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STRUCTURAL DAMAGE ASSESSMENT USING THE ZOOMING METHOD

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Abstract. *The present study describes a formulation for structural damage assessment based on methods of system identification, which results in an unconstrained optimization problem. Response of a damaged structure is different from the original undamaged one. A refined analytical model that predicts the response of the structure subject to static or dynamic loads is available. Using the output error approach of system identification it is possible to determine the necessary changes in the analytical model so that the difference between the experimental results and the response predicted by the analytical model is minimized. As the design space of the resulting optimization problem is highly nonconvex and can possess several local minima a deterministic method of global optimization denominated zooming is used. This method is a good alternative to the stochastic methods of optimization due to its simplicity. Existing software for local optimization can be used with only a minor modification. The method was applied to simple structural systems and the results compared to the ones obtained with a local optimization method and a stochastic global optimization method.*

Keywords*: *Structural Damage Assessment, System Identification Techniques, Zooming Method, Global Optimization Methods

1. INTRODUCTION

All the structures during the service life suffer damage due to unfavorable environmental and mechanical conditions. Damage causes microstructural changes that affect the mechanical properties of such structures, producing a decrease of their strength and rigidity. In several situations, such damage may not be observable. From the standpoint of both safety and performance, it is desirable to monitor the occurrence, location, and extent of such damage.

Several researchers with different approaches have treated this theme. In this work system identification techniques are used for damage assessment, where differences between analytical response and experimental results are employed to characterize damage. These techniques that are classified in the category of non-destructive methods, result essentially, in an optimization problem. As observed in previous works (Hajela & Soeiro, 1990), this problem can present a nonconvex design space, with many local minima. For this reason, the use of traditional methods of optimization, based on mathematical programming and using the gradients of the objective function, can lead to a local minimum, producing a wrong detection of structural damage.

An alternative already presented by Bavaresco & Soeiro, 1997 and Soeiro & Bueno, 1998, is the use of stochastic methods of global optimization, such as the genetic algorithm or the simulated annealing. These methods are basically random methods and require an enormous number of function evaluations with a high computational cost.

With the concern of trying to overcome these shortcomings, the present work uses a deterministic method of global optimization denominated zooming method. The main characteristic of this method is that with small modifications, any method of local optimization can become a method of global optimization and suitable for damage assessment purposes. These local optimization methods are based in mathematical programming and are available in most mathematical computer libraries.

The developed approach was applied for damage detection of simple truss structures with ten and fifteen members. As experimental results static displacements and a reduced set of vibration modes were employed. The use of the zooming method for damage assessment was compared to other optimization algorithms, such as the simulated annealing (a stochastic method of global optimization) and the BFGS (a local optimization method based on gradients), showing promising results.

2. SYSTEM IDENTIFICATION TECHNIQUES IN DAMAGE ASSESSMENT

The classical system identification problem refers to the determination of the parameters of a mathematical model so that the system response obtained experimentally is predicted correctly by the analytical model. In the case of damage assessment, a refined analytical model is already available, which predicts correctly the response of the undamaged structure. Changes in the analytical model will be determined to detect damage.

In a finite element formulation, the characteristics of the structure are defined in terms of the stiffness, damping and mass matrices *K*, *C* and *M*, respectively. Any variations in these matrices such as the ones that may be introduced by damage, would affect the dynamic response characteristics of the structure. A change in the stiffness matrix alone would influence the static response. In the present work, simulated measurements of structure eigenmodes and static deflections under prescribed loads were used for identification of structural damage.

The analytical model describing the eigenvalue problem for an undamped system can be stated in terms of the system matrices defined above, the i-th eigenvalue λ_i, and the corresponding eigenmode ϕ_i as follows.

$$\left[K - \lambda_i M\right]\phi_i = 0 \tag{1}$$

Matrices [K] and [M] may be adjusted to minimize differences between the experimentally observed eigenmodes and values obtained from the analytical model described above. In using static deflections for identification purposes, the analytical model is even simpler, involving only the system stiffness matrix *K* as follows.

$$Kx = f \tag{2}$$

Here *x* is a vector of displacements under applied static loads *f*. Although it is very clear from the previous equations that a variation in the system matrices results in a changed response, it is more important from a damage assessment standpoint to relate these differences to changes in specific elements of the system matrices. Since internal structural damage typically does not result in loss of material, one can assume the mass matrix to be a constant. Typically, the stiffness matrix can be expressed in terms of cross-sectional areas and bending moment of inertia. There is also a dependence on element dimensions and extensional moduli. In the present work, the net changes in these quantities due to damage are lumped into a single coefficient d_i denominated damage variables. These d_i constitute the design variables for the optimization problem and may vary from 0 to 1 representing an undamaged element and a completely damaged (no stiffness) element. In a planar truss the stiffness matrix is modified to incorporate the damage variable as follows.

$$K_i^{(e)} = \frac{(1-d_i)\bullet E_i \bullet A_i}{L_i}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \tag{3}$$

Here *C is cos* α *, S is sin* α *and* E_i, A_i,, L_i *and* d_i, *are* respectively the extensional modulus, area, length, and damage variable related to each truss element.

If the measured and analytically determined static displacements or eigenmodes are denoted by Y^m and Y^a respectively, the optimization problem can be formulated as determining the vector of design variables d_i (and hence the analytical stiffness matrix), that minimize the scalar objective function representing the difference between the analytical and experimental response, and stated as follows.

$$F = \sum_i \sum_j \left(Y_{ij}^m - Y_{ij}^a\right)^2 \tag{4}$$

Here i represents the degree of freedom and j denotes a static loading condition or a particular eigenmode. This minimization requires that Y^a be obtained from the eigenvalue problem or the load deflection equations, using the K matrix that must be identified.

Equation (4) represents the output error approach of system identification (Hanagud et al., 1987). One important advantage of this approach is that the complete set of modes or displacements is not needed since the objective function involves only difference between components of those vectors. Some of the components may be neglected according to its importance in the behavior of the structure. This is more realistic since in large structures only few dominant displacements can be obtained accurately.

3. ZOOMING METHOD

This global optimization method combines a local minimization method with successive truncation of the feasible space to eliminate regions of local minima in the search for the global minimum. The basic idea is to begin the search for a constrained local minimum at any point. Once a local minimum has been found, the problem is redefined by the addition of a constraint that eliminates that solution from a further search. The process is restarted and it continues until no other minimum point can be found.

The addition of the constraint on the objective function is done in the following way:

$$f(z) \le \gamma f(z^*) \tag{5}$$

where $f(z^*)$ is the value of the objective function in the point of local minimum and γ is a constant chosen in the following way:

$$\begin{aligned} 0 < \gamma < 1 \quad \text{if} \quad f(z^*) > 0 \\ \gamma > 1 \quad \text{if} \quad f(z^*) < 0 \end{aligned} \tag{6}$$

The zooming method (Arora *et al*., 1995) appears to be a good alternative to stochastic methods. For its implementation it is just necessary a good local optimizer. In this work a feasible directions method was used for the constrained optimization problem combined with the golden section method to refine bounds and the cubic polynomial interpolation to find the minimum in the one-dimensional search (Vanderplaats, 1984).

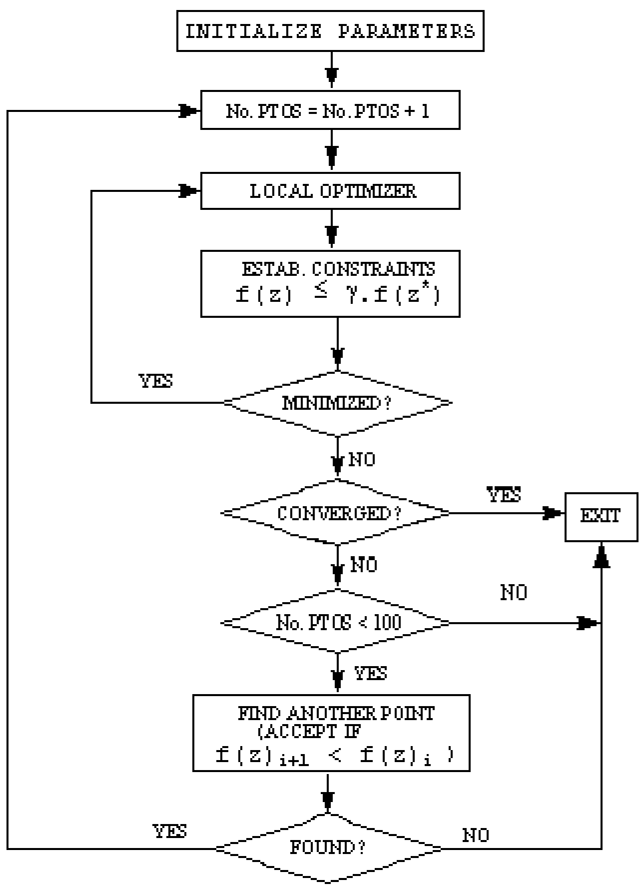

Figure 1: Flowchart of the zooming method

When in the search process, one gets close to the global minimum the feasible region of the redefined problem may become very small. If the starting point is not close to this region, the local minimization method may fail to find a solution. This would lead to the conclusion that the previous local minimum found is the global minimum. To overcome the problem of finding another starting point when the feasible region becomes very small, a random search is performed to find the new starting point. This new starting is only accepted if the value of the current objective function is smaller than the value of the previous one, as follows

$$f(z)_{i+1} < f(z)_i \tag{7}$$

Depending on the design space this may require a lot of function evaluations increasing the computational effort. The flowchart of the zooming method as implemented in this work is shown in Fig. 1.

4. DISCUSSION OF RESULTS

A program in MS FORTRAN PowerStation following the procedure described in the previous section was developed, integrating the optimizer based on the zooming method and the program FEM1DV2 (Reddy, 1993) based on the finite element method. The measured data were simulated by the finite element solution of the structure with some assumed damaged members. The values of the damage variables are then calculated by the described procedure, verifying the assumed damage.

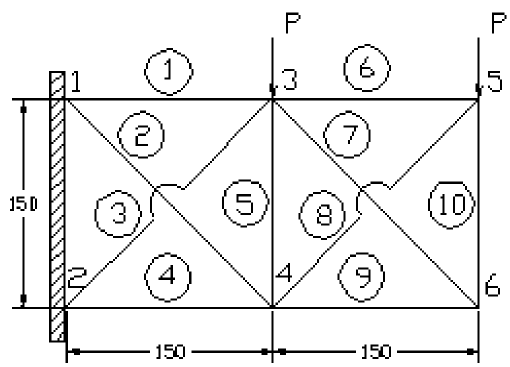

Figure 2: A ten bar planar truss. Dimensions in cm, P=50 kN and A=10cm^2.

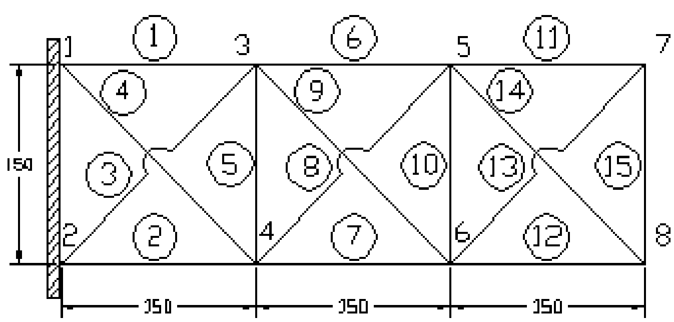

Figure 3: A fifteen bar planar truss. Dimensions in cm and A=10cm^2.

The method for damage detection was applied to representative truss structures with ten and fifteen members. As experimental measurements, static displacements were used for the ten bar truss with the loads indicated in Fig. 1 and four eigenmodes were used for the ten bar and fifteen bar truss shown in Fig. 2. The results were compared to the ones obtained when the optimizer is a local optimization method such as BFGS and a stochastic global method such as the Simulated Annealing (SA).

Table 1: Results for the ten bar planar truss, with d_1=40% e d_5=40%, using static displacements

ELEM.	***Stress (kN/cm²)***	***Exact Solution***	***BFGS***	***S.A.***	***ZOOM.***
1	***10,23***	***0,4***	***0,4027***	***0,3997***	***0,3967***
2	*6,74*	*0,0*	*0,0000*	*0,0002*	*0,0049*
3	*7,39*	*0,0*	*0,0000*	*0,0000*	*0,0004*
4	*9,77*	*0,0*	*0,0000*	*0,0000*	*0,0000*
5	***1,78***	***0,4***	***0,0298***	***0,4007***	***0,4086***
6	*2,99*	*0,0*	*0,0558*	*0,0017*	*0,0152*
7	*2,84*	*0,0*	*0,0089*	*0,0008*	*0,0077*
8	*4,22*	*0,0*	*0,0000*	*0,0004*	*0,0026*
9	*2,01*	*0,0*	*0,0000*	*0,0011*	*0,0121*
10	*2,01*	*0,0*	*0,0263*	*0,0009*	*0,0020*

For the ten bar truss shown in Fig. 1, damage was introduced in members 1 and 5 by reducing their stiffness to 40% of the original value. Here static displacements were used to detect the extent and location of damage. Table 1 shows the results obtained in this case. It is clear that damage in member 1 was detected by all methods with good accuracy. On the other hand, damage in member 5 was detected only by global optimization methods. This is due to the level of stress applied to the member. The design variables corresponding to the more stressed member have a higher sensitivity and are easier to find more accurately which means it is easier to detect damage in such members. This was confirmed choosing another member with low level of stress but with a higher degree of damage (member 10 with 70% of damage).

Table 2: Results for the ten bar planar truss, with d_5=70% and d_{10}=70%, using static displacements.

ELEM.	***Stress (kN/cm²)***	***Exact Solution***	***BFGS***	***S.A.***	***ZOOM.***
1	*10,23*	*0,0*	*0,0000*	*0,0000*	*0,0000*
2	*6,74*	*0,0*	*0,0000*	*0,0000*	*0,0000*
3	*7,39*	*0,0*	*0,0000*	*0,0000*	*0,0000*
4	*9,77*	*0,0*	*0,0000*	*0,0000*	*0,0000*
5	***1,78***	***0,7***	***0,3358***	***0,6999***	***0,6960***
6	*2,99*	*0,0*	*0,0729*	*0,0004*	*0,0036*
7	*2,84*	*0,0*	*0,0000*	*0,0001*	*0,0000*
8	*4,22*	*0,0*	*0,0000*	*0,0000*	*0,0000*
9	*2,01*	*0,0*	*0,0000*	*0,0000*	*0,0000*
10	***2,01***	***0,7***	***0,6608***	***0,6998***	***0,6965***

The ten bar truss with the same type of damage presented in Table 1 was solved using four eigenmodes. Although eigenmodes, usually are capable to capture better the overall behavior of the structure and become more effective in damage assessment, again damage was detected accurately only in member 1 by all methods. Damage in member 5 was only detected by global optimization methods. The results for this example are summarized in Table 3.

For the fifteen bar truss of Fig. 2 damage of 50% was introduced in members 1 and 11. Global optimization methods work clearly better than the local optimization method (BFGS), as can be seen in Table 4.

Table 3: Results for the ten bar planar truss, with d_1=40% and d_5=40%, using the first four eigenmodes

ELEM.	*Exact Solution*	*BFGS*	*S.A.*	*ZOOM.*
1	***0,4***	***0,4305***	***0,4190***	***0,4079***
2	*0,0*	*0,0054*	*0,0313*	*0,0147*
3	*0,0*	*0,0000*	*0,0312*	*0,0158*
4	*0,0*	*0,0019*	*0,0315*	*0,0123*
5	***0,4***	***0,1056***	***0,4179***	***0,4057***
6	*0,0*	*0,0046*	*0,0313*	*0,0152*
7	*0,0*	*0,0000*	*0,0312*	*0,0156*
8	*0,0*	*0,0148*	*0,0317*	*0,0169*
9	*0,0*	*0,0267*	*0,0315*	*0,0226*
10	*0,0*	*0,0159*	*0,0327*	*0,0179*

Table 4: Results for the fifteen bar planar truss, with d_1=50% and d_{11}=50%, using the first four eigenmodes

ELEM.	*Exact Solution*	*BFGS*	*S.A.*	*ZOOM*
1	***0,5***	***0,2371***	***0,5233***	***0,5004***
2	*0,0*	*0,0140*	*0,0356*	*0,0037*
3	*0,0*	*0,0439*	*0,0201*	*0,0345*
4	*0,0*	*0,0184*	*0,0120*	*0,0121*
5	*0,0*	*0,0135*	*0,0099*	*0,0316*
6	*0,0*	*0,0305*	*0,0146*	*0,0383*
7	*0,0*	*0,0910*	*0,0140*	*0,0548*
8	*0,0*	*0,0000*	*0,0343*	*0,0225*
9	*0,0*	*0,0000*	*0,0055*	*0,0115*
10	*0,0*	*0,0331*	*0,0356*	*0,0055*
11	***0,5***	***0,4846***	***0,5069***	***0,5083***
12	*0,0*	*0,0000*	*0,0364*	*0,0154*
13	*0,0*	*0,0007*	*0,0322*	*0,0000*
14	*0,0*	*0,0038*	*0,0135*	*0,0240*
15	*0,0*	*0,0006*	*0,0436*	*0,0013*

In all the above examples the local optimization method (BFGS) presented the lowest level of computational effort but it was poor in correctly detect the location and extent of damage. The global methods worked better than the local one and both performed similarly considering only the results accuracy. The zooming method consumed in average about 10% less computational effort.

6. CONCLUSIONS

The present work presents a method of structural damage assessment using system identification techniques, which results in an optimization problem. Depending on the problem the design space may be nonconvex and present several local minima. For this reason a global optimization method is required to detect damage correctly. The stochastic global optimization methods usually are very costly from the standpoint of computational effort. In this work a deterministic global optimization method was used to reduce the computational cost. Although in average that reduction was obtained, it was observed that a good advantage of this method was the fact that it can use any robust constrained local optimization method with minor modifications. The random methods such as genetic algorithms or simulated annealing usually depend on empirical values that may be problem dependent, which increases considerably the implementation work for a particular structure. The use of neural networks for damage detection as alternative to the method of parameter determination in the structural analytical model is currently in progress.

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