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A SURVEY ON THE RESPONSE OF SUBSEA UMBILICAL CABLES UNDER AXISYMMETRIC LOADS

Anderson B. Custódio, Murilo A. Vaz, Segen F. Estefen
Submarine Technology Laboratory, PEnO/COPPE
Federal University of Rio de Janeiro, UFRJ
Centro de Tecnologia – Bloco C – Sala 203 – Rio de Janeiro, RJ, Brazil

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Abstract. Several authors have addressed the response of subsea cables and flexible pipes under axisymmetric loads. Their methods are used extensively for cross-section design, but only a few examine certain particularities that the most typical formulations incorporate. One of the main difficulties in the analyses is the accurate definition of the cable's geometrical and material properties since the manufacturing processes, mainly polymer extrusion and wire cold conforming introduce systematic and random deviations from the nominal properties. Another issue refers to the ability to treat moderate strains. This paper introduces a model for local axisymmetric analysis of umbilical cables and their response to the variation of important parameters, providing useful information for the cross-section preliminary design. The model derives from Love's theory for thin rods and formulation for thick cylinders, which respectively depict the cross-wound helical wire armours and homogeneous plastic layers. A typical umbilical cross-section employed offshore Brazil is analysed and a parametric study is carried out for surveying the influence of the umbilical's main parameters on stiffness.

Keywords: Umbilicals, Cables, Flexible pipes, Theory of rods.

1. INTRODUCTION

Umbilical cables provide electro-hydraulic signal, power transmission and chemical injection to subsea equipment. The operation of these lines is of vital importance for subsea production systems; the reliability of umbilicals should be kept at the highest level to ensure safe and continuous service. Offshore Brazil, Petrobras has already installed 1100 km of umbilicals and forecasts that 920 km will be additionally laid by the year 2004.

A typical cable cross-section features a number of plastic layers, helical wound armours, electric cables, hydraulic hoses and other elements. The functional elements are protected at the central core from external actions to avoid damage. The designer should provide adequate stiffness and strength to satisfy the cable specified in-service requirements.

Several formulations have been proposed to assess the axisymmetric response, mostly dedicated to flexible pipes, structural and power cables - see Knapp (1979), Oliveira et al (1985), Féret and Bournazel (1987), Berge et al (1992), Feld (1992) and Witz and Tan (1992) -, but it's reasonable that a proper model for umbilical cables must acknowledge some particularities of these structures:

(a) the diameter-to-thickness ratio of the plastic layers is smaller than 15;
(b) the core radial stiffness is small and depends on the response of internal cables and hoses;
(c) there are uncertainties from the geometrical and material properties.

2. MODEL DESCRIPTION

Most of the existing models for assessment of umbilical cable axisymmetric response yield reasonable results in terms of axial stiffness as long as the loading and displacement remain within certain linear limits. Most recent models employ improved solution algorithms from Féret and Bournazel's (1987) method, but few consider material and geometrical nonlinearities.

The armour tendons may be treated as naturally curved rods using relations proposed by Clebsch-Kirchoff (1862), that assumes that no pressure, force or terminal couple is necessary to keep the tendons at place. Some vector handling is necessary for correlating the co-ordinate systems in the rod and umbilical. In order to achieve correct results, initial and final helix angles and radii shall be discerned, as well as the possible lateral contact within the armours shall be treated.

The plastic layers are modelled as thick cylinders under pressures, torque and tension. Since the polymer's modulus of elasticity is small, these layers directly withstand only a fraction of load, but they play an important role in equilibrium in the radial direction.

The central core is described as a set of concentric layers whose stiffness is empirically adjusted as function of the element arrangement and material properties.

Basically the method solves numerically nonlinear equations and verifies if changes in the constitutive laws are required. The number of equations depends on the complexity of the model; it may achieve seven times the number of layers. In the current study, the solution of the axisymmetric problem is obtained by finding the roots of seven groups of equations related to armours, plastic layers, carcasses, cable layers, hoses, materials' secant moduli and relations concerning geometrical continuity, edge and boundary conditions.

When the algorithm computes negative pressure (separation), the apropos geometric continuity equation is substituted by an equation stating that pressure at interface is zero. The algorithm also verifies the lateral contact between tendons, according to Costello and Phillips' (1976) formulation.

The variation of external and internal radii for homogeneous layers as a function of axial deformation and pressures at interfaces is given by:

$$R'_{ext} - R_{ext}\left[1 + \frac{2\,P_{int}\,R_{int}^2}{E\left(R_{ext}^2 - R_{int}^2\right)} - \frac{P_{ext}}{E}\left(\frac{R_{ext}^2 + R_{int}^2}{R_{ext}^2 - R_{int}^2} - \nu\right) - \nu\,\varepsilon_c\right] = 0 \qquad \text{(1a)}$$

$$R'_{int} - R_{int}\left[1 + \frac{P_{int}}{E}\left(\frac{R_{ext}^2 + R_{int}^2}{R_{ext}^2 - R_{int}^2} + \nu\right) - \frac{2\,P_{ext}R_{ext}^2}{E\left(R_{ext}^2 - R_{int}^2\right)} - \nu\,\varepsilon_c\right] = 0 \qquad \text{(1b)}$$

In the group of equations of armours, each one requires three cinematic equations, an equation of lateral loads (that exists only when the tendons contact) and an equilibrium equation in the radial direction. The three cinematic equations depict the relationship between armour's local distension and global deformations, the variation of helix angle and change of tendon height due to stress. The equilibrium in the radial direction is obtained by:

$$\frac{2\pi}{n}\left(p_{ext}R'_{ext} - p_{int}R'_{int}\right) + n_i \left[\begin{array}{l} G_i J_i \left(\dfrac{\sin\alpha'_i \cos\alpha'_i}{R'_i} - \dfrac{\sin\alpha_i \cos\alpha_i}{R_i} \right) \dfrac{\sin\alpha'_i}{R'_i} \\ -E_i I_{y,i} \left(\dfrac{\sin^2\alpha'_i}{R'_i} - \dfrac{\sin^2\alpha_i}{R_i} \right) \dfrac{\cos\alpha'_i}{R'_i} - \varepsilon_{a,i} E_i S_i \end{array} \right] \frac{\sin^2\alpha'_i}{R'_i} - n_i F_{lat} \sin\frac{\pi}{n_i} = 0 \tag{2}$$

where: $F_{lat} = \dfrac{\pi^2 E_i \left(R'_i - R_{cl,i}\right)}{6n_i \cos\alpha'_i \left(1 - v_i^2\right)}$, if lateral contact occurs; zero elsewhere (3)

The thickness reduction and pressure decrease in carcasses, hose and cable layers are computed using empirical formulae that are not fully validated yet. In case of hose layers, the most relevant parameters are the internal fluid pressure, the diameter, the hose wall's thickness and the average modulus of elasticity of the polymer. The response of cables depends mainly on the diameter and modulus of elasticity of both core and insulation.

In order to treat the nonlinear behaviour of the material, there are equations that correlate the material' secant modulus to the von-Mises stress. In reality these equations are best-fit curves that interpolate points furnished by any data source. Suitable equations that calculate the von-Mises stress in each layer are used.

Finally, from the global equilibrium of forces and moments, it is given that:

$$T - \sum_{i=1}^{na} n_i \varepsilon_{a,i} S_i \cos\alpha'_i - \varepsilon_c \sum_{j=1}^{nc} E_j A_j = 0 \tag{4}$$

$$\sum_{i=1}^{na} n_i \left[\begin{array}{l} G_i J_i \left(\dfrac{\sin\alpha'_i \cos\alpha'_i}{R'_i} - \dfrac{\sin\alpha_i \cos\alpha_i}{R_i} \right) \left(\cos\alpha'_i - \sin^2\alpha'_i\right) \\ \qquad + E_i I_{y,i} \left(\dfrac{\sin^2\alpha'_i}{R'_i} - \dfrac{\sin^2\alpha_i}{R_i} \right) \sin\alpha'_i \cos\alpha'_i \end{array} \right] \cos\alpha'_i + \\ \sum_{i=1}^{na} n_i \left(\varepsilon_{a,i} S_i R'_i \sin\alpha'_i\right) - Q + \gamma \sum_{j=1}^{nc} G_j J_j = 0 \tag{5}$$

3. CASE STUDY

A case study of an umbilical cable with nine hydraulic functions (see figure 1) was performed to evaluate the algorithm and the mechanics of the composite structure. Mechanical tests in COPPE's rig supplied data for a numerical-experimental comparison (Vaz et al, 1998). Sample dissection and material tensile tests provided the as-built data summarised in table 1.

Tests showed an axial stiffness of 63 MN (cable free to twist), clockwise and anticlockwise torsional stiffness respectively 28.1 and 12.8 kNm^2 (ends free to elongate). Other relevant parameters such as radial contraction and twist were also recorded.

Table 1. Umbilical characteristics

External diameter:	94±2 mm
Outer sheath thickness:	5±1 mm
Outer sheath material:	HDPE
Young’s modulus of HDPE:	750±150 MPa
Tendon diameter:	4.1 mm
Young’s modulus of steel:	207 GPa
Yield strength of steel:	> 450 MPa
Poisson’s ratio of HDPE:	0.40
Ext. diameter of ext. armour:	84±1 mm
Tendons in external armour:	56 tendons
Helix angle in ext. armour:	20±3^o LH
Ext. diameter of int. armour:	76±1 mm
Tendons in internal armour:	50 tendons
Helix angle in int. armour:	20±3^o RH
Inner sheath external diameter:	68±1 mm
Inner sheath thickness:	8±1 mm
Inner sheath material:	HDPE
Central and non-central hoses:	1×3/8”, 8×3/8”
Internal pressure at hoses:	0 MPa
Core filler material:	Polyurethane
Young’s mod. of polyurethane	200 MPa

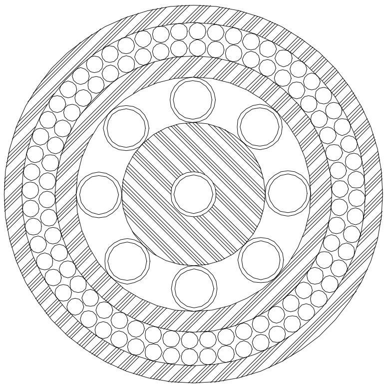

Figure 1 – Umbilical cross-section

3.1 Mechanism of response

Under pure tension the cable elongates, armours contract radially, twist and bend moderately. The armours apply pressure on the inner layers. As the central core and the inner sheath are relatively flexible, there will always be some radial contraction. Due to Poisson’s effect, the outer sheath also suffers radial contraction as it elongates. However, its internal radius reduction is smaller than the inside layer contraction and a gap at the interface between outer sheath and outer armour is formed. If tension is applied and cable twist is prevented, a terminal couple appears. As the cable is usually balanced, torque is not usually excessively large.

If only torsion is applied to the cable termination, there will be twist and elongation but the armours will respond differently. If anticlockwise torque is applied to the anticlockwise wound armour, its radius tends to reduce (see figure 4). Hence, for the cable studied in this paper an anticlockwise torque opens gaps between armours. The inner and outer sheaths physically limit the radial displacement for armour layers. Tension or compression may cause gap closure, because it produces contraction and expansion in both armours against the physical barrier of the inner and outer sheaths, respectively. The behaviour of the cable to axisymmetric loading will be evidenced by the comprehensive results presented next.

3.2 Results

The mechanical behaviour of the umbilical was analysed using an algorithm programmed in Fortran; this algorithm was validated using data provided by Witz (1996). Compressive loads were considered in order to know the mechanics of the composite structure under such loading, although loss of tendon stability and birdcaging is not modelled correctly yet. It is difficult to simulate experimentally the compression keeping the umbilicals straight.

Figure 2 presents the cable tension versus axial strain for end conditions free to twist and fixed. The mean axial stiffness is 73 MN, which reasonably correlates with the experiments. The cable is slightly stiffer in tension and for a fixed end boundary condition.

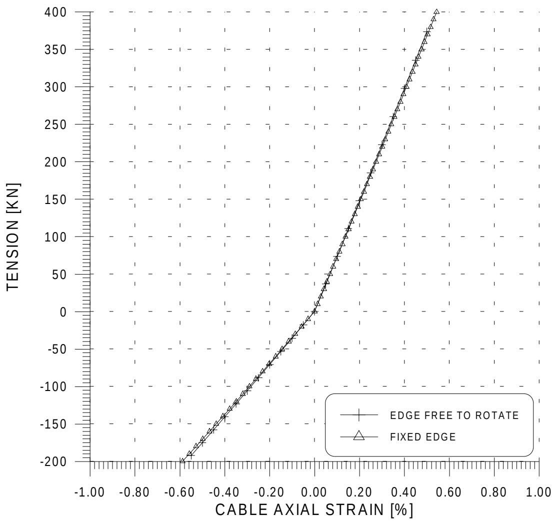

Figure 2 – Axial load versus axial strain

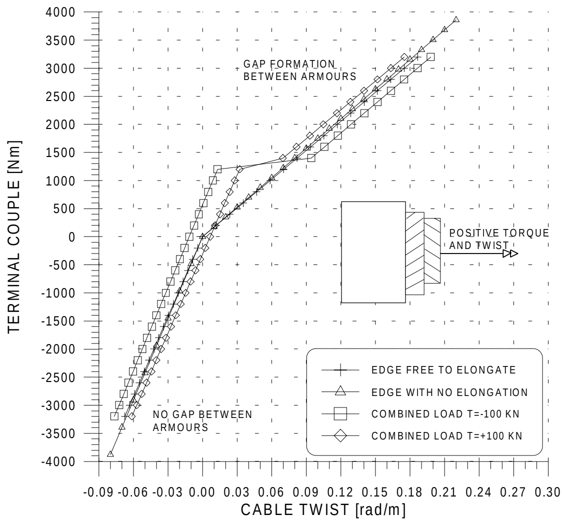

Figure 3- Torque versus twist

The torque-twist response is presented in figure 3. The difference in clockwise and anticlockwise torsional stiffness is due to gap formation in the composite structure (figure 4). Tension and compression prevent gap formation between armours, as torsional stiffness does not reduce unless torque achieves a value large enough to open a gap. Under a combined load of 30 kN (not shown in figure 3), the estimate of clockwise and anticlockwise torsional stiffness are respectively 47.6 and 18.5 kNm^2 whereas the measured stiffness are respectively 40.2 and 22.9 kNm^2. Figures 5 and 6 present charts of axial deformation and twist as a function of tension and torque. These plots also inform the tension-torque lines for zero deformation (torsion test with end prevented from moving) and zero twist (tension test with end prevented from rotating). The algorithm maps the response of the umbilical cable within ranges of loads and strains.

An important parameter for understanding the structure is the pressure at each interface, which is presented in the figures 7, 8 and 9, respectively for the outer sheath-armour, armour-armour and armour-inner sheath interfaces. Gap formation regions clearly exist. The gap between armours is plotted in figure 10. There are clearly three regions where gaps are formed; note that the algorithm perceives it.

The mean axial stress in the armour tendons is plotted in figures 11 and 12. The gap formation phenomenon does not influence these results. The hoop stresses at the sheaths is plotted in the figures 13 and 14. Since the hydraulic hoses are not pressurised, the central core (not including the sheath) is very complacent. Hence, the sheath plays an important structural role as it experiences high pressure at the outer face and small pressure at the inner face.

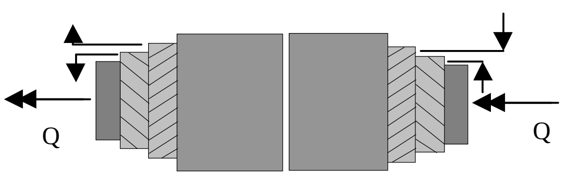

Figure 4 – Response of armours to torque and gap formation

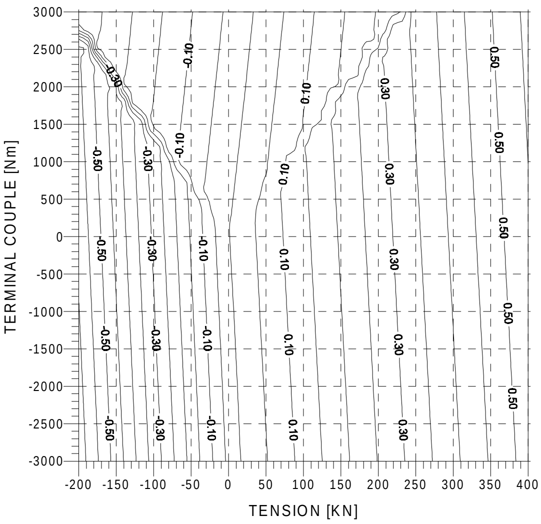

Figure 5 – Axial deformation (%)

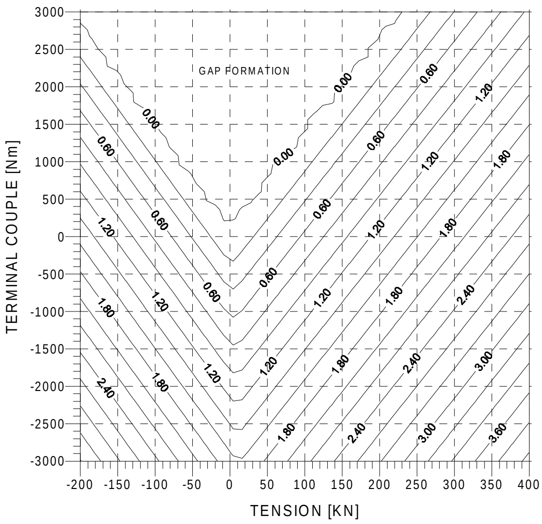

Fig. 8 – Pressure (MPa) between armours

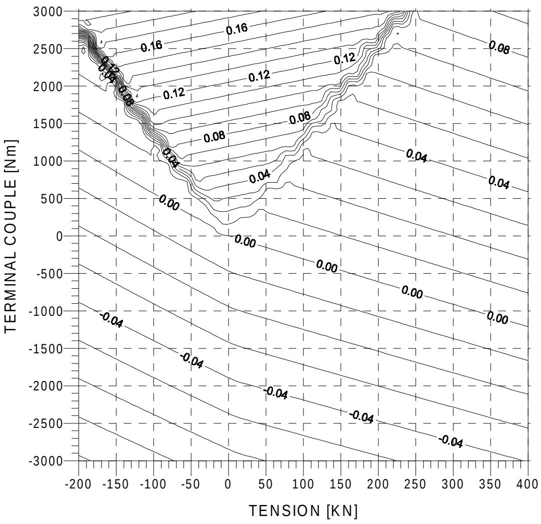

Figure 6 – Twist (rad/m)

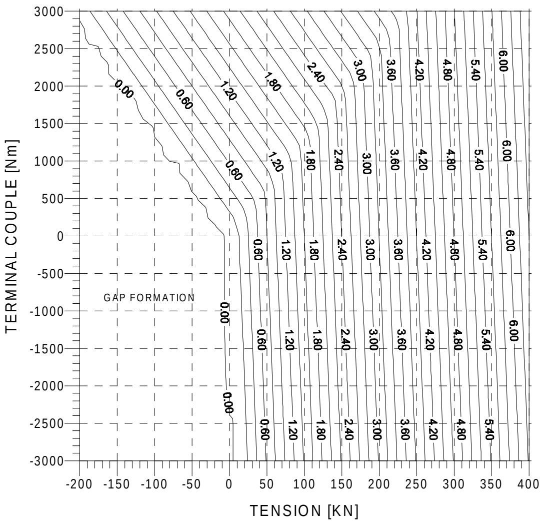

Figure 9 – Pressure (MPa) at interface armour- inner sheath

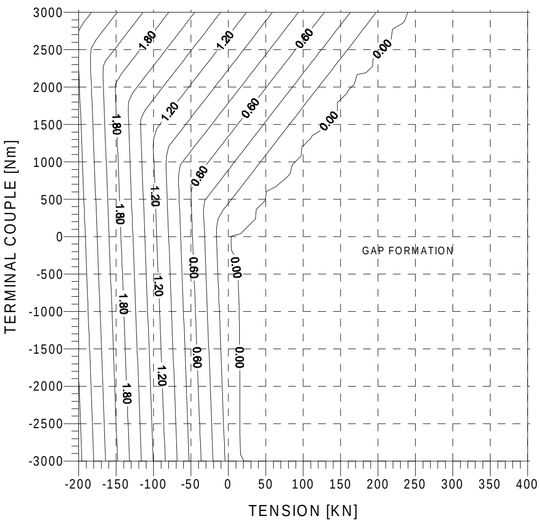

Figure 7 – Pressure (MPa) between outer sheath and outer armour

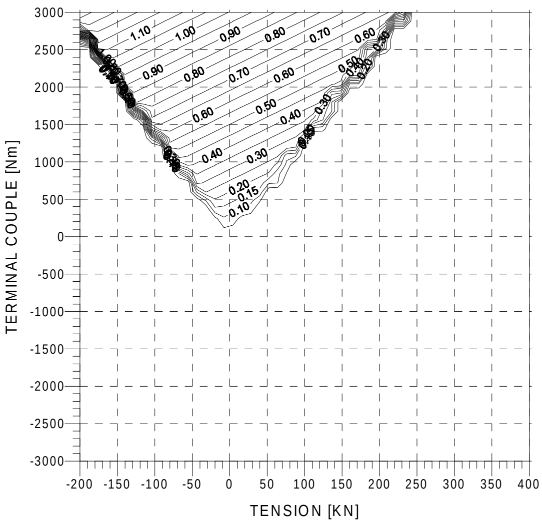

Fig. 10 – Gap (mm) between armours

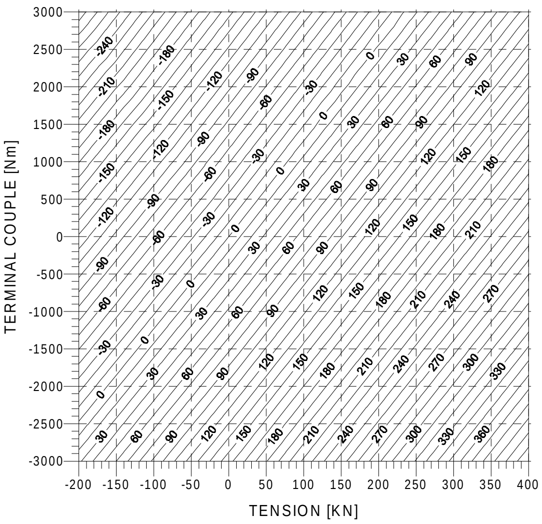

Figure 11 –Mean axial stress (MPa) in outer armour

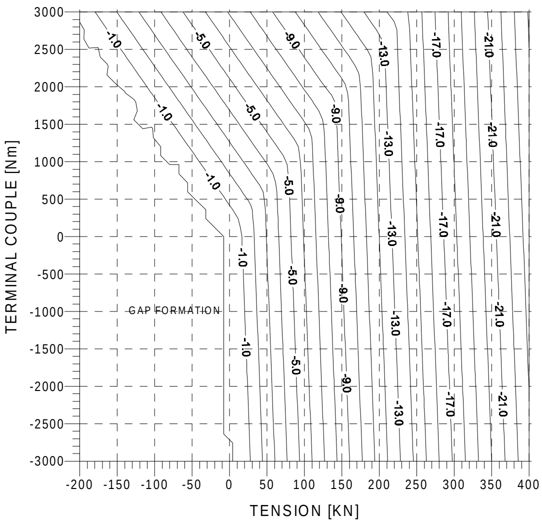

Figure 13– Hoop stress (MPa) in the inner sheath

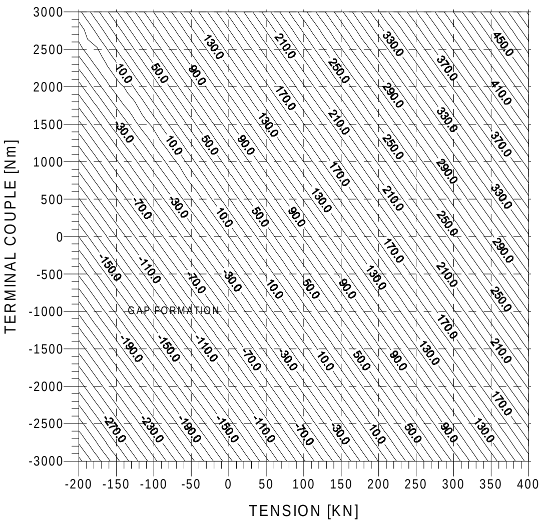

Figure 12– Mean axial stress (MPa) in the inner armour

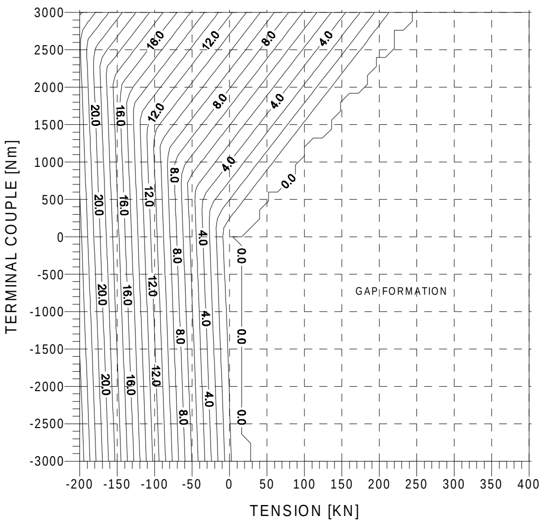

Figure 14– Hoop stress (MPa) in the outter sheath

4. SENSITIVITY STUDY

A good analysis strongly depends on the input data, but relevant parameters obtained from the sample dissection presented significant deviations and the results are influenced by them. Common sources of deviations are poor quality control during fabrication processes, sheath thickness, spacing between layers, inadequate tendon conforming, modulus of elasticity of polymers, inadequate handling. Because of them, a survey on effects of parameter variation was carried out. The sensitivity study requires the algorithm to run iterations using different parameter values. It maps changes in stiffness, stresses, gap formation, etc. Two parameters were chosen, based on their impact in the response and large uncertainty: the armour helix angles and the polymer modulus of elasticity. Such a survey supplies relevant data for the cross-section design and optimisation.

Figures 14 to 16 present the umbilical cable stiffness as a function of the armour helix angles. The axial stiffness (figure 14) is plotted for 100 kN pure tension. It is expected that a larger pitch length increase slightly the axial stiffness; in the extreme case where the pitch angle tends to zero the stiffness is maximum. An uncertainty of 1^o in armour laying angle may cause deviation of 10 MN in the axial stiffness. The stiffness in figures 15 and 16 are calculated at a mean torque of 200 Nm (respectively clockwise and anti-clockwise) with ends free to elongate.

When the cable is unbalanced, it tends to twist as tension is applied and this is represented in figure 17. The cable tends to be better balanced if the pitch lengths are close. This condition is achieved if the inner armour pitch angle is around 2^o smaller than the outer armour pitch angle. The cable is not balanced if the angles are symmetric, because the number of wires and the armour layer radii are different.

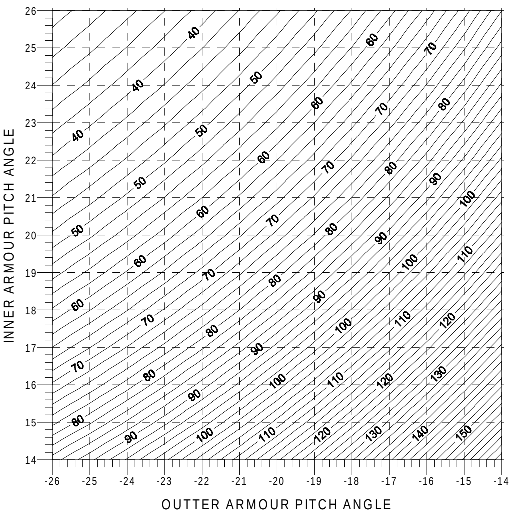

Figure 14– Umbilical's axial stiffness (MN)

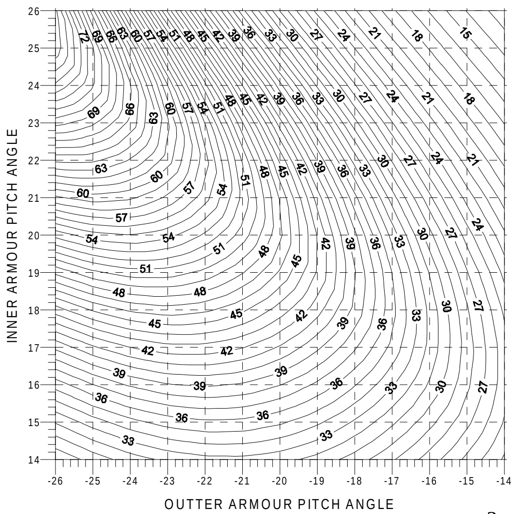

Figure 16- Clockwise torsional (kNm^2) stiffness

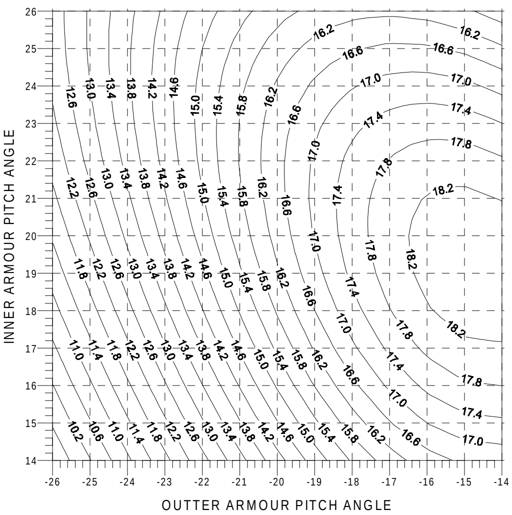

Figure 15 – Anti-clockwise torsional stiffness (kNm^2)

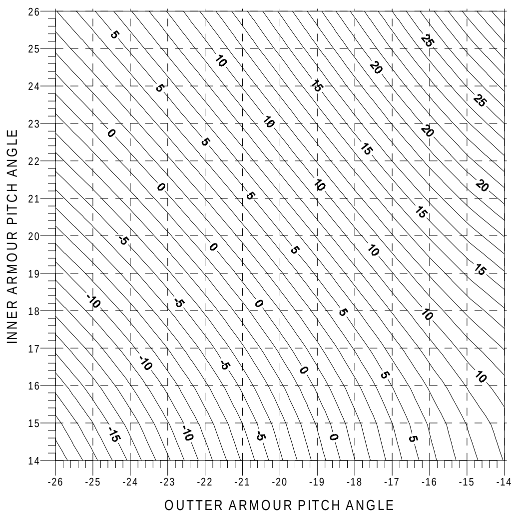

Figure 17– Umbilical cable twist per deformation (ϕ/ε, rad/m) for 100 kN tension

The study for the cable response to change in the polymer modulus of elasticity was studied. Material testing in the sheath samples produced scattered results: the moulded polymer standard-dimension samples frequently produced results 20% greater than actual extruded polymer samples. It is not easy to cut ASTM D638M-93 standard samples from sheaths because of the layer curvature. The umbilical stiffness as a function of the polymer modulus of elasticity is presented in fig. 18 and 19. The material properties of both sheaths were assumed identical. The effect of the polymer modulus of elasticity is large on the axial stiffness, medium on the anticlockwise and small on the clockwise torsional stiffness.

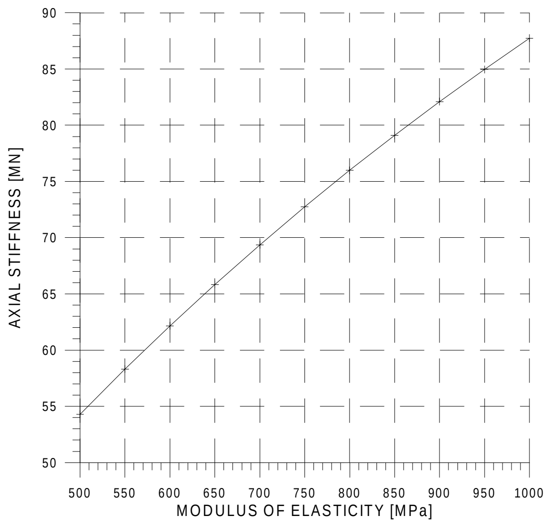

Figure 18– Axial stiffness as a function of the polymer modulus of elasticity

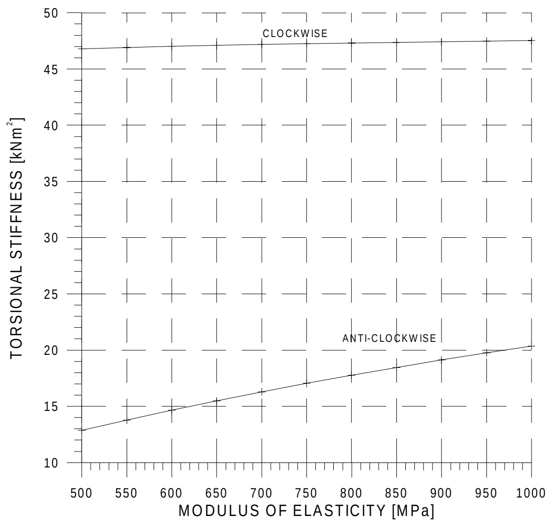

Figure 19- Torsional stiffness as a function of the polymer modulus of elasticity

5. CONCLUSIONS

This paper presents results from an analytical model for the prediction of the axisymmetric response of umbilical cables. It is shown that an accurate response of these multilayer composite structures requires algorithms that consider gap formation and closure. Material non-linearity and large deformations are important when estimating the umbilical ultimate strength and this will be addressed in future work. The aim of the case study was to illustrate the application of the model for an umbilical cable and provide understanding of the behaviour of the composite structure. Furthermore, the analysis pointed out a number of parameters that designers and manufacturers shall evaluate for optimisation. The analysis of complex composite structures is not so reliable as it should be, due to the uncertainties of parameters that control its response.

Appendix A - NOTATION

α	pitch angle
ε_a	local axial deformation in tendon
ε_c	umbilical axial deformation
σ	stress
ν	Poisson's ratio
γ	umbilical twist per unit length
A	cylindrical cross-sectional area
E	modulus of elasticity
ext	external
F_{lat}	força lateral
G	shear modulus of elasticity
int	internal
i	layer number
I	2nd moment of inertia (radial)
J	polar area moment of inertia
L	umbilical length
n_a	number of armour layers
n_c	number of homogeneous layers
n_i	number of tendons in layer *i*
p	pressure
Q	terminal couple
R	radius
R_{cl}	minimum radius for lateral contact
S	wire cross-sectional area
T	terminal axial load
x	x-local direction
y	y-local direction
‘	(apostrophe) final value

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