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FLEXIBLE STRUCTURE VIBRATION CONTROL USING MULTIPLE ACTUATORS

Luiz Carlos Gadelha deSouza,
Silmara Alexandra daSilva
National Institute for Space Research – INPE
Space Mechanics & Control Division - DMC
C.P. 515 – São José dos Campos – SP – 12227-010 – Brazil

Abstract. *The paper presents the results of an active structural vibration control of a flexible satellite performed by a proof-mass actuator (PMA) during an attitude rotation maneuver . The satellite investigated is composed of a rigid rub plus a cantilevered flexible beam with the PMA located at the beam free end. As the satellite manouvers from rest to a pre-defined position, the rigid body motion can excite the flexible part of the satellite. The PMA tasks is to damp-out any residual vibration caused by this manouver efficiently. The rigid/flexible satellite is modeled, using a relatively simple structural dynamics approach.. Lagrange´s equation is then applied to obtain the satellite equations of motion. The PMA gain selection is based on an analytical approach which shows that the pole and zero of the fundamental mode is dominant. The efficiency of the PMA using velocity feedback with a PI control law was examined by numerical simulations for different control maneuvers strategies. It was shown that one such controller has damped the residual flexible vibration successfully. However, it was also shown that the control system efficiency is function of the maneuver strategy.*

Key words: Large Space Structure, Vibration Control, Attitude Maneuver, Proof-Mass Actuator.

1.INTRODUCTION

Although there is an increasing interest in using small satellite Souza (1999) for many different space missions, the space conquest as well as out space missions will not be possible without the use of large space structures (LSS). The construction of International Space Station (ISS), despite technical and funding problems, it appears that it will go ahead, as a result, LSS construction creates new demands on dynamics techniques where the coupling between the rigid and the flexible motion must be take into account in the control system design. On the other hand, the interaction Soares *et al.* (1997) and Silva (1998) between the control system and the flexible structure motion play an important rule in the control system performance. Robust techniques Souza (1997) can also be implemented to consider the parameter variation during the maneuver of rigid/flexible satellite in space.

Investigation in the field of active control of structures vibration in space have covered in the last decades many important topics, among which the electrically drive actuators known as proof mass actuators Mantegazza *et al.* (1993) has captured the interest of many research group. The advantage of this device is that electrical power can be produced directly in space comparing with jet thrusters that require fuel tank. Besides, controllers based on high bandwidth can become a serious concern as for flexible structural motion and control system interaction.

In this the paper one concentrates on the specific problem of damping out flexible motion when rotating a satellite composed of a rigid rub plus a cantilevered flexible beam with a proof-mass actuator located at the beam free end.. A simple structural dynamics approach is applied to investigate the dynamics and control of such maneuvers.

2.EQUATIONS OF MOTION

Initially the satellite is modeled by a rigid central rub plus an uniform cantilever beam with a tip-mass M at its free end. Using the Euler-Bernoulli theory with the assumptions, that the shear deformation is small compared to the bending deformation. The transverse displacement of the beam from its equilibrium position x at time t is given by y(x,t). See Fig.1.

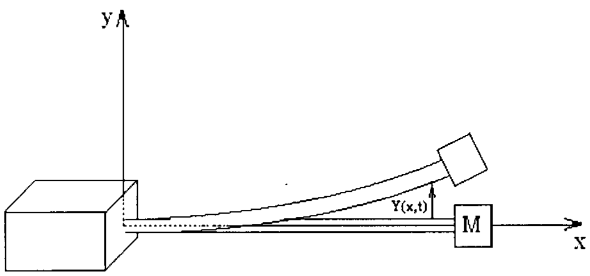


Figure 1 : Rigid/Flexible Satellite with a tip-mass M

The equation of motion of a uniform cantilever beam is a fourth order partial differential equation given by:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\sigma\frac{\partial^2 y(x,t)}{\partial t^2}=0 \tag{1}$$

where E is the Young´s modulus of elasticity, I is the beam moment of inertia and σ is the mass per unit length.

The following boundary conditions are associated with the clamped-free beam motion

$$y(0,t)=0,\ \frac{\partial y(x,t)}{\partial x}\Big|_{x=0}=0 \tag{2}$$

$$\frac{\partial^2 y(L,t)}{\partial x^2} = 0 \quad (3)$$

$$M\frac{\partial^2 y(L,t)}{\partial t^2} - EI\frac{\partial^3 y(L,t)}{\partial x^3} = 0 \quad (4)$$

In order to find the modal equations governing the vibration motion, one needs to find the natural frequencies and the modes shapes of the beam plus tip-mass. Assuming the solution of Eq.(1) in the following separable form:

$$y(x,t) = Y(x)\phi(t) \quad (5)$$

where Y(x) and $\phi(t)$ are the nth mode shape and generalized coordinate, respectively.

Substituting Eq.(5) into Eq.(1) and using the previously boundary conditions, after some manipulation the natural frequencies is found solving:

$$1 + c\lambda L ch\lambda L + \frac{M\lambda}{\sigma}\left[c\lambda L sh\lambda L - s\lambda L ch\lambda L\right] = 0 \quad (6)$$

where $\lambda^4 = \sigma\omega^2/EI$ and c=cos, s=sin, ch=cosh sh=sinh.

Using the data from Wie (1988) to solve Eq.(6) numerically, the first three natural frequencies for different values of tip-mass are given by Table 1. It is possible to note that the frequencies decrease as the mass increase. Considering that $\phi(x) = 1$, the expression for the the nth mode shape is given by

$$Y(x) = \frac{\left[(\sin\lambda x - \sinh\lambda x) + (\cosh\lambda x - \cos\lambda x)\psi\right]}{\left[(\sin\lambda L - \sinh\lambda L) + (\cosh\lambda L - \cos\lambda L)\psi'\right]} \quad (7)$$

where $\psi = \frac{\sin\lambda x + \sinh\lambda x}{\cos\lambda x + \cosh\lambda x}$ $\quad \psi' = \frac{\sin\lambda L + \sinh\lambda L}{\cos\lambda L + \cosh\lambda L}$

Table 1: Natural Frequencies in rad/s

Mode	Mass 0kg	Mass 10kg	Mass 40kg	Mass 80kg	Mass 162kg
	ω	ω	ω	ω	ω
1	2.6851	2.4027	1.8963	1.5451	1.1882
2	16.8277	15.3369	13.6454	12.9167	12.4090
3	47.1183	43.4931	40.5068	39.4959	38.8678

3.MODAL EQUATION OF MOTION

The attitude maneuver is performed by the satellite angular acceleration generated by any type of torque actuator (reaction wheel or thruster).

To find the equations of motion one uses the Lagrange´s equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right)-\frac{\partial T}{\partial q_r}+\frac{\partial V}{\partial q_r}=Q_r \tag{8}$$

where T and V correspond to the kinetic and potential energy of the system. The rth generalized coordinate is q_r and Q_r is the generalized force which is applied at the beam free end in the same direction as y(x,t).

The expression for the total kinetic energy of a beam with a tip-mass M is:

$$T(t)=\frac{\sigma L^3}{6}\dot{\theta}^2(t)+\sigma\dot{\theta}(t)F_n\sum_{n=1}^{N}\dot{\phi}_n(t)+\frac{\sigma}{2}\sum_{n=1}^{N}\dot{\phi}_n^2(t)G_n+\frac{ML^2}{2}\dot{\theta}^2(t)+ML\dot{\theta}(t)\sum_{n=1}^{N}Y_n(L)\dot{\phi}_n(t)+\frac{M}{2}\sum_{n=1}^{N}Y_n^2(L)\dot{\phi}_n^2(t) \tag{9}$$

where $F_n=\int_0^L xY_n(x)dx$, $G_n=\int_0^L Y_n^2(x)dx$

The potential energy due to the strain energy is given by:

$$V(t)=\sum_{n=1}^{N}\left[\omega_n^2\left(\frac{\sigma G_n+MY_n^2(L)}{2}\right)\phi_n^2(t)\right] \tag{10}$$

Substituting the expressions for the kinetic and potential energy in the Lagrange´s equation, the modal equations of motion for the beam under rotation is:

$$\ddot{\phi}_n(t)+\omega_n^2\phi_n(t)=-\frac{L_n}{M_n}\ddot{\theta}(t)+\frac{Y_n(L)}{M_n}F(t) \tag{11}$$

where $M_n=\sigma G_n+MY_n^2(L)$, $L_n=(\sigma F_n+MLY_n(L))$

Note that in order to use these equations the values of the constants L_n , M_n, , F_n and G_n must be found for the first N modes of interest. In short , given a particular maneuver with acceleration given by $d^2\,\theta/dt^2$, the PMA function is damp-out some eventual residual flexible vibration (q(t)) using the tip force F(t), which from mow on can also be represented by U(t).

4.TIP-MASS AND POLE-ZERO LOCATION

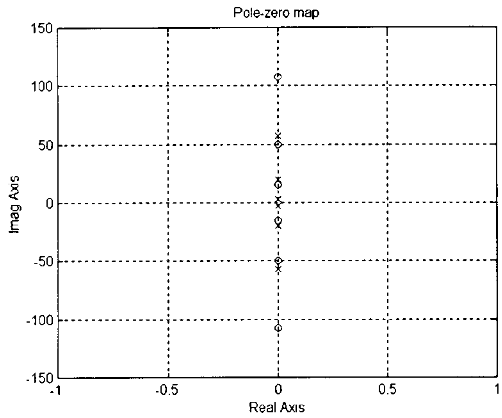
An analytical transfer function from the control force input to the deflection at various points on the beam can be determined using the analytical approach Wie (1988). This analysis provide some physical insights of the rigid/flexible satellite behavior which helps in the modeling and control system design. In order to do that, the transcendental transfer function from the tip force U(s) to the beam deflection Y(x,s) at location x is:

$$\frac{Y(x,s)}{U(s)} = \frac{(c\mu L + ch\mu L)(s\mu x - sh\mu x) - (s\mu L + sh\mu L)(c\mu x - ch\mu x)}{2EI\mu^3\left(1 + ch\mu Lc\mu L + \frac{M\mu}{\sigma}(sh\mu Lc\mu L - s\mu Lch\mu L)\right)} \quad (12)$$

In particular, the transcendental transfer function from the tip force to the tip deflection at x=L , with $\lambda=\mu L$ is:

$$\frac{Y(1,s)}{U(s)} = \frac{s\lambda ch\lambda - sh\lambda c\lambda}{\lambda^3(1 + c\lambda ch\lambda) + m\lambda^4(c\lambda sh\lambda - s\lambda ch\lambda)} \quad (13)$$

By determining the roots of the numerator and denominator of these transfer functions for given tip-mass, these transcendental transfer function can be represented as infinite products of pole and zeros. Eq.(12) shows that the transfer-function zeros are independent of the tip-mass , while the poles represents the natural frequencies are dependent on the tip-mass. The poles and zeros of Eq.(13) for tip-mass of 10kg and 162Kg are shown in Fig 2 and Fig. 3. One observes, that when M increases, the poles and zeros associated with the second and higher modes became closely spaced and the first mode becomes a single dominant mode.



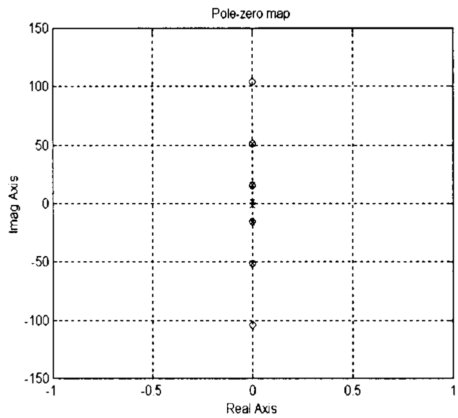


Figure 2: Poles and zeros for tip-mass 10kg **Figure 3: Poles and zeros for tip-mass 162kg**

5.PROOF-MASS ACTUATOR DYNAMICS

The proof-mass actuator consists of a mass m connected to a spring of constant k. The mass is acted on by a linear dc motor supplying a force U(t). This in turn, affects the tip of the beam as shown inn Fig. 4.

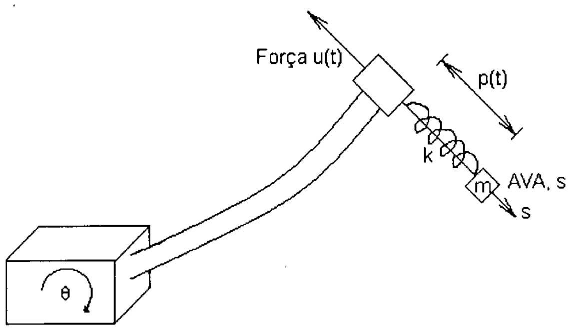


Figure 4 : The proof-mass actuator

The control force expression for the PMA is

$$\begin{aligned} u(t) &= -m\ddot{p}(t) - m\left(\ddot{y}(L,t) + L\ddot{\theta}(t)\right) - kp(t) \\ &= u_1(t) - mL\ddot{\theta}(t) \end{aligned} \tag{14}$$

where p(t) is the displacement of the spring from its equilibrium length. The term $mLd^2\theta/dt^2$ may be regarded as modifying the excitation due to the satellite rigid motion, therefore, this term is not a control force from the PMA.

Assuming a control law given by

$$u_1(t) = -Km\dot{y}(L,t) \tag{15}$$

where K is the gain of the actuator which is associated to the gains of a PI controller (K_p and K_I) by the expression [16] and m is a mass of PMA.

$$\frac{K_p}{K_I} = \frac{m}{K} \tag{16}$$

The tip velocity of the beam for the first three mode considering $Y_n(L)=1$ is given

$$\begin{aligned} \dot{y}(L,t) &\cong Y_1(L)\dot{\phi}_1(t) + Y_2(L)\dot{\phi}_2(t) + Y_3(L)\dot{\phi}_3(t) \\ &= \dot{\phi}_1(t) + \dot{\phi}_2(t) + \dot{\phi}_3(t) \end{aligned} \tag{17}$$

Substituting the previous equations into Eq.(11) and rearranging the terms, the general modal equation for flexible motion including the light structural damping β and compensated by the PMA is given by

$$\ddot{\phi}_n(t) + 2\beta_n\omega_n\dot{\phi}_n(t) + \gamma_n\left(\dot{\phi}_1(t) + \dot{\phi}_2(t) + \dot{\phi}_3(t)\right) + \omega_n^2\phi_n(t) = -\left(\frac{L_n + mL}{M_n}\right)\ddot{\theta}(t) \tag{18}$$

where $\gamma_n = \dfrac{mK_I}{M_n}$

6.PMA ACTUATOR GAIN SELECTION

In practice the gain selection might be done just for the fundamental mode, since its pole and zero are dominant, as shown previously. Thus K_I can be estimate considering only the first mode in Eq.(18) to achieve a compensated damping of ζ by the following expression[19]

$$K = \frac{2M_1\omega_1}{m}(\zeta - \beta_1) \qquad (19)$$

Then the value of K_I may be used in a model of the system involving the first N modes. One has assumes a design damping of 0.05 (ζ) and a structural damping of 0.002 (β_1) for the fundamental mode

7.CONTROL SYSTEM SIMULATIONS

7.1 ONE ACTUATOR

All simulation are based on Eq.(18), which must be solved numerically for particular acceleration profile with n=3.

The maneuvers were considered to star with the satellite at rest, following a angular motion and bring the satellite back to rest at their end. The overall satellite angle rotation is found by integrating its acceleration. The maneuvers strategies simulated are shown in table 2 and its respective results with a tip-mass of 10Kg are shown in Figures 5 to 13.

Table 2: Maneuvers Strategies Profile

Maneuver	Description (s and rad/s^2)
1	$t_0=0$ $t_1=4$ $t_2=8$
	$\ddot{\theta}(t)=0.0167$ $\ddot{\theta}(t)=-0.0167$
2	$t_0=0$ $t_1=5.3$ $t_2=10.6$
	$\ddot{\theta}(t)=0.0167$ $\ddot{\theta}(t)=-0.0167$
3	$t_0=0$ $t_1=2.5$ $t_2=7.5$ $t_3=10$
	$\ddot{\theta}(t)=0.0167$ $\ddot{\theta}(t)=0$ $\ddot{\theta}(t)=-0.0167$

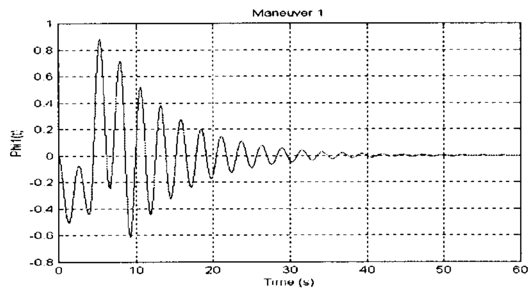


Figure 5 : First mode - Maneuver 1

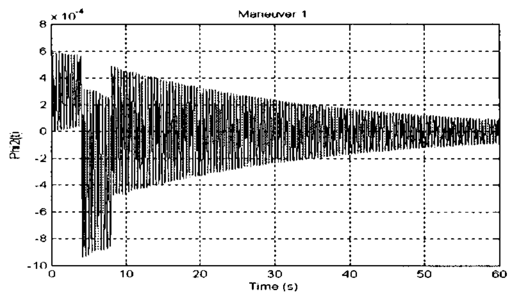


Figure 6 : Second mode - Maneuver 1

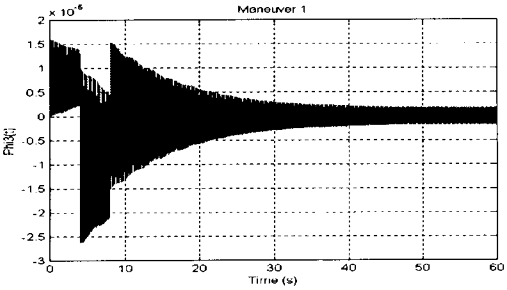


Figure 7 : Third mode - Maneuver 1

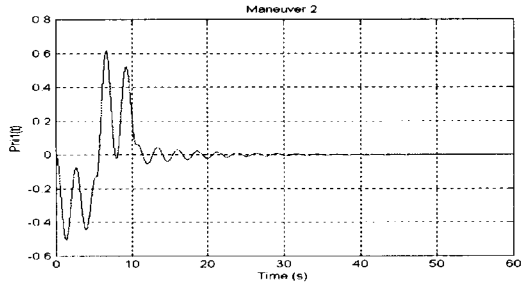


Figure 8 : First mode - Maneuver 2

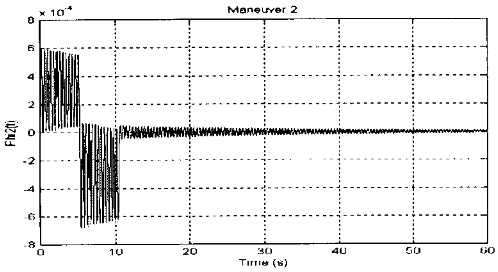


Figure 9 : Second mode - Maneuver 2

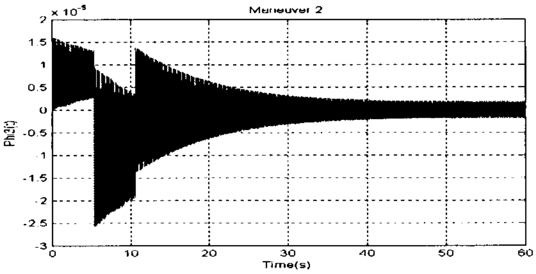


Figure 10 : Third mode - Maneuver 2

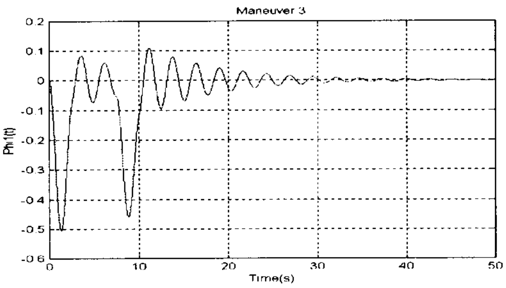


Figure 11 : First mode - Maneuver 3

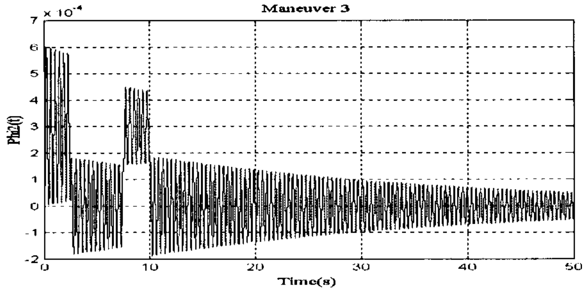


Figure 12 : Second mode - Maneuver 3

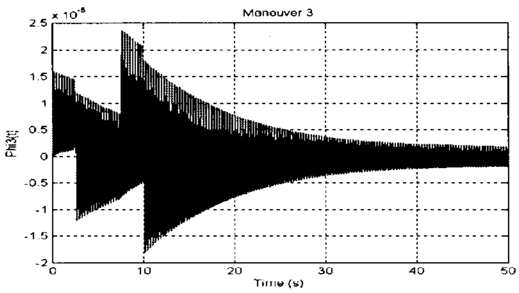


Figure 13 : Third mode - Maneuver 3

7.2 MULTIPLE ACTUATORS

In this case we use N actuators to control N modes of vibration. The law of control is:

$$u_m(t) = -\alpha_m \dot{y}(x_m, t) \tag{20}$$

each α_m is a gain of each mode of vibration and the equation of motion is:

$$\ddot{\phi}_n(t) + 2\beta_n\omega_n\dot{\phi}_n(t) + \omega_n^2\phi_n(t) = -\left(\frac{L_n + Y_n(L)mL}{M_n}\right)\ddot{\theta}(t) - \frac{\sum_{m=1}^{N}\left(Y_n(x_m)\alpha_m\sum_{r=1}^{N}Y_r(x_n)\dot{\phi}_r(t)\right)}{M_n} \tag{21}$$

The maneuvers were considered the same for one actuator and the strategies are shown in the table 2 and its respective results with a tip-mass of 10kg are shown in the figures 14 to 16.

Case 3
α_1=100
α_2=100
α_3=100

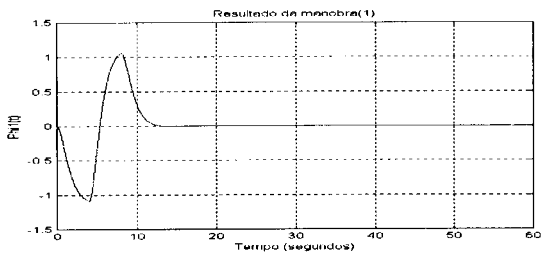


Figure 14 : First mode - Maneuver 1

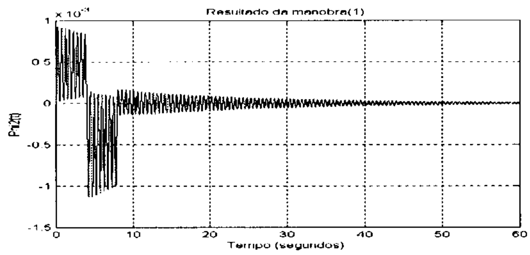


Figure 15 : Second mode - Maneuver 3

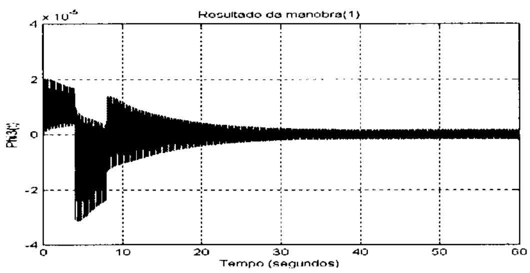


Figure 16 : Third mode - Maneuver 3

8.COMENTS

From the previously Figures one can observes that the fundamental mode is predominant, since it moves the beam from its equilibrium position much more than the second and third modes. Therefore, unless a very fine control of the beam is required it may be sufficient to control the fundamental mode. The results also demonstrate that the PMA does provide sufficient damping , since at the end of each maneuver all the mode are gradually damped out. This confirm that gain selection based on the fundamental mode, neglecting the higher modes, is a very good approximation. As for the maneuvers strategy , one observes that after the first maneuver has ended the first mode oscillations still reach the amplitude of over 0.8, while after the second maneuver the first mode oscillations do not exceed 0.5 and after the third maneuver the first mode do not exceed 0.1. This suggest that the complete success of the control system is very dependent upon the maneuver strategy.

9.SUMMARY

In this paper the study of dynamics and control for rigid/flexible space system is presented. It is investigated the particular problem of rotating a flexible beam attached to a rigid hub and the performance of the proposed control system based on a PMA. The dynamics of the beam were modeled, and its modes shapes and natural frequencies found. Lagrange´s equation was then applied to find the modal equations of motion for the rotating flexible structure. The efficiency of the proof-mass actuator using velocity feedback with a PI control law was examined. It was shown that one such controller has damped the residual flexible vibration successfully. However, it was also shown that the control system efficiency is function of the maneuver strategy.

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