

FLIGHT CONTROL LAW DESIGN WITH AND WITHOUT PRE-DECOUPLING

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Abstract. *A study about how to design a control law that not only keep following a reference attitude but also maintains attitude decoupling due to the vehicle manoeuvres has been performed. It has been noticed that in order to achieve this objective it is necessary to use a linear coupled model to design the control law instead of the usual linear uncoupled model used to design the tracking control law. The design is performed directly by linear quadratic optimal control method. An alternative approach was also studied. In this alternative approach, the design first decouples the system and then the tracker is designed by the same linear quadratic optimal control method used in the first approach. Both designs were assessed with respect to flying qualities in the time domain and with respect to stability in the frequency domain. The robustness of both designs was also assessed. By the end of the work some suggestions about how to implement this control law has also been studied. In this work the design method is applied as a case study for the Brazilian VLS.*

Key words: *Attitude Control, Decoupling, Control Algorithm*

1. INTRODUCTION

It is well known that when a satellite launcher performs some angular manoeuvre a coupled response is obtained, that is, if the vehicle performs a roll and yaw manoeuvre this will cause also a response in pitch oscillation, a feature that is undesirable from the point of view of vehicle performance. The traditional control law design is not able to suppress this undesirable response since it is designed based on a decoupled linear model of the vehicle. In this way it is not possible to take into account this undesirable flight dynamics characteristic. So, in order to suppress this coupling it is necessary to design the control law based on a vehicle linear coupled model. The design will result in a control law that has the capability of decoupling the three axis, that is a roll and yaw manoeuvre will result in zero pitch response, a roll and pitch manoeuvre will result in zero yaw response and finally a pitch and yaw manoeuvre will result in zero roll response. The stability of the vehicle with this new control law will also be checked in order to guarantee that it has been preserved. Some other methods

of decoupling can also be investigated as in D’Azzo & Houpis (1988), Ridgely et al.(1985) , Speyer et al.(1984) and Sobel & Shapiro(1985). Another approach is to use eigenstructure assignment, as described for example, in Andry et al.(1983) et al. A very good design example can also be found in Stevens & Lewis (1992) and Park & Nagati(1997), that shows how to design with output feedback.

2. VEHICLE MATHEMATICAL MODEL

In order to design the extended control law it is necessary to use a linear coupled model, which can be obtained from the linearization of the non linear equations of the vehicle given by

$$\dot{v} = \frac{-Cn_\alpha \bar{q} S_{ref}}{mU_0} v - g \sin(\theta) \sin(\phi) + g \cos(\theta) \sin(\psi) \cos(\phi) - rU_0 + pw + \frac{F_{coy}}{m} r - \frac{2F_E}{m} \beta_y \quad (1)$$

$$\dot{w} = \frac{-Cn_\alpha \bar{q} S_{ref}}{mU_0} w - g \sin(\theta) \cos(\phi) - g \cos(\theta) \sin(\psi) \sin(\phi) - pv + U_0 q + \frac{F_{coz}}{m} q + \frac{2F_E}{m} \beta_z \quad (2)$$

$$\dot{p} = - \left[\frac{Cl_p \bar{q} S_{ref} D_{ref}^2}{2U_0 I_{xx}} + \frac{\dot{I}_{xx}}{I_{xx}} \right] p - \frac{(I_{zz} - I_{yy})}{I_{xx}} qr + \frac{4d_1 F_E}{I_{xx}} \beta_r \quad (3)$$

$$\dot{q} = \frac{la_x Cn_\alpha \bar{q} S_{ref}}{I_{yy} U_0} w + \left[\frac{M_{coy}}{I_{yy}} - \frac{\dot{I}_{yy}}{I_{yy}} - \frac{Cm_q \bar{q} S_{ref} D_{ref}^2}{2U_0 I_{yy}} \right] q - \frac{(I_{xx} - I_{zz})}{I_{yy}} pr - \frac{2F_E lc_x}{I_{yy}} \beta_z \quad (4)$$

$$\dot{r} = - \frac{la_x Cn_\alpha \bar{q} S_{ref}}{I_{zz} U_0} v + \left[\frac{M_{coz}}{I_{zz}} - \frac{\dot{I}_{zz}}{I_{zz}} - \frac{Cm_q \bar{q} S_{ref} D_{ref}^2}{2U_0 I_{zz}} \right] r - \frac{(I_{yy} - I_{xx})}{I_{zz}} pq - \frac{2F_E lc_x}{I_{zz}} \beta_y \quad (5)$$

$$\dot{\theta} = \frac{\cos \phi}{\cos \psi} q - \frac{\sin \phi}{\cos \psi} r \quad (6)$$

$$\dot{\psi} = (\sin \phi) q + (\cos \phi) r \quad (7)$$

$$\dot{\phi} = p - (tg \psi \cos \phi) q + (tg \psi \sin \phi) r \quad (8)$$

where the parameters are:

U_0 Vehicle velocity in the x-body axis

Cn_α derivative of normal aerodynamic force with respect to angle of attack (α)

Cl_p derivative of roll damping moment with respect to roll rate (p)

Cm_q derivative of pitch or yaw (damping) moment with respect to pitch-rate, q or (yaw-rate, r)

\bar{q} dynamic pressure

S_{ref} vehicle reference area

D_{ref} vehicle reference diameter

m vehicle mass

d_1 control moment in roll arm

I_{xx} I_{yy} I_{zz} moments of inertia with respect to the x, y and z axis, respectively

la_x aerodynamic moment arm

lc_x control moment in pitch and in yaw arm

F_E thrust force

F_{coy} F_{coz} y and z axis coriolis force

M_{coy} M_{coz} y and z axis coriolis moment

The linear coupled model can be obtained in Greensite (1970). This model has been obtained from the linearization of these non linear equations for the states $U_0, v_0, w_0, p_0, q_0, r_0, \phi_0, \theta_0, \psi_0$. The model can be represented as follows

$$\dot{x} = Ax + Bu + Ex_{ref} \quad (9)$$

the E matrix is simply

$$E^T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

the state vector is given by

$$x^T = [w \ q \ \theta \ e_\theta \ v \ r \ \psi \ e_\psi \ p \ \phi \ e_\phi] \quad (11)$$

the control vector is given by

$$u^T = [\beta_z \ \beta_y \ \beta_r] \quad (12)$$

and the reference input vector is given by $x^T_{ref} = [\theta_{ref} \ \psi_{ref} \ \phi_{ref}]$ (13)

Now it is possible to notice that the coupling between the states has been taken into account. It is necessary to notice that it is required the previous knowledge of both, the vehicle trajectory and the vehicle parameters as a function of flight time in order to carry out this design.

3. DESIGN WITHOUT PRE DECOUPLING

This control law can be obtained either by optimal control method as described in Friedland (1986). In this way the control law gains has been obtained using the following performance index

$$J = \int_t^T (x^T Q x + \beta^T R \beta) d\tau \quad (14)$$

with a Q (diagonal weighting matrix) for the states and also a R(diagonal weighting matrix) for the controls. The resulting gains are obtained by

$$u = -R^{-1} B^T M_1 x - R^{-1} B^T M_2 x_{ref} \quad (15)$$

where M_1 is obtained from the steady state solution for the algebraic Ricatti equation

$$M_1 A + A^T M_1 - M_1 B R^{-1} B^T M_1 + Q = 0 \quad (16)$$

This equation will have a solution if the pair (A,B) is controllable. In this way the gains can be expressed by

$$G_{fb} = R^{-1} B^T M_1 \quad (17)$$

and

$$G_{ff} = R^{-1} B^T M_2 \quad (18)$$

with M_2 obtained from

$$M_2 = -(A_{CL}^T)^{-1} M_1 E \quad (19)$$

where A_{CL} is the closed loop matrix given by

$$A_{CL} = A - B G_{fb} \quad (20)$$

Then it will be possible to obtain a control law given by

$$u = -G_{fb} x - G_{ff} x_{ref} \quad (21)$$

where the gain matrices can be represented as

$$G_{fb}^T = [G_z \ G_y \ G_r] \quad \text{and} \quad G_{ff} = [G_{0z} \ G_{0y} \ G_{0r}]$$

that can be noticed in figure 1, where there is a block diagram of the vehicle with this control law. The design has been performed using the MATLAB (1987) software very easily. This

control law can be designed for the complete trajectory of the vehicle and the resulting gains can be scheduled as a function of the flight time in the same way as it was performed in the traditional design.

4. DESIGN WITH PRE DECOUPLING

Here, the design will be based on the approach developed by Falb & Wolovich (1967) and Wonham & Morse (1970) and shown in Brogan (1991). In a first step the feedback and feedforward gains for decoupling are found, then in a second step the feedback and feedforward gains for regulation and tracking are found. Considering a system described by

$$\dot{x} = Ax + Bu \quad (22)$$

with output given by :

$$y = Cx \quad (23)$$

and assuming that the number of inputs m is equal to the number of outputs the problem of reducing such a system to a decoupled system using a state feedback control law given by

$$u = -K_d x + F_d v \quad (24)$$

was studied in Falb & Wolovich (1967) and in Wonham & Morse (1970). The transfer function matrix for the system given by equations (22) and (23) with the state feedback given by equation (24) is

$$H(s) = C[sI - A + BK_d]^{-1} BF_d \quad (25)$$

The system will be decoupled if the matrices F_d ($m \times m$) and K_d ($m \times n$) are selected in such a way that the resulting $H(s)$ will be diagonal and non-singular. Following Brogan (1991), considering the inverse transform of equation (25) and the Cayley-Hamilton theorem applied to

$$e^{[A-BK_d]t} \quad (26)$$

an alternative statement of the decoupling problem is obtained, that is, the matrices,

$$C[A - BK_d]^j BF_d \quad j = 0, 1, \dots, n-1 \quad (27)$$

must all be diagonal for the system to be decoupled. By naming the i th row of C as c_i and defining a set of m integers by

$$d_i = \min\{j | c_i A^j B \neq 0, j = 0, 1, \dots, n-1\} \quad \text{or} \quad d_i = n-1 \quad \text{if} \quad c_i A^j B = 0 \quad \text{for all } j$$

The system can be decoupled using equation (24) if and only if the N matrix ($m \times m$) given by (28) is non-singular,

$$N^T = [c_1 A^{d_1} B \quad c_2 A^{d_2} B \quad \dots \quad c_m A^{d_m} B] \quad (28)$$

One set of decoupling matrices is given by

$$F_d = N^{-1} \quad (29)$$

and

$$M^T = [c_1 A^{d_1+1} \quad \dots \quad c_m A^{d_m+1}] \quad (30)$$

$$K_d = N^{-1} M \quad (31)$$

So, finding F_d and K_d and using (24) the original system will be decoupled. Then for the resulting decoupled system the main control law can be designed.

To design the control system for tracking and decoupling it is first necessary to find the K_d and F_d matrices. Considering the system given by equation (22) and the decoupling control law given by equation (24) the decoupled closed loop system is given by

$$\dot{x} = (A - BK_d)x + BF_d v \quad (32)$$

In order to design the tracking system it is necessary the inclusion of the reference state, and

$$G = [R^{-1}B_1^T M_1; R^{-1}B_1^T M_2] \quad (43)$$

with

$$G_1 = R^{-1}B_1^T M_1 \quad (44)$$

and

$$G_0 = R^{-1}B_1^T M_2 \quad (45)$$

where the M_1 and M_2 matrices are given by

$$M_1 A_1 + A_1^T M_1 - M_1 B_1 R^{-1} B_1^T M_1 + Q = 0 \quad (46)$$

$$M_2 = -(A_{CL}^T)^{-1} M_1 E_1 \quad (47)$$

with A_{CL} given by

$$A_{CL} = A_1 - B_1 R^{-1} B_1^T M_1 \quad (48)$$

5. CASE STUDY

5.1 Vehicle data

To study how the application of both designs will perform working on a vehicle, the data of the Brazilian satellite launcher (VLS) was used. Using an arbitrary point in the vehicle trajectory, the following values are obtained for the A and B matrices :

$$A = \begin{bmatrix} -0.0162 & 87.9 & -9.48 & 0 & 0.0006 & 0 & -2.013 & 0 & -0.687 & 0.399 & 0 \\ 0.0022 & 0.0148 & 0 & 0 & 0 & -0.0005 & 0 & 0 & 0.0042 & 0 & 0 \\ 0 & 0.98 & 0 & 0 & 0 & -0.2084 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0006 & 0 & -2.11 & 0 & -0.0162 & -87.9 & 9.47 & 0 & -1.965 & 1.3272 & 0 \\ 0 & 0.0005 & 0 & 0 & -0.0022 & 0.0151 & 0 & 0 & -0.0024 & 0 & 0 \\ 0 & 0.2078 & 0 & 0 & 0 & 0.9782 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0289 & 0 & 0 \\ 0 & 0.0704 & 0 & 0 & 0 & -0.015 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 10.87 \\ 0 & 0 & 4.08 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -10.87 & 0 \\ 0 & 4.08 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 25.89 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Note that these matrices can be taken at any point of the vehicle trajectory, that is, one can take the values of the states at any time and obtain the A and B matrices for that particular time.

5.2 Design Parameters

The design parameters used in both designs were the following:

$Q = \text{diag} (0, 0.1, 0.1, 1, 0, 0.1, 0.1, 1, 0.1, 0.1, 1)$, for the design without pre decoupling

$Q = \text{diag} (0, 0.1, 0.1, 5, 0, 0.1, 0.1, 1, 0.1, 0.1, 5)$, for the design with pre decoupling

$R = \text{diag} (0.1, 0.1, 0.1)$, for both designs

The above choices were obtained by observing the system performance after several simulations, with respect to stability and flying qualities performance. The detailed design for the case without decoupling can be found in Oliva and Leite (Sep. 1998), and the detailed design with pre decoupling can be found in Oliva and Leite (Nov.1998). A comparison of the obtained gains showed that the design with pre decoupling resulted in gains of higher magnitude than the gains obtained for the design without pre decoupling. For this, it can be expected that the control effort required by the design with pre decoupling will probably be higher than the corresponding control effort for the design without pre decoupling. This fact can also lead to a worst flight condition in the event of a sensor failure.

6. STABILITY CHARACTERISTICS

An assessment of the stability performance in the frequency domain was carried out in accordance with the procedure outlined in Stevens and Lewis (1992), as also in Maciejowski (1989). In figures 3 and 4 the singular values plots are reported for the design with pre decoupling, with and without feedforward. Similar results were obtained for the design without pre decoupling. The analysis of these results showed that at low frequencies the lower singular value presented by the design with feedforward is larger than the lower singular value given by the design without feedforward, and so the design with feedforward is able to give a better robustness with respect to performance. By the other side at high frequencies, the higher singular value presented by the design without feedforward is lower than the higher singular value of the design with feedforward. In this case the design without feedforward is able to give a better robustness with respect to stability.

The results for the design without pre-decoupling showed that at low frequencies the lower singular value presented by the design with feedforward is higher than the lower singular value presented by the design without feedforward. So, the design with feedforward will result with a better robustness with respect to performance than the design without feedforward. In the case of high frequencies the higher singular value presented by the design without feedforward is lower than the higher singular value presented by the design with feedforward. Then, here the design without feedforward will result with a better robustness with respect to stability than the design with feedforward.

From the reported analysis it is clear, in both cases, that the design without feedforward is giving a better performance than the design with feedforward, since in the case of low frequencies the differences between both design are very small. The analysis of the results also showed the following:

At low frequencies the lower singular value presented by the design without pre decoupling is a little bit larger than the lower singular value presented by the design with pre decoupling. So the design without pre decoupling will result with a robustness with respect to performance a little bit better than the design with pre decoupling. At high frequencies the higher singular value of the design without pre decoupling is lower than the higher singular value of the design with pre decoupling. Then, the design without pre decoupling will result with an improved robustness with respect to stability regard to the design with pre decoupling.

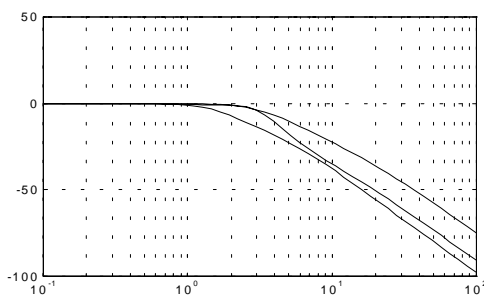


Figure 3 - Singular Values for the Design with pre decoupling and without feedforward

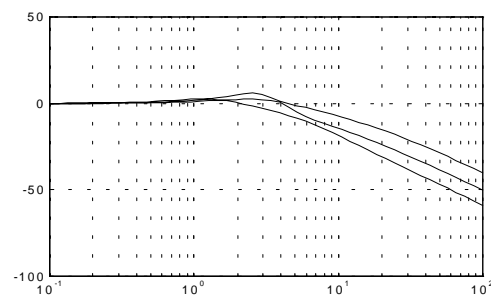


Figure 4 - Singular Values for the Design with pre decoupling and with feedforward

7. TRACKING PERFORMANCE

In figures 5 and 6 the tracking response of both designs is reported, where the symbol PD will mean pre decoupling. From these two figures it was noticed that the responses given by the

design without PD are very close to each other in the three channels, as previewed by the singular value analysis. By the other side the responses returned by the design with PD are not so close to each other in the three channels.

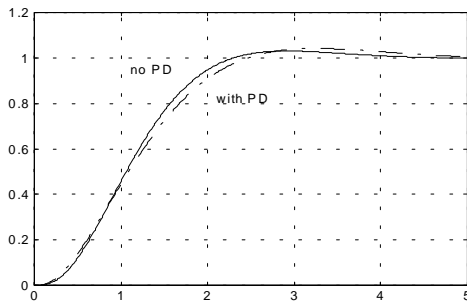


Figure 5 - Pitch-Attitude response after a pitch attitude step input - no feedforward.

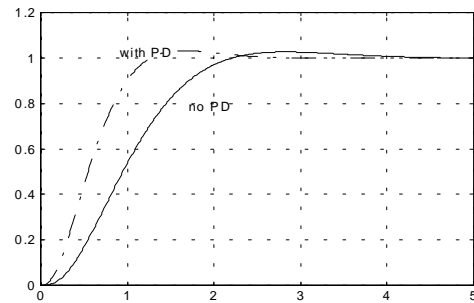


Figure 6 - Roll-Attitude response after a roll attitude step input - no feedforward.

8. DECOUPLING PERFORMANCE

In figures 7, 8, 9 and 10 the decoupling performance of both designs is reported. From these figures it was noticed that the design without PD showed a better decoupling response than the design with PD.

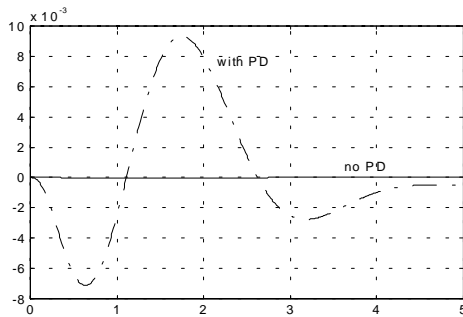


Figure 7- Pitch-Attitude response after a yaw attitude step input - no feedforward.

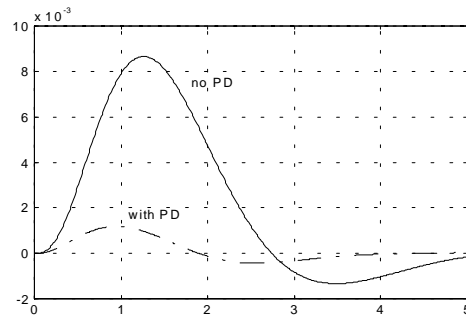


Figure 8 - Pitch-Attitude response after a roll attitude step input - no feedforward.

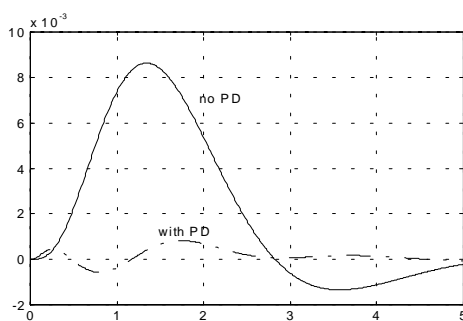


Figure 9 - Roll-Attitude response after a pitch attitude step input - no feedforward.

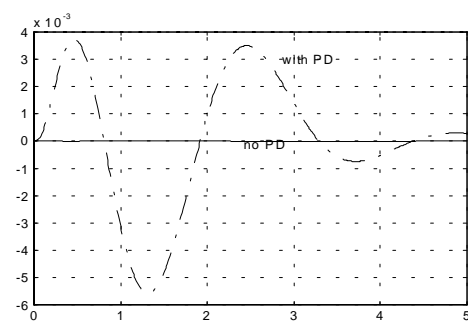


Figure 10 - Roll-Attitude response after a yaw attitude step input - no feedforward.

9. NON LINEAR RESPONSES

To access the robustness of both designs with respect to parameter variations, the obtained gains were implemented into a vehicle non-linear model, that is, a model where the parameters are varying with respect to flight time, and the same flight manoeuvres performed for the linear cases, reported in section 7 and 8 were executed.

9.1 Tracking Performance

In figures 11 and 12 the tracking performance of both designs were reported. From these figures it can be noticed again that the design without PD still giving a very uniform response in the three channels. The analysis of the results does not show any difference at all in the predicted performance with respect to tracking.

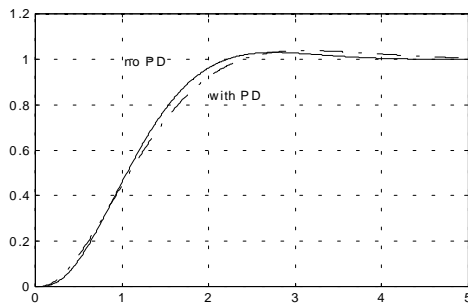


Figure 11 - Pitch-Attitude response for a pitch attitude step input

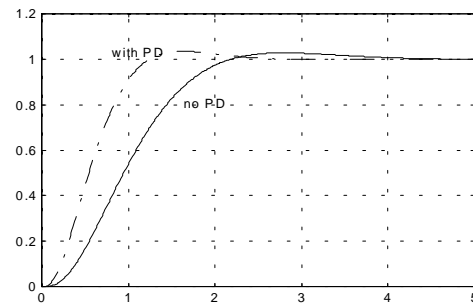


Figure 12 - Roll- attitude response for a roll attitude step input

9.2 Decoupling Performance

The decoupling performance of the system working with the vehicle parameters varying was also studied. The study showed that in both cases the performance is deteriorated, however much more in the case without PD.

It was noticed that in the case of no PD the performance is maintained while in the case with PD the performance is deteriorated.

From the study it was noticed that the worst cases are those where the coupling was increased 10 times with respect to the linear case, order of 10^{-3} . However, due to the small coupling values obtained, the system still working quite well. It can also be remembered that in the simulations the gains were maintained fixed during the flight time, which does not represent the actual case. In fact, in the real system the gains will be obtained for each flight condition, and scheduled with flight time, which of course will offer a better performance with respect to that showed on the reported figures of this work.

10. CONTROL EFFORT

The control effort required to perform a tracking manoeuvre and also required to perform a decoupling manoeuvre was studied for both designs, with the following results:

10.1 Control effort required for tracking manoeuvre

The control effort required to perform a tracking manoeuvre was analysed, and it was noticed that the design without pre decoupling is requiring a lower control effort than the design with pre decoupling, as previewed before.

10.2 Control effort required for decoupling manoeuvre

The analysis of the control effort required for decoupling showed that the design without pre decoupling offers a better performance than the design with PD. In any case in both designs the control effort required during the decoupling is very small.

11. CONCLUSIONS AND COMMENTS

From the performed study it can be concluded that the control law designed without pre decoupling can offer a better performance than the control law designed with pre decoupling with respect to all features and mainly regard to stability robustness. It is also much easier to carry on the design without pre decoupling than with pre decoupling considering the design work. It can also be notice from the obtained gain matrices, that both control laws can be used with some simplification, that is, some of the small gains can be neglected without affecting the original performance. Of course as showed in figures 18-B and 16-B the control law design with pre decoupling has not an acceptable degree of stability robustness. So, the design without pre decoupling is much more attractive in all aspects.

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