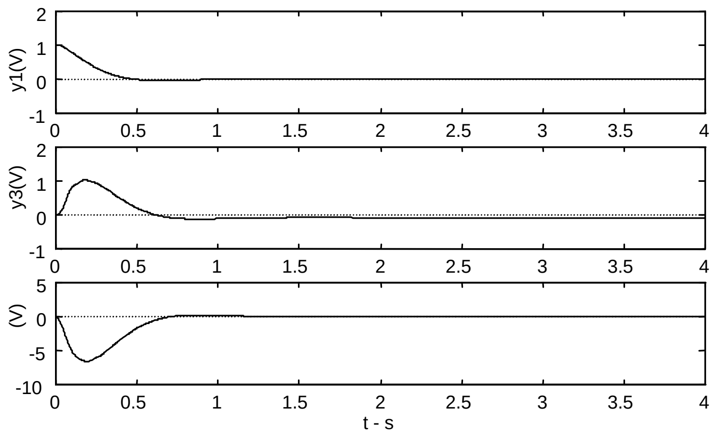

Figure 10 - Closed-loop response.

6. CONCLUSION

The results obtained show that the H_∞ approach can be used to control a flexible structure using a truncated model. Some characteristics of this design are: tracking a reference signal; 1st bending mode suppression; use of a low order design model with only the dynamics to be controlled; robustness to residual modes; and low order additive uncertainty model.

Some fundamental problems we have to face in this approach are: to translate the design specifications into a set of weighting functions; adequately model the uncertainties; and to choose the appropriated plant input and output signals.

REFERENCES

Berkman, F. & Karnop, D., 1969, Complete response of distributed system controlled by a finite number of linear feedback loops, ASME Journal of Engineering for Industry, 91, November, pp. 1062-1068.

Góes, L.C. S. & Adade Filho, A., 1999, Robust control of a slewing flexible structure, Proceedings of the 6^{th} Pan American Congress of Applied Mechanics, Rio de Janeiro, Brazil, 04-08/Jan/1999, Vol. 8, pp. 1139-1142.

Inman, D. J., 1989, Vibration with Control, Measurement, and Stability, Prentice-Hall, New Jersey.

Juang, J-J. & Pappa, R. S., 1988, An eigensystem realization algorithm for modal parameter identification and modal reduction, Journal of Guidance, Control, and Dynamics, 8 (5), pp. 620-627.

Maciejowski, J. M., 1989, Multivariable Feedback Design, Addison-Wesley, Wokingham, England.

Middleton, R. H., 1991, Trade-offs in linear system design, Automatica, vol. 27, n. 2, pp. 281-292.

Moreira, F. J. O., 1998, A limited-band H_∞ controller for the active control of structural vibration (in Portuguese). Ph. D. Thesis, State University of Campinas, Campinas, São Paulo, Brazil.

Moreira, F. J. O. & Arruda, J. R. F., 1997, Model identification using ERA and Q-Markov COVER: an experimental example, Proceedings of the VII Symposium on Dynamic Problems of Mechanics, DINAME 97, pp. 242-244.

Moreira, F. J. O. & Arruda, J. R. F., 1998, Vibration control using band-limited H_∞ controllers (in Portuguese), XII Brazilian Automatic Control Congress (XII CBA), pp. 917-922, Sept. 1998.

Moreira, F. J. O. & Arruda, J. R. F., 1999, Active vibration control with an H_∞ controller robust to residual and parametric uncertainties: an experimental implementation, Proceedings of the 6^{th} Pan American Congress of Applied Mechanics, Rio de Janeiro, Brazil, 04-08/jan/1999, Vol. 8, pp. 1147-1150.

Soares, A. M. S. & Góes, L. C. S., 1993, Digital control of a one link flexible manipulator, Proceedings of the DINAME-93, Florianópolis-SC, Brazil, March 1993.