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HYPERSONIC LAMINAR BOUNDARY LAYERS WITH ADIABATIC WALL CONDITION

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Abstract. A new set of self-similar solutions of a compressible laminar boundary layer is used for air as perfect gas and where the viscosity is a power function of temperature. Modified Levy-Mangler and Dorodnitsyn-Howarth transformations are presented to solve the flow in a thin laminar boundary layer with no external pressure gradients on a smooth flat plate. This result in an explicit relation between the stream function and the enthalpy fields described by a closed-coupled system of nonlinear ordinary differential equations. In the present work, boundary layer flows with external Mach numbers 4 and 10 over in adiabatic wall are studied. The present solution methodology provides a straightforward way of comparing results using the viscosity-temperature linear relation, Sutherland law, and the relation according to the kinetic theory. Also, the results may provide important data needed for the design of future hypersonic vehicles.

Keywords: Self-similar equations, Compressible laminar boundary layer, Supersonic and hypersonic flows.

1. INTRODUCTION

Supersonic and hypersonic flows with low enthalpy conditions can be considered and may be modeled by a calorically or thermally perfect gas equation of state. In a calorically perfect gas the specific heats, c_p and c_v, are considered as constant. A thermally perfect gas is one in which the specific heats are functions of temperature only. This is a result of the vibrational energy within the gas molecules and the electronic energy associated with the electron motion within the atoms or molecules. In the more general case of a purely compressible gas at thermodynamic equilibrium, the specific heats are functions of two thermodynamic properties, for example pressure and temperature. An equation of state for real gases should be used in such cases. This relation may be applied in high enthalpy flows where dissociation and ionization occur.

A hypersonic flow over a flat plate can be divided into four distinct regions, Figure 1.

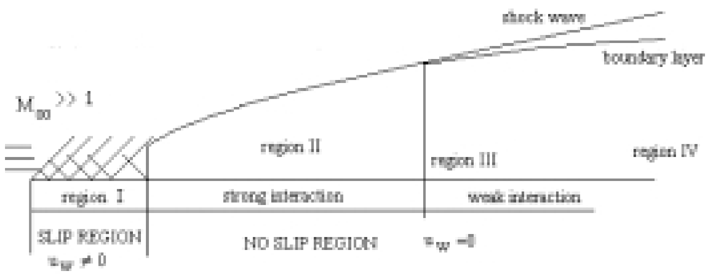

Figure 1.Flowfield on a flat plate at hypersonic flow

Near the plate leading edge, there exists a delay in the formation of the shock layer and the boundary layer as a result of the slip phenomena in this region (Nagamatsu and Sheer, 1960). Close to the leading edge, region I, the slip condition is stated and the flow is not continuum, so that the Navier-Stokes equations are not valid, and the first order kinetic flow theory should be applied (Nagamatsu and Li, 1960).

Immediately after the noncontinuum region, there exists a strong interaction continuum region, where the shock layer and the boundary layer are merged and the no-slip condition prevails at the surface of the plate, region II. In this region, the pressure gradient in the y-direction may be ignored, $\frac{\partial p}{\partial y} = 0$, but the pressure gradient in the x-direction can not be neglected, due to the presence of the shock wave inside the viscous layer.

Far from the leading edge region, a weaker interaction region may be found. In this region, which is close to the strong interaction (region III) the pressure gradient in the x- and y-directions inside the boundary layer are very small and may be ignored. However, outside the boundary layer, in the inviscid layer between the shock wave and the boundary layer, the pressure gradient in the y-direction can not be neglected.

Downstream of the strong interaction region, in region IV, the classical approach of Prandtl incompressible boundary layer theory can be applied to the compressible boundary layer. In this region the pressure gradients in the x- and y-directions may be neglected, both inside as well as outside the boundary layer.

Van Driest (1952) used the Crocco’s method and derived a set of ordinary differential equations to describe the compressible laminar boundary layer. He studied flows with Mach numbers up to 25 on a flat plate assuming a perfect gas obeying Sutherland viscosity law. The main results presented are the skin-friction and heat-transfer coefficients as function of Reynolds number, Mach number and wall-to-free stream temperature ratio.

Cohen and Reshotko (1957) studied self-similar solutions for a two-dimensional steady compressible laminar boundary layer with heat transfer and pressure gradients. They used the perfect gas assumption, with a unit Prandtl number and a linear viscosity-temperature relation across the boundary layer.

Mirels (1955) studied the shock wave as it advances into a stationary fluid bounded by a wall. A boundary-layer flow is established along the wall behind the shock. He employed a Blasius equation to solve the heat transfer behind the shock wave, with a modified boundary condition at $f'(0)=\frac{u_w}{u_e}$, where u_w and u_e are the velocity of the shock wave relative to the wall and the velocity outside the boundary layer, respectively. He stated that with increasing the Reynolds numbers, the laminar boundary layer behind the shock wave becomes unstable and the transition to turbulent flow occurs.

Toro et al. (1997) have recently developed new self-similar solutions for a compressible laminar boundary layers over a flat plate with the constant wall temperature boundary condition. The influence of the fluid Prandtl number, wall to free-stream temperature ratio, and the power of the viscosity-temperature law for supersonic flows with external Mach numbers up to 4 have been investigated.

The purpose of this work is to apply the new methodology of Toro et al. (1997) to study self-similar supersonic and hypersonic compressible laminar boundary layers over in adiabatic wall boundary condition. The work is limited to low enthalpy flows, where the dissociation phenomena is not present, and where the viscosity is a power function of the temperature.

2. MATHEMATICAL MODEL

A steady compressible flow of a viscous, heat conducting, Newtonian fluid is considered. For this flow at the limiting case of a high Reynolds number, or a small dynamic viscosity, the Navier-Stokes equations may be simplified to the classical Prandtl laminar boundary layer. Following Van Driest (1952), the steady, compressible, viscous, thin boundary layer, with two-dimensional flow, and zero pressure gradients in the x- and y-directions, over a smooth flat plate may be described by the following conservative equations:

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0, \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \qquad \frac{\partial p}{\partial y}=0, \tag{2}$$

$$\rho u\frac{\partial i}{\partial x}+\rho v\frac{\partial i}{\partial y}=\frac{\partial}{\partial y}\left(\frac{1}{Pr}\mu\frac{\partial i}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2. \tag{3}$$

Here, ρ is the flow density, p is the pressure, p=const., (u, v) are the axial and transverse velocity components, μ is the viscosity, Pr is the Prandtl number, $Pr=\frac{\mu c_p}{k}$, c_p is the gas specific heat coefficient, k is the gas thermal conductivity, and i is the specific enthalpy.

The boundary conditions needed to solve the problem are no penetration and no slip condition on an adiabatic wall (at $y=0$, $x\geq 0$) and the free stream conditions as $y\to\infty$:

$$u(x,0)=0, \qquad v(x,0)=0, \qquad \frac{\partial i}{\partial y}(x,0)=0, \quad u(x,\infty)=u_\infty, \qquad i(x,\infty)=i_\infty. \tag{4}$$

We assume a power law for the viscosity change as function of temperature given by

$$\frac{\mu}{\mu_r} = \left(\frac{T}{T_r}\right)^{\alpha}. \tag{5}$$

Here μ_r and T_r are reference viscosity and temperature, respectively, and α is a given power.

3. SELF-SIMILAR SOLUTION

We seek self-similar solutions of equations (1) - (5). The boundary layer equations may be reduced to self-similar solutions by introducing a modified change of variables that combines the Levy and Mangler and the Dorodnitsyn-Howarth transformations, and the power α that accounts for the variation of viscosity with temperature. Let:

$$\xi(x) = \int_0^x \rho_w \mu_w u_\infty d\xi', \qquad \eta(x,y) = \frac{\rho_w u_\infty}{\sqrt{2\xi}} \int_0^y \left(\frac{\rho}{\rho_w}\right)^{\alpha} dy'. \tag{6}$$

The stream function $\psi(x,y)$ is defined by

$$\rho u = \rho_w \frac{\partial \psi}{\partial y} \qquad \rho v = -\rho_w \frac{\partial \psi}{\partial x}. \tag{7}$$

We assume a self-similar relation:

$$\psi(x,\eta) = \sqrt{2 u_\infty \nu_w x}\, f(\eta), \qquad \frac{i(x,\bar{y})}{i_\infty} = \theta(\eta). \tag{8}$$

Replacing equations (6) - (8) into the momentum and energy equations, we obtain the self-similar compressible boundary layers given by:

$$f''' + ff'' = \frac{\alpha - 1}{\theta}\left[f'\theta'' + 2\theta' f'' + ff'\theta' - \alpha \frac{f'(\theta')^2}{\theta}\right], \tag{9}$$

$$\theta'' + \Pr f\theta' = -(\gamma - 1) M_\infty^2 \Pr\left[\left(\frac{\theta_w}{\theta}\right)^{\alpha-1}\right]^2 \left[f'' - (\alpha - 1)\frac{f'\theta'}{\theta}\right]^2. \tag{10}$$

Here $M_\infty = \frac{u_\infty}{a_\infty}$ is the Mach number of the external flow. The boundary conditions (4) result in:

$$f(0) = 0, \qquad f'(0) = 0, \qquad f'(\infty) = \theta_W^{-(\alpha-1)}, \qquad \theta'(0) = 0, \quad \theta(\infty) = 1. \tag{11}$$

For more detail of this theoretical approach see Toro et al. (1997). Toro et al. (1997) also presented results for various hot and cold walls with Mach numbers of external flow up to 4, with the constant wall temperature boundary condition.

$$f(0)=0, \qquad f'(0)=0, \qquad f'(\infty)=\theta_w^{-(\alpha-1)}, \qquad \theta(0)=\theta_w, \qquad \theta(\infty)=1. \qquad (12)$$

The system (9), (10) and (11) is a system of nonlinear ordinary differential equations that explicitly describes the relations between the stream function and the temperature fields, for the adiabatic wall case, while the system (9), (10) and (12) for the constant wall temperature case. Due to the compressibility effects, density is used here as a variable. The momentum equation in the x-direction (9) and the energy equation (10) are coupled by enthalpy θ, as one would expect. For the special case where viscosity changes as a linear function of temperature (i.e., $\alpha=1$), the compressible laminar boundary layer represented by Blasius and Pohlhausen equations are recovered.

4. RESULTS

The system of equations (9), (10) and (11) or (12) may now be used to study the behavior of self-similar compressible boundary layers. This system is solved by a standard Runge-Kutta fourth order integration technique using MATLAB. There are four parameters that affect the solution: the Mach number of the external flow, M_∞; the temperature ratio, θ_w; the Prandtl number, Pr; and the power α of the viscosity-temperature relation. Only the external flow Mach number influence will be presented.

Figure 2 displays the variation of viscosity-temperature law at pressures between 0.1 and 1 atmosphere. Note that $\alpha=0.69$ matches the numerical real gas values of Brower (1990) better than the other powers for a wide range of temperatures, up to 3000 K. The power $\alpha=0.76$ is a typical value used for air at relatively low temperatures, less than 1000 K. Also, notice that the power $\alpha=0.63$ provides a close match with Sutherland law,

$$\frac{\mu}{\mu_{ref}}=\left(\frac{T}{T_{ref}}\right)^{3/2}\frac{T_{ref}+S}{T+S}$$

with T_{ref} = 288.15 K and S = 110 K, that is commonly used for air at temperatures, up to 2000 K.

A typical solution of the self-similar equations (9-10) for $M_\infty=4$, $\alpha=0.69$, Pr=0.71, with constant wall temperature (equation 12, $\theta_w=3.0$, hot wall) and with adiabatic wall (equation 11) boundary conditions are shown in Figures 3 and 4, respectively. Note that in Figure 3 the temperature (θ) increases within the boundary layer, which demonstrates that viscous dissipation is an important aspect at high Mach number flows. It results in this case in a heat flux into the wall (θ'), if $\theta_w<\theta_{max}$ within the boundary layer. For the adiabatic wall case (Figure 4), there is no gradient temperature at $\eta=0$ ($\theta'(0)=0$), the maximum temperature is located at the wall ($\theta(0)=3.67175$), and the temperature (θ) decreases within the boundary layer.

Figure 5 presents the influence of Mach number on streamwise velocity and enthalpy profiles, for constant wall temperature (hot wall, $\theta_w=3$) and adiabatic wall cases, using the power viscosity-temperature law with $\alpha=0.69$ and Pr = 0.71.

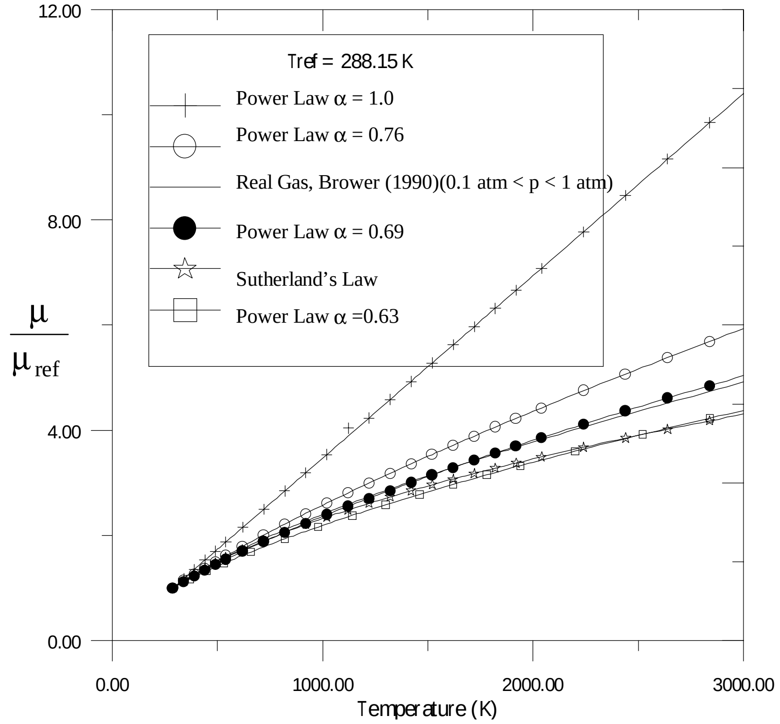

Figure 2. Viscosity as function of temperature.

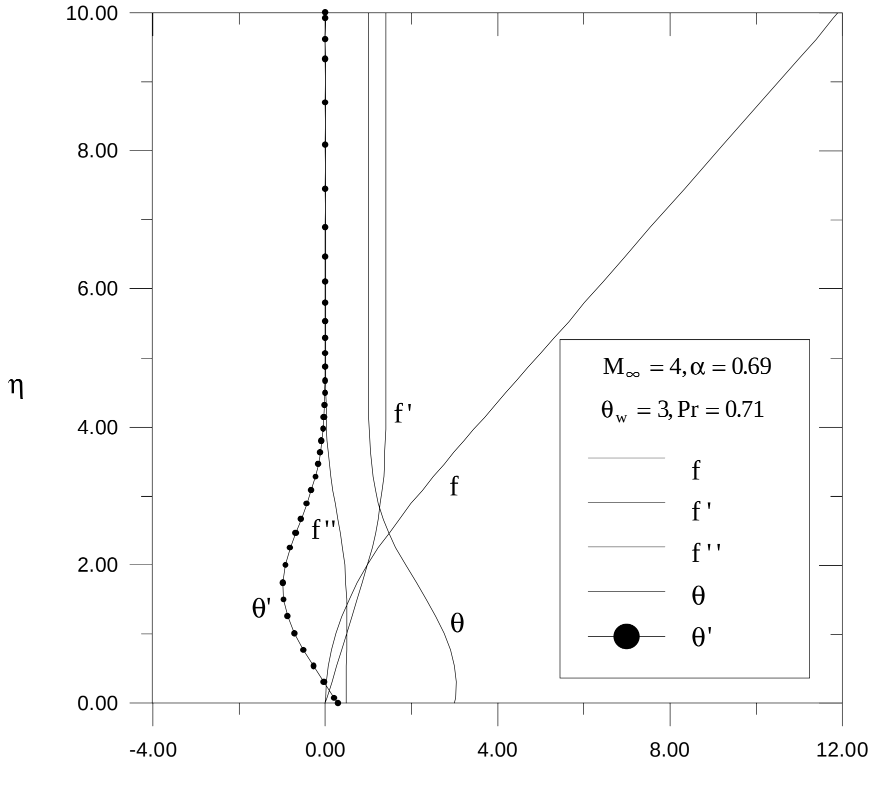

Figure 3. Typical solution for constant hot wall (θ(0)=3) boundary condition

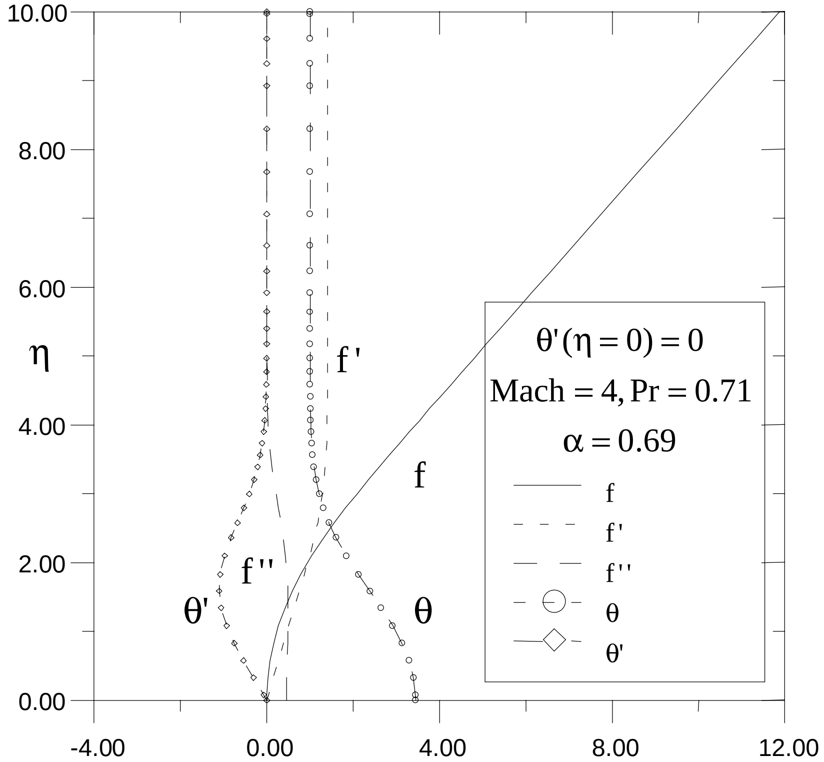

Figure 4. Typical solution for adiabatic wall (θ'(0)=0) boundary condition

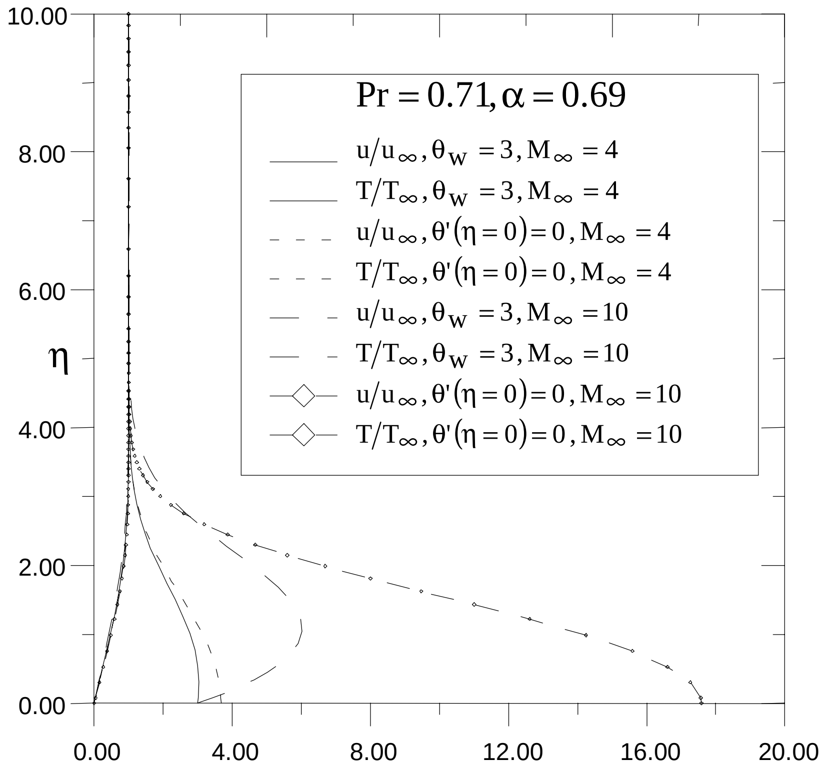

Figure 5. Streamwise velocity and temperature profiles

For the constant temperature wall case, when the Mach number is 4, the viscous dissipation creates a heat flux into the wall (see Table I and Figures 3 and 5). For Mach number greater than 4, at the free stream and at the wall, the fluid is colder than inside the boundary layer. This means that there exists a heat flux from the boundary layer to the wall and to the free stream. In the supersonic cases, the heat flux is dominated not only by the temperature ratio but also by the viscous dissipation, since the temperature increases within the boundary layer.

For adiabatic wall case, Mach numbers 4 and 10, there is no heat flux from the boundary layer to the wall. The temperature decreases, in both cases, within the boundary layer. Yet, the temperatures in the boundary layer and on the wall are much higher than in the constant wall temperature cases (Figure 5 and Table I).

Table I and equation (11) or (12) shows that $f'(\infty)$ is function of θ_w. For constant wall temperature, $\theta_w=3$, $f'(\infty)$ is constant. For adiabatic wall boundary condition the $f'(\infty)$ changes. For both Mach numbers 4 and 10, the hydrodynamic and thermal boundary layer thickness for adiabatic wall case are smaller than those for constant wall temperature as can be seen in Table I.

Table I. Values of $f''(0)$, $\theta'(0)$, $f'(\infty)$, $\theta(\infty)$ and η_∞ for various M_∞.

M_∞	$f''(0)$	$\eta_{\infty f}$	$f'(\infty)=\theta_w^{-(\alpha-1)}$	$\theta'(0)$	$\eta_{\infty\theta}$	$\theta(0)=\theta_w$
	constant	wall	temperature			
4	0.48363	5.5327	1.4057	0.29055	5.7998	3.
10	0.43445	6.2696	1.4057	5.68680	6.2696	3.
	adiabatic	wall				
4	0.48976	5.4081	1.4966	0	5.6599	3.67175
10	0.500327	5.434	2.4324	0	5.4340	17.59

5. CONCLUSION

A new methodology for calculating self-similar solutions of a compressible laminar boundary layer, considering the viscosity as a power function of the temperature, is applied. The modified Levy-Mangler and Dorodnitsyn-Howarth transformations described the similarity variables in terms of a power of the density that takes into account the viscosity-temperature power law. These transformations result in an explicit relation between the stream function and the temperature fields. Solutions are presented for boundary layer flows over adiabatic wall with external flow Mach numbers 4 and 10. The present solution methodology also provides a straightforward way of comparing results using the viscosity-temperature linear relation, Sutherland's law and the relation according to the kinetic theory.

These self-similar solutions are applicable to subsonic and supersonic flows. They may also apply to hypersonic flows with low enthalpy, as long as the flow is far from the leading edge and is described by a continuum calorically or thermally perfect gas relation.

These self-similar solutions may provide important data needed for the design of future hypersonic vehicles. It may also be applied in two important areas, as follows:

1) Most of the Hypersonic Shock Tunnels operate at the cold flow condition, where the total enthalpy is low enough and there are no chemical reactions within the boundary layer.

Here, the real gas effects may be ignored. Experiments carried out in those facilities, can be compared to the present analyses.

2) The Navier-Stokes equations for supersonic or hypersonic flows must be solved by numerical methods since there are no exact solutions for them. The present self-similar solutions can be used as a tool to validate these numerical simulations.

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