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FREQUENCY RESPONSE MEASUREMENT OF VIBRATION ELECTROMAGNETIC MICRO EXCITERS BY MEANS OF MLS AND THE WIGNER DISTRIBUTION FUNCTION

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Abstract.
The experimental determination of properties or the active control of vibrational systems require the application of shaking forces to convenients points of the structure, what can be done by a vibration electromagnetic exciter. Important features of such a transducer can be determined by its impulse response. Although this function fully determines the transient behavior of the exciter, the most relevant information is not easily visualized by inspection. So, in the past, many transforms or measurements have been developed for evaluation of the frequency response of such devices. This paper explores the application of the Wigner distribution to the analysis of vibration exciters, based in their impulse response measured by maximum length sequences.

Keywords: MLS, Maximum Length Sequence, Wigner Distribution,
Time-frequency Analysis, Impulse esponse, Vibration Exciters

1. INTRODUCTION

To find the dynamic properties of a structure, the response to a vibrational force is of interest rather than the actual vibration level. To produce a defined vibration an electromagnetic exciter, also called a shaker, is used because it is versatile, its operating principle is simple, and its experimental assemblage is fairly easy. Consequently it can be built in a large variety of sizes and shapes, suiting many operating conditions. In principle the electromagnetic vibration exciter operates like a loudspeaker, where the movement of the cone is produced by the force produced in the moving coil, immersed in a magnetic field, when a current is applied to it. The force used to accelerate the moving element is proportional to the input current and to the magnetic flux density. Such a magnetic flux can be produced by magnets, usually of ferrite, but now the use of rare earth magnets is spreading because they produce a much stronger magnetic flux density per unit mass. Thus, a fairly strong force can be produced with very small electromagnetic exciters.

2. THE VIBRATION MINI EXCITER

An small electromagnetic vibration exciter was used to illustrate the capabilities of the Wigner distribution for the evaluation of a transducer performance. This exciter has a total mass of 0.260 kg only, and is 60 mm high. It uses rare earth magnets in its magnetic base, for producing a fairly high magnetic flux density B, which is uniformly distributed across an annular air gap, where a coil can move transversally to the magnetic flux. This coil is fixed to a structure, which has a built in piezoelectric accelerometer, to an external cover, and to an elastic suspension, constituting the moving head of the exciter. Figure 1 gives an idea of the exciter main components layout. The built in accelerometer is useful for monitoring the vibrations of the moving head. The elastic suspension allows for an easy axial displacement of the moving head while suppressing its transversal displacements, to an extent that the exciter can work in any position, fixed either by the magnetic base or by the moving head. A detailed account of the performance of such an electromagnetic exciter, but without the built in accelerometer, is found in Vasconcelos (1996).

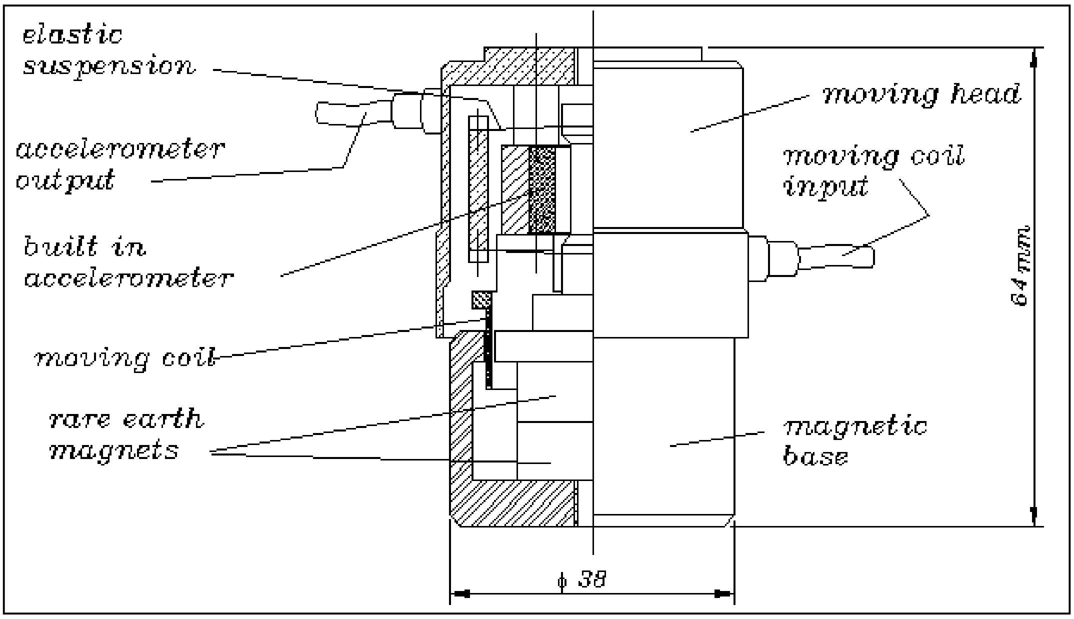

Figure 1 - Electromagnetic Exciter

The exciter elastic suspension was considered linear in this investigation, since the displacements involved were small, in comparison with the maximum peak displacement of 1.5 mm, which can be reached when working with very low frequencies. Besides this non-linearity of the elastic suspension, it is worth to mention that some non-linearity may occur also in the translation of the moving coil input current into force, since, even though the magnetic flux density decreases rapidly outside of the air gap (Vasconcelos, 1996), its distribution in the direction of the coil movement is asymmetrical during large excursions of the latter.

3. MLS - Binary Maximum Length Sequence for Measuring the Impulse Response

As mentioned by Borish and Angell (1983), in measuring the impulse response of a linear system, the most direct approach is to apply an impulsive excitation to the system and observe the response. There are two basic difficulties with this approach. The first is generating the impulsive excitation, and the second is obtaining adequate dynamic range. If one is dealing with an electronic system, generating an impulse is not a severe problem. But there are settings for which this is difficult, such as in measuring the impulse response of a concert hall. Although techniques exist for producing an impulsive acoustical excitation - electronic spark gaps, pistol shots, or exploding balloons are often used - it is difficult to assure that the energy is equally distributed over all frequencies of interest. Because the duration of the impulse, by definition, is very short it is difficult to deliver enough energy to the system to overcome the noise that is present. The amplitude of the impulse is limited by the range of linearity of the system and its duration by the range of frequencies of interest. This problem is exacerbated by a non-uniform distribution of energy in the impulsive excitation because the linearity limitation to the amplitude is imposed by the frequency range where most of the energy falls in ranges that are shortchanged it is not even possible to obtain the dynamic range that is theoretically possible.

A different approach is to excite the system with noise. Because the excitation is applied for a longer period of time, more energy is delivered to the system for a given amplitude of signal, circumventing the dynamic range problem. Further, it is easier to assure the uniformity of the energy distribution over frequency. The response is the convolution of the excitation with the impulse response. The impulse response can be extracted from the measurement by cross-correlating the noise input with the output ("Fig. 5A"). The implementation of this approach on a digital computer is based upon a proposal by Schroeder (1979), who observed that techniques developed in Hadamard spectroscopy by Nelson and Fredman (1970) and Harwit&Sloane (1979) could be applied to this problem. Schroeder originally described the use of pseudo-random noise for measuring the impulse response of a concert hall. In this work he performed the cross-correlation operation in the frequency domain. Because the length of the pseudo-random noise sequence was one less than a power of 2, it was necessary to interpolate a sample in order to exploit the fast Fourier transform. Cabot (1979) also reported using a cross-correlation technique based upon analog bucket-brigade delay lines. Basing the measurement technique upon a digital computer offers the greatest potential for maximizing the dynamic range. Also, having the final result in digital form makes additional manipulations more convenient.

3.1 Additional Improvement of Dynamic Range

When measuring the impulse response of a linear system using an impulsive excitation, a technique that is often used to improve the signal-to-noise ratio is to average together the response to a number of impulses (Berman et. al, 1977). The desired response adds coherently, but the noise adds incoherently, providing an improvement in the signal-to-noise ratio of $N^{1/2}$, where N is the number of periods being averaged together. This procedure has the disadvantage of consuming considerable time due to the fact that one must wait until the system has returned to its quiescent state before initiating the next measurement. If the impulse response has a duration of T seconds, a total of NT seconds will be required for the measurement. The cross-correlation method starts with a tremendous advantage in dynamic range. The advantage is exactly the same as that which would be obtained by averaging together n impulse responses, but the measurement only requires time T to perform. However, in extremely noisy environments it would be possible to combine the averaging technique with the cross-correlation method in order to obtain an additional improvement in dynamic range.

Summarizing, the impulse response of a linear system can be determined by exciting the system with noise and cross-correlating the input and output. As contrasted with the straightforward technique using an impulsive excitation, this approach is capable of providing vastly superior dynamic range.

In order to minimize the amount of computation required by the cross-correlation step, the system can be excited by a binary maximal length sequence ("Fig. 2"), and the cross-correlation performed using the fast Hadamard transform. By this means, only additions are required and the number of additions is approximately 2.5n $\log_2 n$, where n is the length of the sequence.

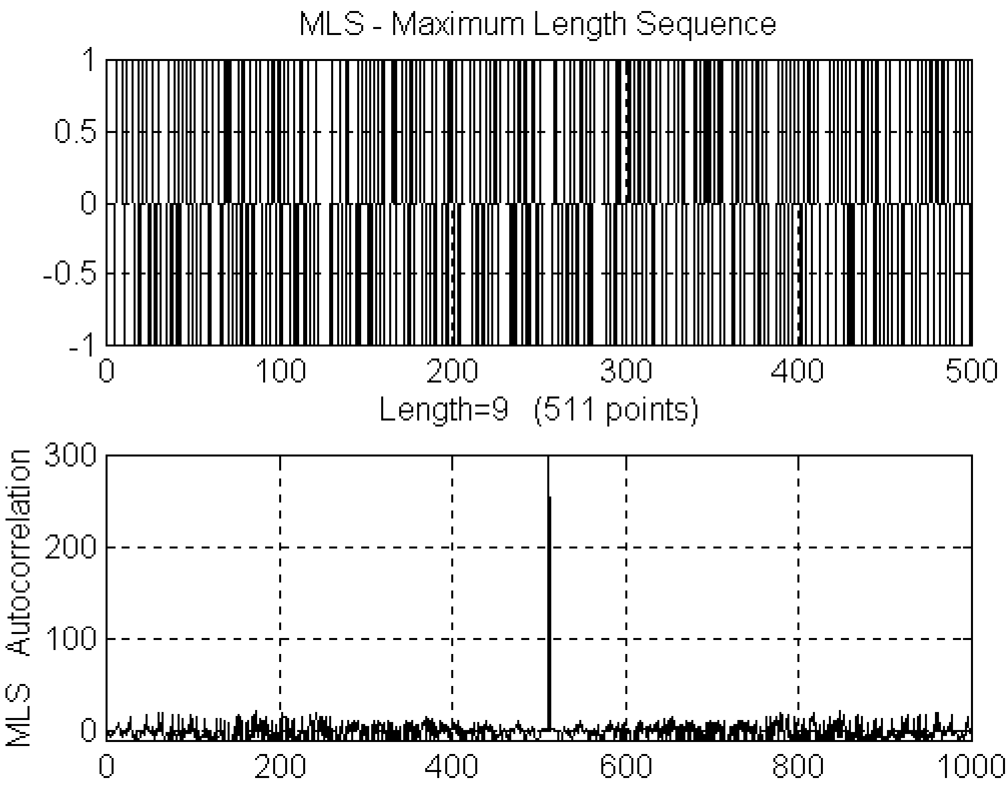

Figure 2 – MLS signal and its auto-correlation function

4. TRANSIENT SIGNAL ANALYSIS – TIME FREQUENCY ANALYSIS

In calculating the frequency response of linear systems, traditional spectral analysis techniques based on the Fourier Transform or Digital Filtering provide a good description of stationary and pseudo-stationary signals. Unfortunately, these techniques face some limitations when the signals to be analyzed are transient or highly non-stationary (i.e. signals with time varying spectral properties). For a time-varying signal, whether a deterministic transient or a non-stationary random signal, a single-domain representation does not reveal all the information content and may lead to erroneous results such as the loss of desired characteristic features or misinterpretation for the conclusion. To alleviate the shortcomings of the single-domain representation, some other efforts have been directed toward developing a joint time-frequency representation (**TFR**). The solution would be to deliver an instantaneous spectrum for each time index of the signal. The tools which attempt to do so are called time-frequency analysis techniques. Three of the most important and widely known time-frequency functions are the Short Time Fourier Transform (STFT), the Wavelet Transform (WT) and the Wigner Distribution (WD).

4.1 The Short Time Fourier Transform

The STFT is defined as the Fourier Transform (using FFT) of a windowed time signal, for various positions *b* of the window, which can also be stated in terms of an inner product (< >) between the signal and the window. So, s is the signal, g is the window, b is the time parameter, *f* is frequency and *t* is time. The inner product between two time functions *f(t)* and *h(t)* is defined as the time integrated (from minus infinity to plus infinity) product between the two time signals, where the second signal has been complex conjugated. Time functions that are real can be converted into complex functions by use of the Hilbert Transform.

Definition:

$$S_b = \int_{-\infty}^{+\infty} s(t) g^*(t-b) e^{-j2\pi f(t-b)} dt$$

$$= \langle s, g_{b,f} \rangle \qquad (1)$$

where $g_{b,f}(t) = g(t-b) e^{j2\pi f(t-b)}$

The idea of the STFT is to split a non stationary signal into fractions within which stationary assumptions apply and to carry out a Fourier Transform on each of these fractions. The signal s(t) is split by means of a window g where the index b represents the time location of this window (and therefore the time location of the corresponding spectrum).

The series of spectra, each related to a time index, form a time-frequency representation of the signal. The length (and the shape) of the window and also its translation steps, are fixed. Values for these choices have to be made before starting the analysis.

STFT provides constant absolute bandwidth analysis, which is often preferred with vibration signals in order to identify harmonic components. STFT offers constant resolution in the time as well as in the frequency domain irrespective of the actual frequency.

4.2 The Wavelet Transform

The Wavelet Transform (WT) is defined from a basic wavelet ψ which is an analysing function located in both time and frequency. From the basic wavelet, a set of analysing functions is found by means of scalings (parameter a) and translations (parameter b).

Definition:

$$S(a,b) = a^{-1/2} \int_{-\infty}^{+\infty} s(t) \psi^* \frac{(t-b)}{a} dt$$

$$= \langle s, \psi_{b,a} \rangle \qquad (2)$$

where $\psi_{b,a}(t) = a^{-1/2} \psi \left[(t-b)/a \right]$

The Wavelet is an alternative tool that deals with transient or non-stationary signals. The analysis is carried out by means of the special analyzing function ψ, called the basic wavelet. During the analysis this wavelet is translated in time (for selecting the part of the signal to be analyzed), then dilated/expanded or contracted/compressed using a scale parameter a (in order to focus on a given range or number of oscillations). When the wavelet is expanded, it focuses on the signal components which oscillate slowly i.e., low frequencies. When the wavelet is compressed, it shows the fast oscillations i.e., high frequencies. Due to this scaling process (compression-expansion of the wavelet), the WT leads to a time-scale decomposition. It can be said that both STFT and WT are local transforms using an analyzing (weighting)-function.

4.3 The Wigner Distribution (WD)

The Wigner-Ville Distribution is a global transform and is regarded as being the most fundamental of all time-frequency distributions. It also provides an energy distribution of the signal in both the time and frequency domain. The main characteristic of this transformation is that it does not place great restriction about the simultaneous resolution in time and in frequency. In other words, one can say that the WD is less limited by the uncertainty principle (Cohen, 1995) because it is a more general transform, not using an analyzing function.

Definition:

$$W\ (t,\omega) = \int_{-\infty}^{+\infty} s(t+\tau/2)\ s^*(t-\tau/2)e^{-j\omega\tau}\,d\tau \qquad (3)$$

As can be seen from its definition, the WD is a kind of combined Fourier Transform and auto-correlation calculation (i.e., auto-spectrum estimate as a function of time t or auto-correlation estimate as a function of frequency ω).

The properties of the Wigner Distribution (Cohen, 1995) are summarized here as follows:

(i) The WD is a real valued function:

$$W^*(t,\ \omega\ ) = \int_{-\infty}^{+\infty} e^{j\omega\tau} s^*(t+\frac{\tau}{2})\ s(t-\frac{\tau}{2})\ d\tau = W(t,\ \omega\ )$$

(ii) The integration of the WD with respect to the frequency yelds the instantaneous signal power and the time integral of the WD produces the signal's power density spectrum:

$$\int_{-\infty}^{+\infty} W(t,\omega)\ d\omega = 2\pi|s(t)|^2$$

$$\int_{-\infty}^{+\infty} W(t,\omega)\ dt = 2\pi|S(\omega)|^2$$

(iii) A time shift or frequency modulation in the signal will have the corresponding shift in the WD:

If s(t) $\rightarrow$ s(t+t_0) then W(t, ω) $\rightarrow$ W(t+t_0, ω),

if s(t) $\rightarrow$ $e^{j\omega_0 t}$s(t) then W(t, ω) $\rightarrow$ W(t, $\omega + \omega_0$).

(iv) The WD is symmetrical in time for a given signa:

If s(t) $\rightarrow$ s(-t) then W(t, ω) $\rightarrow$ W(-t, ω),

if s(t) $\rightarrow$ s^*(t) then W(t, ω) $\rightarrow$ W(t, -ω).

(v) The first-order moments of the WD with respect to ω and t give the instantaneous frequency ω(t) and group delay $\tau(\omega)$ of a given signal, respectively:

$$\omega(t) = \int_{-\infty}^{+\infty} \omega \, W(t,\omega) \, d\omega$$

$$\tau(\omega) = \int_{-\infty}^{+\infty} t \, W(t,\omega) \, dt$$

(vi) The integration of the square of the WD equals the square of the time integration of the signal's power. This is the counterpart of Parseval's relation for the WD, called Mayol's formula.

$$\int_{-\infty}^{+\infty} \left| W(t,\omega) \right|^2 dt d\omega = \left| \int_{-\infty}^{+\infty} s^2(t) dt \right|^2$$

In essence, a TFR – time-frequency representation – is a function that describes the distribution of the energy of the signal in time and frequency. Its graphical representation may be constructed using a perspective view of the surface described by the function (three-dimensional display) or by sectional planes parallel to the joint time-frequency domain (contour plot), where the amplitudes are represented by contours or associated to color maps.

5. APPLICATION OF THE WIGNER DISTRIBUTION TO THE EVALUATION OF A TRANSDUCER

A vibration exciter is an ideal transducer if its impulse response is a Dirac pulse. This means that its frequency response is constant over all the frequency axis and the vibration response is a delayed copy of the electrical signal. However, this definition is not very realistic, since a transducer with an infinitely flat frequency response cannot be physically realized. Therefore, a more practical definition states that an "ideal" transducer is one of which the time-frequency behavior resembles that of a smooth band-pass filter. The comparison with such an "ideal" band-pass filter gives a manageable criterion for the time-frequency behavior of the system by an easy visual inspection of the function. So, in the ideal situation, the Wigner Distribution of the filter and the transducer should be very similar.

6. NUMERICAL SIMULATIONS AND EXPERIMENTAL MEASUREMENTS

For the purposes of this work the following simulations were done:

a) Impulse response of a band-pass filter similar to the exciter, by digital filtering;
b) Wigner Distribution of the previous impulse response; “Fig. 4”.

For the comparison with the previous signals the following measurements were done:

A) Impulse response of the vibration exciter, by mechanical impact; “Fig. 3”.
B) Impulse response of the vibration exciter, by MLS; “Fig. 5A".
C) Wigner Distribution of the impulse mentioned in item B; “Fig. 5B”.

As stated in the text, one can observe from “Fig. 3” that, in fact, impulse responses measured by applying an impulsive excitation to the system face some practical limitations enhanced by traditional analysis tools based on Fourier methods. Fortunately, by means of techniques based on joint time-frequency representations it is possible, by inspection of the TFR function - (as shown in Figs. 4 and 5), to determine the frequency response of a linear system, as well other dynamical properties, by comparison with the Wigner Distribution of the related “ideal” band-pass filter.

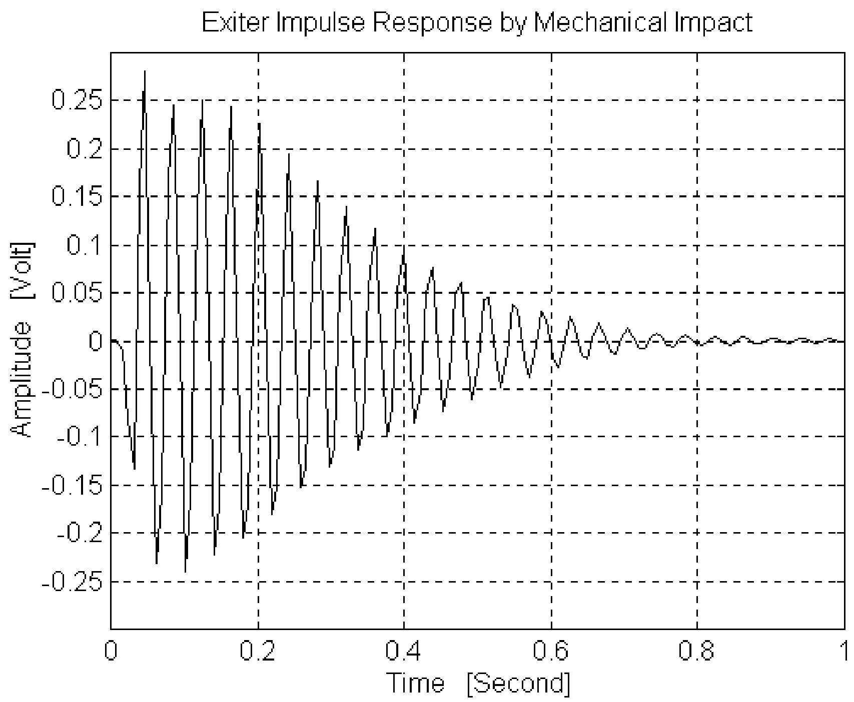

Figure 3 - Exciter Impulse Response by mechanical impact. The signal was captured by the built in accelerometer of the transducer.

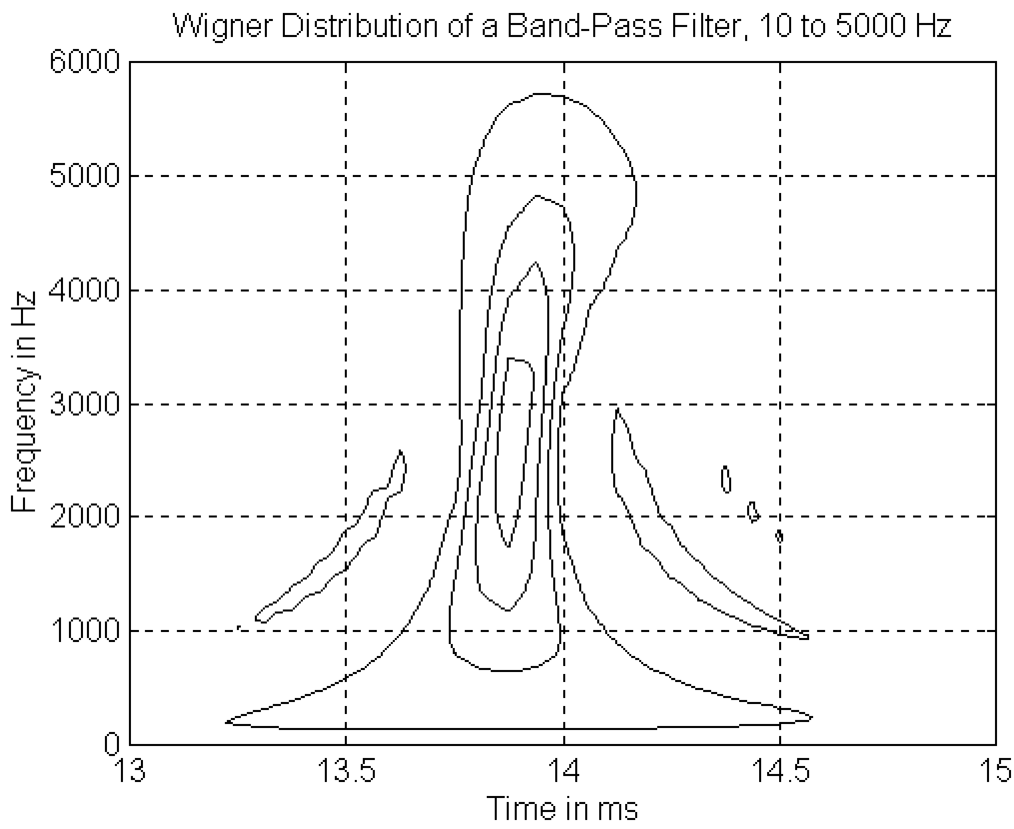

Figure 4 - WD of an ideal filter similar to the vibration exciter analysed.

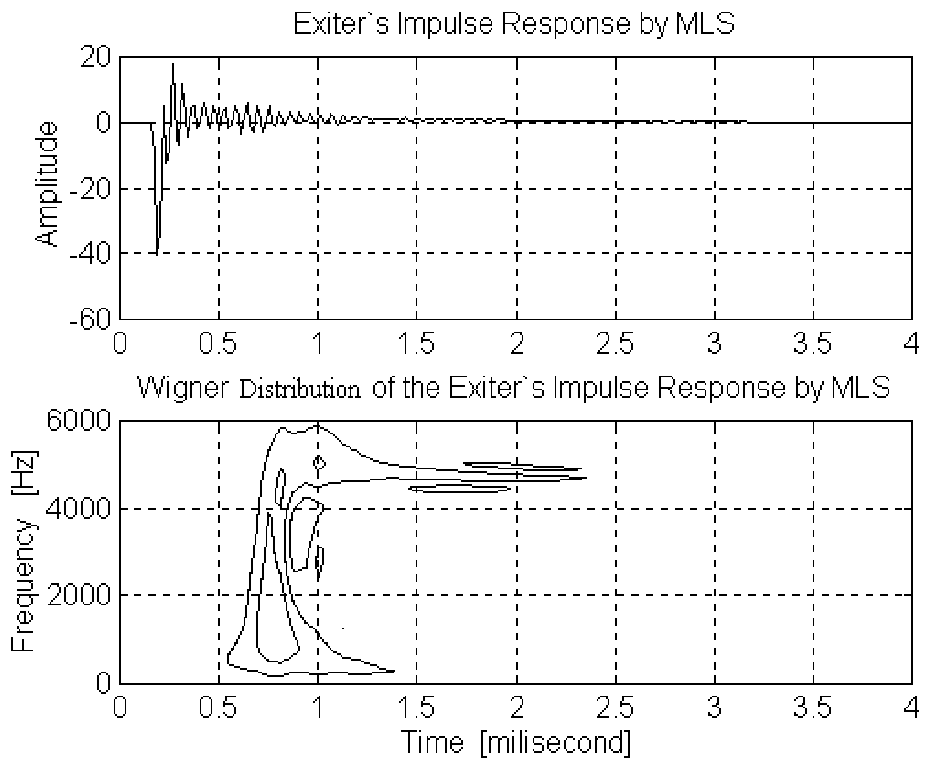

Figure 5 - A) Exciter’s IR by MLS; B) TFR of the exciter’s IR by MLS.

7. CONCLUSIONS

Although the research started in this paper is far from an exhaustive survey of the topics *Impulse Response Measurements* and *Time-frequency Analysis*, it can be seen from the initial results presented here that there is much to be gain from the application of both techniques to the field of transient or non-stationary signals, as well to the analysis of vibrating systems.

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REFERENCES

Berman, J.M. and Finchan, L. R., 1977, The application of Digital Techniques to the Measurement of Loudspeakers, J. Audio Eng. Soc., vol. **25**, pp. 298-309.

Borish, J., Angell, J. B., 1983, An efficient Algorithm for Measuring the Impulse Response Using Pseudorandom Noise, JAES **31**, 478-487.

Cabot, R. C., 1979, Acoustic Applications of Cross Correlation, J. Audio Eng. Soc., **27**, 1020.

Harwit, M., Sloane, N. J. A., 1979, Hadamard Transform Optics, Academic Press, N. Y.

Nelson, E. D., Fredman, M. L., 1970, Hadamard Spectroscopy, J. Opt. Soc. Am., **60**, 1664-1669.

Schroeder, M. R., 1979, Integrated-Impulse Method Measuring Sound Decay Without using Impulses, J. Acoust. Soc. Am., **66**, 497-500, Aug.

Vasconcelos, H. F.,1996, Vibrations Eletromagnetic Micro Exciters Using Rare Earth Magnets, Proceedings of the 14th International Workshop on Rare Earth Magnets and their Applications, **1**, 607-616, São Paulo, SP.

Cohen, L., 1995, *Time-Frequency Analysis*, Englewood Cliffs, NJ: Prentice-Hall.