COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

SIMULATION OF TURBULENT HEAT TRANSFER IN POROUS MEDIA USING A SPATIALLY PERIODIC CELL AND THE k-ε MODEL

Francisco Dias Rocamora Jr.
Instituto de Estudos Avançados-IEAv, Centro Técnico Aeroespacial-CTA
12228-900 – São José dos Campos, SP, Brasil
Marcelo J.S. de Lemos
Departamento de Energia - IEME
Instituto Tecnológico de Aeronáutica - ITA
12228-900 – São José dos Campos, SP, Brasil - E-mail: *mdelemos@tecsat.com.br*

Abstract. *A spatially periodic array is used to simulate the flow field inside the elementary control volume representing a porous medium. For simplicity, two-dimensional geometry is considered. Boundary treatment includes symmetry lines and spatially periodic conditions as well as prescribed temperatures or heat fluxes at the walls. This arrangement is aimed at the analysis of the medium representative cell in order to give insight to the macroscopic treatment of flow in porous media. A Generalized Coordinate system is used to generate the grid inside the computational domain. Solution of the flow equations is obtained with the SIMPLE method. The value of medium porosity is varied and the analysis covers a broad range of Reynolds number. The energy equation considers no source term and wall proximity is handled by means of the wall-function of Kader and Yaglom (1972). Temperature pattern and cell Nusselt number are reported.*

Key-words: *Porous Media, Heat Transfer, Generalized Coordinates, Turbulence Modeling*

1. INTRODUCTION

Due to its broad range of applications in science and industry, the study of flow through porous media has gained extensive attention lately. Engineering systems based on fluidized bed combustion, enhanced oil reservoir recovery, combustion in an inert porous matrix, underground spreading of chemical waste and chemical catalytic reactors are just a few examples of applications of this interdisciplinary field. In a broader sense, the study of porous media embraces fluid and thermal sciences, materials, chemical, geothermal, petroleum and combustion engineering.

In order to accurately describe turbulence phenomena occuring in a porous media, it is important to determine both flow and heat transfer characteristics. That can be accomplished either by volume integrating distributed microscopic flow properties or by time averaging equations for classical models for porous media. Example of the former approach is the work of Pedras and de Lemos (1998) whereas Anthohe and Lage (1997) followed the latter path. In a previous work Rocamora and de Lemos (1998) have dealt with flow characteristics at high

Reynolds numbers using an elementary cell from a periodic array in order to represent the flow in a porous medium. The objective was to obtain integral values of macroscopic flow properties. In this work we use the same approach (microscopic approach) in order to study the heat transfer characteristics in a porous matrix at high Reynolds numbers. Two types of boundary conditions at the wall boundaries are considered for the energy equation, *a*) prescribed temperature and *b*) prescribed heat flux. In addition, the temperature wall-functions of Kader and Yaglom (1972) for turbulent heat transfer are used in conjunction with the high Reynolds K-ε model. Temperature maps and cell Nusselt numbers are presented.

2. TRANSPORT AND CONSTITUTIVE EQUATIONS

The flow equations (continuity, momentum and the K-ε equations) have already been presented in Rocamora and de Lemos (1998), so they will be only stated in the next section. We will attain our attention mostly to the energy equation and its boundary conditions for turbulent flow.

2.1 Flow Transport and Constitutive equations

The vector form of the transport equations governing fluid flow can be written as:

Mass :

$$div\left(\tilde{n}\vec{v} \right) = 0 \tag{1}$$

Momentum :

$$div\left(\tilde{n}\vec{v}U_i - \vec{t}_i \right) = s_{u_i} \tag{2}$$

where $\vec{t}_i$ in Eq. (2) contains the total stress acting in the i-direction, and s_{u_i} represents all source terms, including the pressure gradient and gravitational forces. Expressions for $\vec{t}_i$ and s_{u_i} are readily obtained as,

$$\vec{t}_i = \hat{o}_{ij}.\vec{i}_j \tag{3}$$

$$s_{u_i} = -\frac{\partial P}{\partial y^i}\ ,\ P = p + \tilde{n}gh \tag{4}$$

The total stress in Eq. (3) comprises both laminar plus turbulent contributions and can be written as:

$$\tau_{ij} = -\frac{2}{3} k \delta_{ij} + (\mu_t + \mu) S_{ij} \tag{5}$$

where δ_{ij} is the delta Kronecker, $k = \overline{u_i' u_i'}/2$ is the turbulent kinetic energy per unit mass, μ and μ_t, are the molecular and turbulent viscosity, respectively, and

$$S_{ij} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{6}$$

Equation (5) already embodies the Eddy Diffusivity Model (EDM) and an expression for μ_t, calculated through the standard K-ε model of Jones and Launder (1972), reads :

$$\mu_t = C_\mu \rho \frac{k^2}{\varepsilon} \tag{7}$$

where C_μ is a constant. Since transport equations for standard k and ε are presented elsewhere, they are not repeated here.

2.2 The Energy equation

In general, the transport equation for a scalar quantity can be expressed as follows :

$$div\left(\rho \vec{v} \phi - \vec{q}_\phi \right) = s_\phi \tag{8}$$

where ϕ is a scalar quantity and $\vec{q}_\phi$ is its diffusive flux given, e.g., by the Fourier-type law:

$$\vec{q}_\phi = \Gamma_\phi \, grad \, \phi = \Gamma_\phi \left(\frac{\partial \phi}{\partial x} \vec{i} + \frac{\partial \phi}{\partial y} \vec{j} \right) \tag{9}$$

and s_ϕ represents the source or sink of ϕ. For the energy equation the scalar quantity ϕ is replaced by the temperature T and the Γ_ϕ coefficient in Eq. (9) already contains the turbulent thermal diffusivity.

The wall boundary conditions make use of the wall-functions of Kader & Yaglom (1986) for high Reynolds turbulent heat transfer expressed by :

$$\frac{(T_c - T_w) c_p \rho c_\mu^{1/4} k_c^{1/2}}{Q_w} = \frac{Pr_t}{\kappa} ln \, n_c^* + C_Q (Pr) \tag{10}$$

where C_Q is a function of the molecular Prandtl number Pr. A best-fitting to the available experimental data made by Kader and Yaglom (1972) yielded :

$$C_Q = 12.5 Pr^{2/3} + 2.12 \ln Pr - 5.3 \qquad if \quad Pr > 0.5$$

$$C_Q = 12.5 Pr^{2/3} + 2.12 \ln Pr - 1.5 \qquad if \quad Pr \le 0.5$$

where the non-dimensional distance n_c^*, which is used for switching between laminar and turbulent treatment for the first node adjacent to the wall, is given by :

$$n_c^* = \frac{\rho\, c_\mu^{1/4}\, \kappa_c^{1/4}\, n_c}{\mu}$$

3. PROBLEM DESCRIPTION

An elementary cell as shown in Fig. 1 is used to represent the porous matrix as an infinite periodic array. Also shown is the non-orthogonal (generalized coordinate) grid used to solve the flow equations inside the cell. The grid dimensions (NIxNJ) used for all the geometries was (43x44).

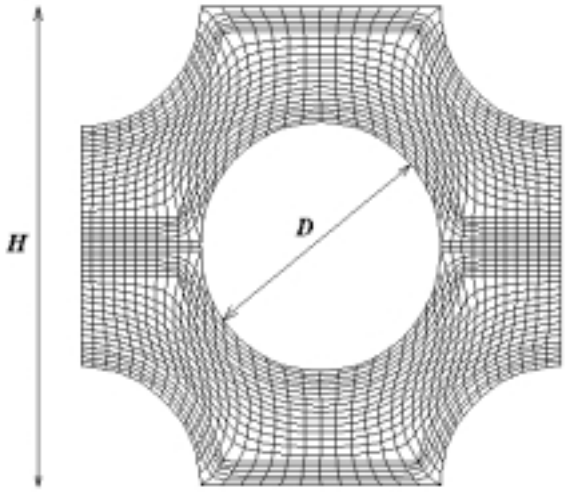

Figure 1- Model of R.E.V. Periodic cell and elliptically generated grid.

The flow equations are solved such that a periodic condition for the pressure gradient and velocity fields is achieved in the cell and the resulting temperature field is then analyzed. Here two situations are considered, *a*) prescribed wall temperature and *b*) prescribed wall heat flux.

The dimensions of the cell for the cases considered in this work were H=0,1m, D_{rod}=0,03m (ϕ=0,86), 0,05m (ϕ=0,61) and 0,06m (ϕ=0,43), were ϕ is the corresponding medium porosity.

4. RESULTS AND DISCUSSION

4.1 Preliminary results

Some preliminary cases were run using the wall-functions of Kader and Yaglom (1972) described above, applied to flow inside tubes. Some results are shown in Table 1.

As can be seen from Table 1, the Nusselt numbers obtained for the tube with uniform wall heat flux are in good agreement with the literature for both laminar and turbulent flows. For turbulent flows the Nusselt numbers are compared to the Gnielinski (1976) correlation. The deviation of the numerical value of the heat transferred to the fluid, $Q_{L,\text{num.}}$, from the

theoretical value, $Q_{L,\text{theor.}}$, obtained by integrating the heat flux over the tube wall area, should also be noted.

Table 1 - Nusselt numbers for developed flow in tubes with uniform heat flux.

	Laminar			Turbulent		
Case	1	2	3	4	5	6
grid	(200x20)	(200x20)	(200x20)	(100x10)	(100x10)	(100x10)
Re	360	720	1080	6480	12960	19440
q_w (w/m^2)	1,0x10^3	1,0x10^3	1,0x10^3	1,0x10^5	1,0x10^5	1,0x10^5
T_w (° C)	61,4	58,7	59,7	132,0	98,0	83,1
T_b (° C)	55,742	53,384	54,120	65,182	60,157	55,884
h (w/m^2 K)	176,74	188,11	179,21	1496,6	2642,5	3674,3
Nu	4,59	4,88	4,66	38,87	68,63	95,44
Nu-Gi				38,27	71,82	101,92
%ΔNu	5,18	11,82	6,78	1,57	-4,44	-6,36
$Q_{L,\text{num.}}$ (w)	12,57	13,24	12,30	591,69	597,70	581,68
%ΔQ_L	-0,08	5,25	-2,23	-5,83	-4,98	-7,42

$Q_{L\text{-theor..}}$ = 12,58 w (q_w = 1,0x10^3 w/m^2); 628,32 w (q_w = 1,0x10^5 w/m^2).
Tube diameter (D) = 0,02 m; Tube length (L) = 0,10 m.

4.2 Periodic cell results

With the model of the elementary cell shown in Fig. 1 several runs were made varying the Reynolds number and the medium porosity. As an example of the temperature fields obtained for a periodic cell with uniform surface temperature on the rods, Figs. 2, 3 and 4 are presented.

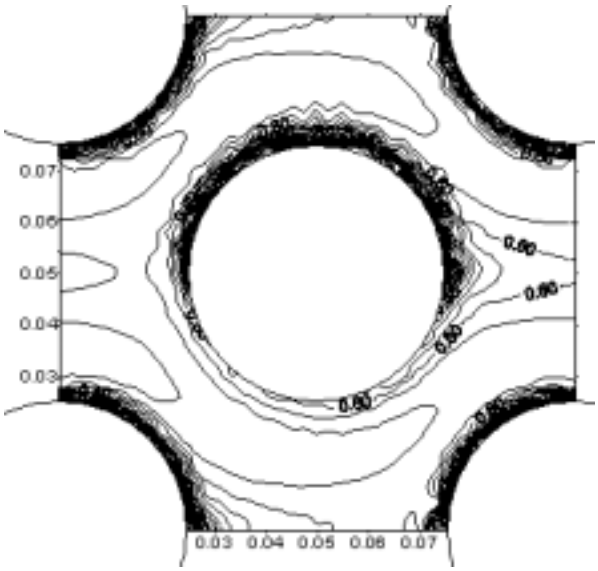

Figure 2 - Temperature Field for Case P3

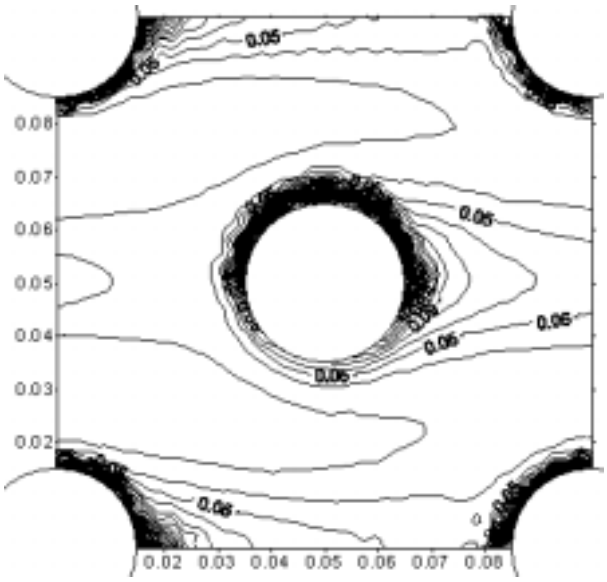

Figure 3 - Temperature Field for Case P4

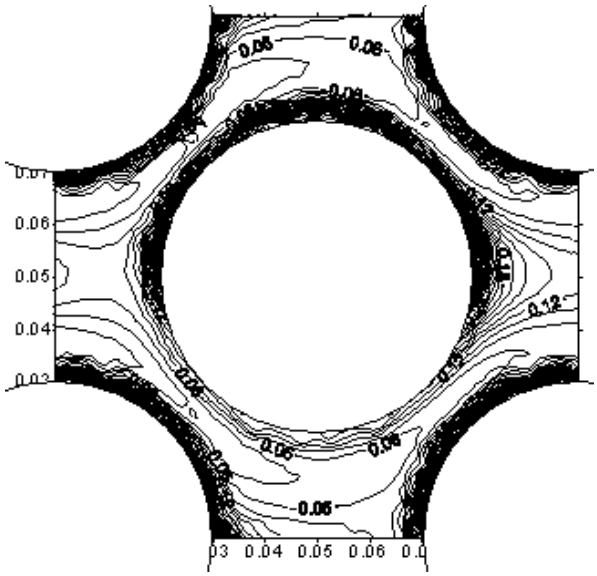

Figure 4 - Temperature Field for Case P5

From these figures one can notice that the pattern of the temperature fields established in the cell as periodicity is achieved for the velocity and pressure gradient are very much alike.

Although the interpolation technique used to obtain the isotherms seems not to work properly very close to the wall were the temperature gradients are high, a symmetry with respect to the horizontal center line of the cell is observed to occur as expected. Other interpolation techniques were tried but the results were even worse. Perhaps a better grid refinement could overcome this problem, but we have not tried it.

The results obtained are shown and commented upon as follows. Table 2 shows the values of the Reynolds numbers for maximum average cell velocity as required by the Zhukauskas (1972) correlation for the calculation of the Nusselt number for a rod bundle. For all the cases analyzed a uniform wall temperature of T_w=100 ^{o}C was considered.

Table 2 – Variables used for the calculation of the cell Nusselt numbers

Case	Porosity (ϕ)	$Re_{D,max}$	Q_{TOTAL} (w)	$T_{bulk,cell}$ (^{o}C)
P3	0,61	$1,51x10^{5}$	4299	58,32
P4	0,86	$7,50x10^{4}$	2599	58,61
P5	0,43	$5,60x10^{5}$	5707	59,51
P6	0,61	$7,55x10^{4}$	2332	59,64
P7	0,61	$3,02x10^{5}$	8082	59,22

The Nusselt number for the cell was obtained as follows:

a) Calculation of the average heat transfer coefficient and Nusselt number for the cell based on the cell bulk temperature.

The average cell heat transfer coefficient is calculated as :

$$\bar{h}_{cell} = \frac{Q_{TOTAL}}{\left(T_w - T_{bulk,cell}\right)} \tag{11}$$

The bulk temperature for the cell is obtained integrating over the flow volume of the cell using the velocity as a weighting function, i.e.,

$$T_{bulk,cell} = \frac{\int_{V_{cell}} T\left|\vec{V}\right| dV}{\int_{V_{cell}} \left|\vec{V}\right| dV} \tag{12}$$

The Nusselt number is then obtained as :

$$\overline{Nu}_{calc.} = \frac{\bar{h}_{cell}\, D_{rod}}{k_f} \tag{13}$$

where D_{rod} is the rod diameter and k_f is the conductivity of the fluid.

b) Calculation of the cell Nusselt number using the Zhukauskas (1972) correlation for rod bundles.

The Zhukauskas correlation is given by :

$$\frac{\bar{h}\, D_{rod}}{k_f} = C\, Re_{D,max}^{m}\, Pr^{0,36} \left(\frac{Pr_{\infty}}{Pr_s}\right)^{1/4} \tag{14}$$

where

$$Re_{D,max} = \frac{\overline{U}_{max} D_{rod}}{\nu} \tag{15}$$

and the constants C and m are obtained as functions of the rod arrangement (aligned or staggered) and the maximum Reynolds number (i. e., using the maximum average velocity, U_{max}, in any section between two rods).

The results obtained by both methods are shown in Table 3 bellow. Also shown are the constants C and m for each case.

Table 3 – Nusselt numbers for elementary cells.

Case	$\overline{Nu}_{cell,calc.}$	C	m	$\overline{Nu}_{cell,theor.}$ $(N \geq 20)$	% Deviation	$\overline{Nu}_{cell,theor.}$ $(N = 3)$	% Deviation
P3	403	0,40	0,60	450	-10,44	373	8,04
P4	147	0,40	0,60	296	-50,33	246	-40,24
P5	771	0,022	0,84	989	-22,04	820	-5,98
P6	226	0,40	0,60	297	-23,91	246	-8,13
P7	774	0,022	0,84	776	-0,26	644	20,19

The Nusselt numbers obtained using Eq. (14) are valid for a number of rows greater or equal to 20. To correct for a lower number of rows a correction factor given by Zhukauskas (1972) was used and the Nusselt numbers obtained are also shown in Table 3. The value of the correction factor was CR=0,84.

Concerning the Nusselt numbers obtained for the cells and shown in Table 3, there are many doubtful points that deserve some considerations. First of all, the definition of the cell average heat transfer coefficient based on the cell bulk temperature, Eq. (12), might not be the most appropriate. Secondly, the Zhukauskas (1972) correlation is appropriate for a rod bundle with many rows, typically N>10, and its validity for a periodic cell, even considering the cell as a bundle with a small number of rows, i. e., N=3, is questionable. Nevertheless, the heat transfer analysis for the rod bundle was the closest situation to a periodic cell we could gather data to compare with.

From Table 3 and Figs. 2, 3 and 4 we can observe that the largest deviation of the numerical value obtained for the Nusselt number from the ones obtained through the Zhukauskas (1972) correlation occurred for the case P4, whose porosity is the lowest. This suggests that, for this case, each rod behaves more like an isolated rod than as part of a rod bundle, therefore suffering less influence of its neighbors and making the correlation for the rod bundle worse.

5. CONCLUSION

In this work it a numerical analysis of heat transfer in porous media using a spatially periodic cell to represent the medium was presented. Nusselt numbers as well as temperature fields were obtained for the cell as periodicity is achieved for the velocity and pressure gradient. The results were compared to Nusselt numbers obtained using a correlation for heat transfer in cross flow through rod bundles given by Zhukauskas (1972) and, despite the dissimilarities, agreed reasonably well for the low porosity cases.

6. REFERENCES

Antohe, B. V., Lage, J. L., 1997 A general two-equation macroscopic turbulence model for incompressible flow in porous media, Int. J. Heat Mass Transfer, vol. 40, pp. 3013-3024.

Bear, J., 1972, Dynamics of Fluids in Porous Media, American Elsevier Pub. Co., New York.

Gnielinski, V., 1976, New Equations for Heat and Mass Transfer in Turbulent Pipe and Channel Flow, Int. Chem. Eng., vol.16, pp. 359-368.

Jones, W. P., Launder, B. E., 1972, The Prediction of Laminarization with Two-Equation Model of Turbulence, Int. J. Heat Mass Transfer, Vol. 15, pp. 301 – 314.

Kader, B. A., Yaglom, A. M., 1972, Heat and Mass Transfer Laws for Fully Turbulent Wall Flows, Int. J. Heat Mass Transfer, vol. 15, pp. 2329-2351.

Rocamora, F. D. Jr., de Lemos, M. J. S., 1998, Numerical Solution of Turbulent Flow in Porous Media using a Spatially Periodic Array and the k-ε Model, ENCIT-98 - Proc. of 7th Braz. Cong. Eng. Th. Sci., vol. 2, pp. 1265-1271.

Pedras, M. H. J., de Lemos, M. J. S., 1998, Resultados da Modelagem da Turbulência Macroscópica em Meios Porosos, ENCIT98 - Proc. of 7th Braz. Cong. Eng. Th. Sci., vol. 2, pp. 1272-1277.

Zhukauskas, A.,1972, Heat Transfer from Tubes in Cross Flow, Adv. Heat Transfer, vol. 8 , pp. 93-160.