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HYBRID APPROACH FOR TRANSIENT FORCED CONVECTION IN FULLY-DEVELOPED TURBULENT FLOW INSIDE PARALLEL-PLATE CHANNELS

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Abstract. *Transient turbulent forced convection of Newtonian fluids within parallel-plates channels subjected to third kind boundary conditions and periodic variation of inlet temperature is considered. The well-established Cebeci-Smith's turbulence model is used in conjunction with turbulent Prandtl number models, valid for a wide molecular Prandtl number range for the closure of the forced convection formulation. A hybrid approach is employed to handle a class of parabolic-hyperbolic problem, that appears in connection with transient forced convection in channels. A integral transformation process eliminates the independent variables in which the diffusion phenomena occurs, while the resulting system of coupled hyperbolic equations is solved by a second-order explicit finite-difference scheme. Results are presented for average fluid temperature and wall heat flux over a wide range of the dimensionless axial coordinate.*

Keywords: Turbulent forced convection, Integral transforms, Algebraic turbulence models, Transient convection

1. INTRODUCTION

Studies on transient forced convection in channel flow define a very broad field in heat transfer research. Several procedures available in the literature have been developed in order to represent important characteristics of the heat flow. However, they are concentrated mainly on laminar fluid flow situations (Cotta et al., 1986, Sucec and Radley,1990, Yan, 1993, Guedes and Ozisik, 1994).

Some early attempts to solve unsteady turbulent heat transfer in channels with periodically varying inlet temperature focused the solution only on the periodic thermal response of the system, after the initial transients have disappeared (Cotta, Kim and Ozisik, 1989; Guedes et al., 1994). The integral transform technique was employed to transform the original problem in a system of coupled ordinary differential equations in the complex domain due to the periodic regime, which could then be readily solved analytically. Cotta and Gerk (1994) developed a mixed finite-difference and integral transform approach used to solve parabolic-hyperbolic partial differential equations that appear in transient laminar forced convection. The procedure consists of handling the original partial differential equation by using the integral transform technique to eliminate the independent variables in which the diffusion phenomena predominates. Then, the transformed coupled hyperbolic equations system is solved numerically through a modified upwind second-order finite difference scheme.

In the present study, a hybrid solution is developed for transient turbulent forced convection through a parallel-plates channel with time varying inlet temperature and third kind boundary conditions. The well-established Cebeci-Smith's turbulence model is used in conjunction with a turbulent Prandtl number model in formulating the forced convection equations. The goal is to extend the study undertaken by Guedes et al. (1994), developed for periodic turbulent forced convection, following the fully transient analysis of Guedes and Ozisik (1994) for the laminar flow situation.

2. ANALYSIS

Consider unsteady turbulent forced convection of a incompressible, fully developed Newtonian flow in the thermal entrance region of a parallel-plates channel subjected to third kind boundary conditions and periodic time variation of the inlet temperature. The mathematical formulation of the problem in dimensionless form is similar to one presented by Guedes and Ozisik (1994), including the turbulent thermal diffusivity, ε_h :

$$\frac{\partial \theta}{\partial \tau} + U_{fd} \frac{\partial \theta}{\partial Z} = \frac{\partial}{\partial Y}\left[(1+\varepsilon_h) \frac{\partial \theta}{\partial Y} \right], \quad 0 < Y < 1 \;, \quad \tau > 0, \quad Z > 0 \tag{1.1}$$

with the initial, inlet and boundary conditions given respectively by:

$$\theta(Y,Z,0) = \theta_0 \tag{1.2}$$

$$\theta(Y,0,\tau) = \sin(\Omega\tau) \tag{1.3}$$

$$\frac{\partial \theta}{\partial Y} = 0 \qquad Y = 0 \tag{1.4}$$

$$Bi\theta + \frac{\partial \theta}{\partial Y} = 0, \qquad Y = 1 \tag{1.5}$$

The turbulent thermal diffusivity is defined as

$$\varepsilon_h = \frac{\Pr \varepsilon_{fd}}{\Pr_t \nu} \tag{2}$$

where ε_{fd} is the turbulent viscosity and Pr_t is the turbulent Prandtl number.

The turbulent thermal diffusivity is represented by using the well-established Cebeci-Smith's turbulence model in conjunction with a turbulent Prandtl number model (Pimentel et al., 1996) in order to close the turbulent forced convection equations. The velocity field is obtained as described in Pimentel et al. (1999).

The first step in the solution of this problem, as proposed by Cotta and Gerk (1994) for laminar flow, is to apply the integral transform technique (GITT) on Eq. (1.1) to eliminate the independent variables in which the diffusion phenomena occurs. Then the resulting system of coupled hyperbolic equations is solved by a second-order explicit finite-difference scheme.

3. INTEGRAL TRANSFORM TECHNIQUE

The basic steps in applying the integral transform technique to the solution of the system (1.1 – 1.5) are presented in Cotta (1993).

The following auxiliary problem is chosen, which is a special case of the classical Sturm-Liouville system:

$$\frac{d}{dY}\left[(1+\varepsilon_h)\frac{d\,\Gamma_i}{dY}\right]+\mu_i^2\Gamma_i=0\,,\qquad 0<Y<1 \tag{3.1}$$

with the following boundary conditions

$$\frac{d\,\Gamma_i}{d\,Y}=0,\qquad Y=0 \tag{3.2}$$

$$Bi\Gamma_i+\frac{d\,\Gamma_i}{d\,Y}=0,\qquad Y=1 \tag{3.3}$$

The sign-count method (Cotta, 1993) is used to solve the eigenvalue problem above and determine as many eigenvalues and eigenfuctions as needed for convergence of the integral transform solution (Cotta, 1993).

By utilizing the eigenfunction orthogonality property, we define the following integral transform pair:

$$\bar{\theta}_i(Z,\tau)=\int_0^1\bar{\Gamma}_i(Y)\theta(Y,Z,\tau)\,dY \qquad \textit{Transform} \tag{4.1}$$

$$\theta(Y,Z,\tau)=\sum_{i=1}^{\infty}\bar{\Gamma}_i(Y)\bar{\theta}_i(Z,\tau) \qquad \textit{Inverse} \tag{4.2}$$

where $\bar{\Gamma}_i=\frac{\Gamma_i}{N_i^{1/2}}$ are the normalized eigenfunctions and the normalization integral is given by

$$N_i=\int_0^1[\Gamma_i]^2dY \tag{5}$$

We now operate on Eq. (1.1) with the operator $\int_0^1\bar{\Gamma}_i\;dY$ to obtain the following infinite hyperbolic partial differential system:

$$\frac{\partial\bar{\theta}_i}{\partial\tau}+\sum_{k=1}^{\infty}A_{ik}\frac{\partial\bar{\theta}_k}{\partial Z}+\mu_i^2\bar{\theta}_i=0\,,\qquad \tau>0\,,\quad Z>0 \tag{6.1}$$

and the transformed inicial and inlet conditions are given respectively by:

$$\bar{\theta}_i(Z,0)=\int_0^1\theta_0\,\bar{\Gamma}_i(Y)\,dY \tag{6.2}$$

$$\bar{\theta}_i(0,\tau)=\int_0^1\sin(\Omega\tau)\bar{\Gamma}_i(Y)\,dY \tag{6.3}$$

and

$$A_{ik}=\int_0^1 U(Y)\bar{\Gamma}_i(\mu_i,Y)\bar{\Gamma}_k(Y)\,dY \tag{6.4}$$

The resulting system of coupled hyperbolic equations, Eqs.(6.1 – 6.3) is solved by a second-order explicit finite-difference scheme based on a modification to the upwind method for the classical wave equation. This method was developed by Beam and Warming (1976). The proposed scheme involving a predictor-corrector procedure is given by:

Predictor

$$\bar{\theta}_{i,j}^{\overline{n+1}}=\bar{\theta}_{i,j}^{n}-\lambda\sum_{k=1}^{N}A_{ik}\left(\bar{\theta}_{i,j}^{n}-\bar{\theta}_{i,j-1}^{n}\right)-\mu_i^2\Delta\tau\bar{\theta}_{i,j}^{n} \tag{7.1}$$

Corrector

$$\bar{\theta}_{i,j}^{n+1}=\frac{1}{2}\Bigg[\bar{\theta}_{i,j}^{n}+\bar{\theta}_{i,j}^{\overline{n+1}}-\lambda\sum_{k=1}^{N}A_{ik}\left(\bar{\theta}_{k,j}^{\overline{n+1}}-\bar{\theta}_{k,j-1}^{\overline{n+1}}\right)$$
$$-\lambda\sum_{k=1}^{N}A_{ik}\left(\bar{\theta}_{k,j}^{n}-2\bar{\theta}_{k,j-1}^{n}+\bar{\theta}_{k,j-2}^{n}\right)-\mu_i^2\Delta\tau\bar{\theta}_{i,j}^{\overline{n+1}}\Bigg] \tag{7.2}$$

with

$$\lambda=\frac{\Delta\tau}{\Delta Z} \tag{7.3}$$

where the subscripts i and k represent terms in the eigenfunction expansion, j is related to the space coordinate discretization and n to the time step advanced. $\Delta\tau$ is the time step, ΔZ the spatial mesh size in the finite difference scheme and N is the truncation order of the eigenfunction expansion. The present scheme is second order accurate in both the time and space variables, as demonstrated by Cotta and Gerk.(1994).

The stability of this scheme is given through the Courant number (γ), according to the restriction

$$0 \le \gamma_{max} \le 2 \tag{8.1}$$

where

$$\gamma_{max} = c_{i\,max} \frac{\Delta\tau}{\Delta Z} \tag{8.2}$$

and

$c_{i\,max}$ is the maximum eigenvalue of A^* and

$$A^*(\tau) = \left\{A^*_{ik}(\tau)\right\} = A_{ik} \exp\left[\left(-\mu_k^2 - \mu_i^2\right)\tau\right] \tag{8.3}$$

4. RESULTS AND DISCUSSION

The numerical results obtained through the strategy analyzed in the previous section are discussed below. The computational code was written in Fortran 77 and implemented in a Pentium 166 MHz microcomputer. All the cases were evaluated with $\theta_0 = 0$ and first type boundary condition. In the comparison with previous laminar flow results a step change at the inlet temperature was used, while in the turbulent flow situation a sinusoidal time variation of the inlet temperature was considered. It should be noted that the following Z dimensionless coordinate makes the results, in terms of Z, independent of Reynolds number for the laminar situation, while for the turbulent flow case, a typical value for the Reynolds number (Re=1.0E+05) was adopted.

Figure 1 shows the average temperature plotted as a function of the axial coordinate Z, at different times τ, for the laminar flow case with a step change at the inlet temperature. Comparison with the numerical results presented by Cotta et al. (1986) clearly confirms the validity of the present code.

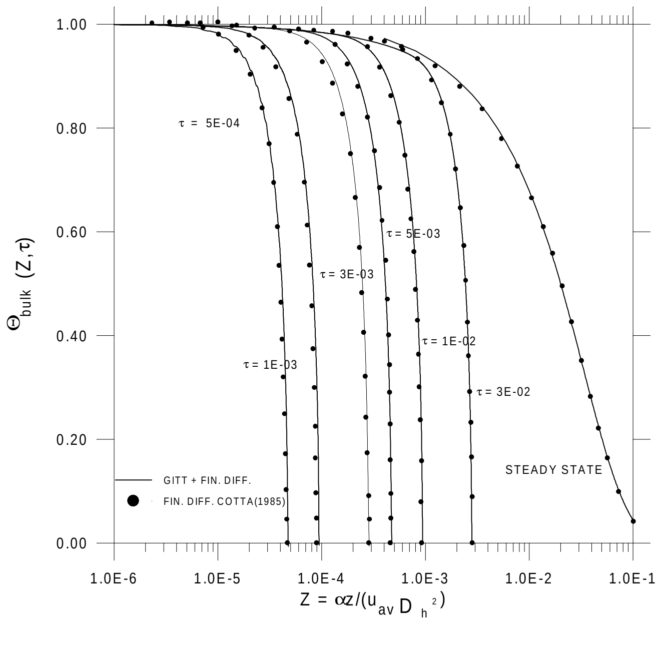

Figure 1 – Bulk temperature distribution in laminar flow for a step change in inlet temperature. Comparison with purely numerical solution in Cotta et al. (1986).

Similarly, Fig. 2 shows the comparison between the local Nusselt number calculated from a second-order accurate numerical solution (Cotta et al., 1986) and the results obtained with the present hybrid approach. Again, a perfect validation of the proposed hybrid scheme and computer code is achieved.

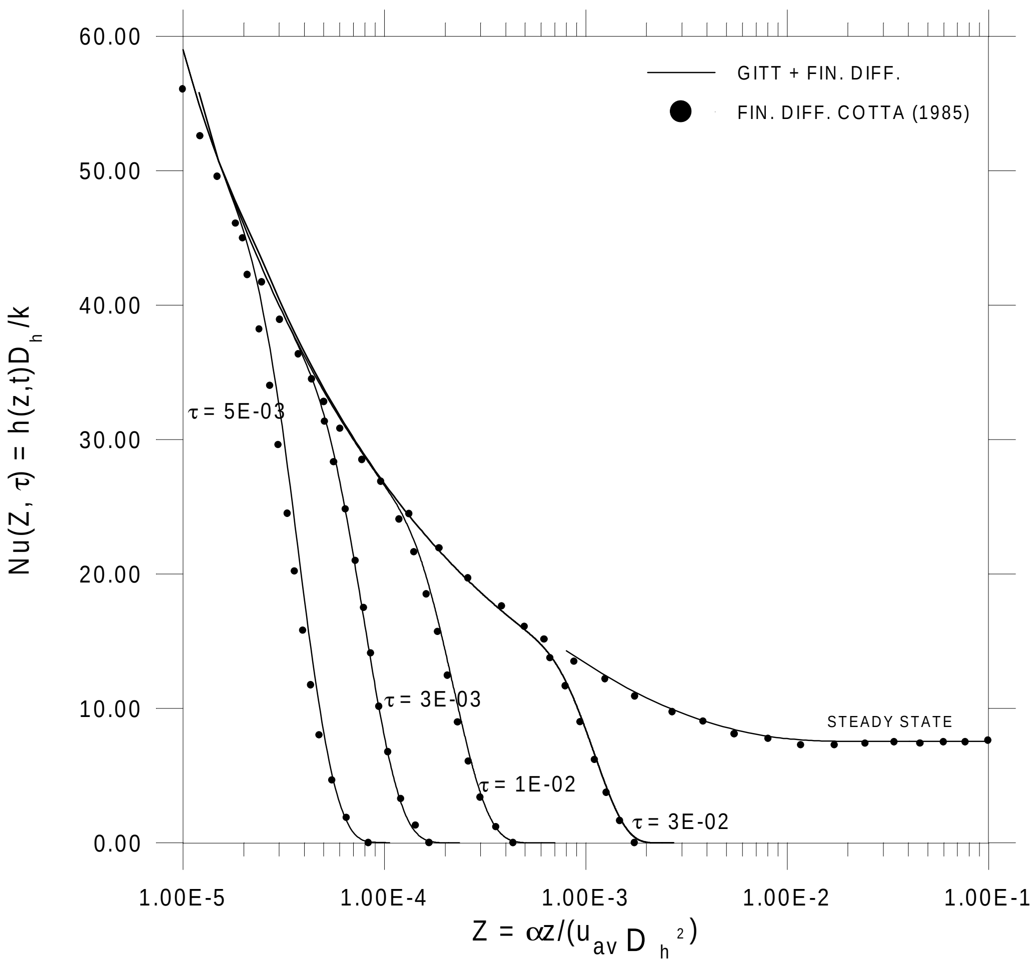

Figure 2 – Nusselt number distribution in laminar flow for a step change in inlet temperature. Comparison with purely numerical solution in Cotta et al. (1986).

Table 1 presents the convergence behaviour of dimensionless bulk temperature and wall heat flux at several axial coordinate and time values for transient turbulent forced convection and sinusoidal time variation of inlet temperature.

One can notice that the eigenfunction expansion is already fully converged to four significant digits for truncation order $N \cong 15$, within the range of independent variables considered, for both quantities of practical interest.

The axial coordinate and time increments were also carefully varied and analyzed, to ensure the four significant digits reported, with ΔZ as low as 5E-04 and $\Delta\tau$ as low as 1E-05.

Table 1 – Convergence of dimensionless bulk temperature and wall heat flux for transient turbulent forced convection and sinusoidal variation of inlet temperature with $\Omega = 0.5$.

N	Z = 0.005		Z = 0.01		Z = 0.02		Z = 0.05	
	θ_{bulk}	Heat Flux	θ_{bulk}	Heat Flux	θ_{bulk}	Heat Flux	θ_{bulk}	Heat Flux
	$\tau = 0.50$							
5	0.2228	-11.18	0.2040	-9.787	0.1710	-8.082	0.1007	-4.756
10	0.2229	-11.19	0.2041	-9.777	0.1711	-8.092	0.1008	-4.762
12	0.2229	-11.18	0.2041	-9.777	0.1711	-8.091	0.1008	-4.762
15	0.2229	-11.18	0.2041	-9.776	0.1711	-8.090	0.1008	-4.761
17	0.2229	-11.18	0.2041	-9.776	0.1711	-8.090	0.1008	-4.761
	$\tau = 1.00$							
5	0.4320	-21.69	0.3959	-18.96	0.3324	-15.71	0.1968	-9.295
10	0.4323	-21.70	0.3961	-18.98	0.3326	-15.73	0.1969	-9.306
12	0.4323	-21.69	0.3961	-18.98	0.3326	-15.73	0.1969	-9.306
15	0.4323	-21.69	0.3961	-18.98	0.3326	-15.73	0.1969	-9.304
17	0.4323	-21.69	0.3961	-18.98	0.3326	-15.73	0.1969	-9.304
	$\tau = 2.00$							
5	0.7586	-38.08	0.6954	-33.31	0.5845	-27.63	0.3469	-16.39
10	0.7591	-38.10	0.6959	-33.35	0.5849	-27.67	0.3472	-16.41
12	0.7591	-38.10	0.6958	-33.34	0.5848	-27.67	0.3471	-16.41
15	0.7590	-38.10	0.6958	-33.34	0.5848	-27.66	0.3471	-16.41
17	0.7590	-38.10	0.6958	-33.34	0.5848	-27.66	0.3471	-16.41
	$\tau = 5.00$							
5	0.5399	-27.11	0.4953	-23.73	0.4170	-19.72	0.2487	-11.75
10	0.5402	-27.12	0.4956	-23.76	0.4172	-19.74	0.2488	-11.76
12	0.5402	-27.12	0.4956	-23.76	0.4172	-19.74	0.2488	-11.76
15	0.5402	-27.12	0.4956	-23.75	0.4172	-19.74	0.2488	-11.76
17	0.5402	-27.12	0.4956	-23.75	0.4172	-19.74	0.2488	-11.76
	$\tau = 7.50$							
5	-0.5151	25.86	-0.4721	22.61	-0.3965	18.75	-0.2349	11.10
10	-0.5155	25.87	-0.4724	22.64	-0.3968	18.76	-0.2351	11.11
12	-0.5154	25.87	-0.4724	22.63	-0.3968	18.77	-0.2351	11.11
15	-0.5154	25.87	-0.4723	22.63	-0.3967	18.77	-0.2351	11.11
17	-0.5154	25.87	-0.4723	22.63	-0.3967	18.77	-0.2351	11.11

The behaviour of the dimensionless bulk temperature at different axial positions can be viewed through Figs. 3a and 3b for two different inlet temperature oscillation frequencies. The results were obtained for turbulent flow with sinusoidal oscillation of the inlet temperature and first kind wall boundary conditions. The wall heat flux results are also represented in Figs. 4a and 4b. The expected physical behaviour, where the amplitudes decrease with increasing distance from the inlet is observed for both quantities. Besides, the comparison of the results in Figs. 3 and 4 indicates that the maximum heat flux occurs at minimum average temperature position and vice-versa.

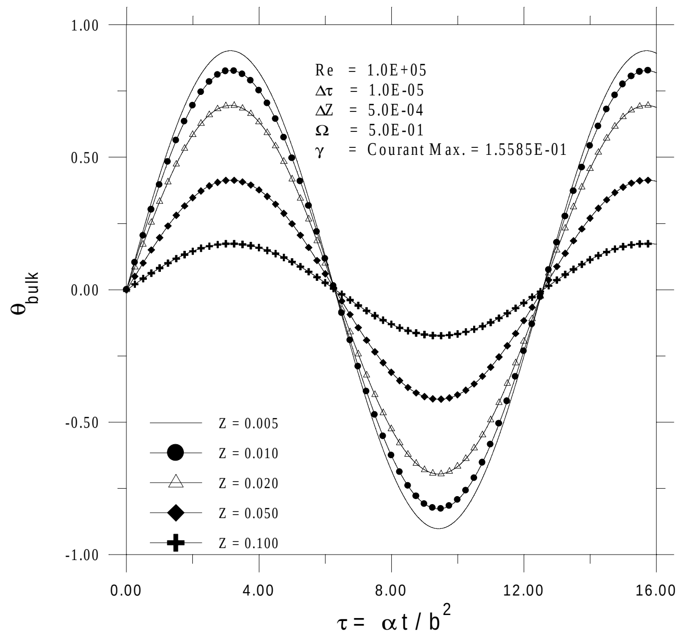

Figure 3a – Variation of the dimensionless bulk temperature as a function of time for Ω = 0.5.

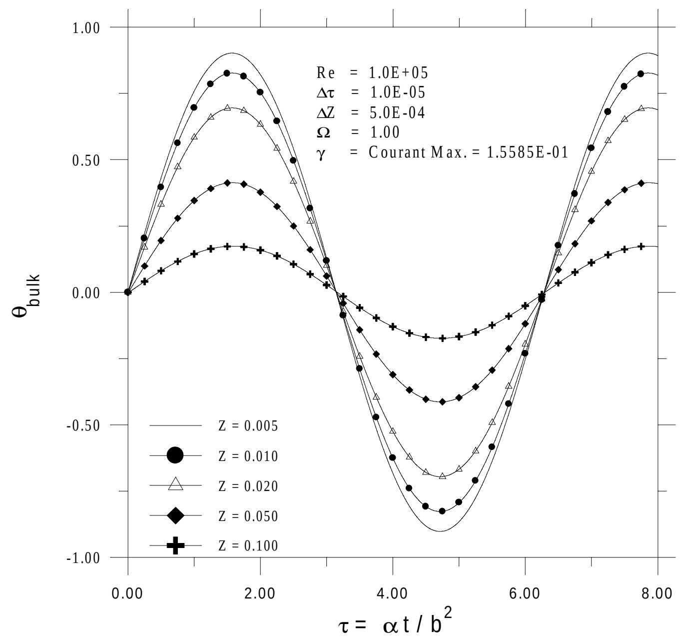

Figure 3b – Variation of the dimensionless bulk temperature as a function of time for Ω = 1.0

Comparing the behaviour of both sets of curves for the two different frequencies, it can be concluded that the periodic regime has been already attained within the time range considered. Then, the amplitudes decay is essentially the same for each axial position, while the phase lag with respect to the inlet temperature oscillation increases as the frequency is made larger.

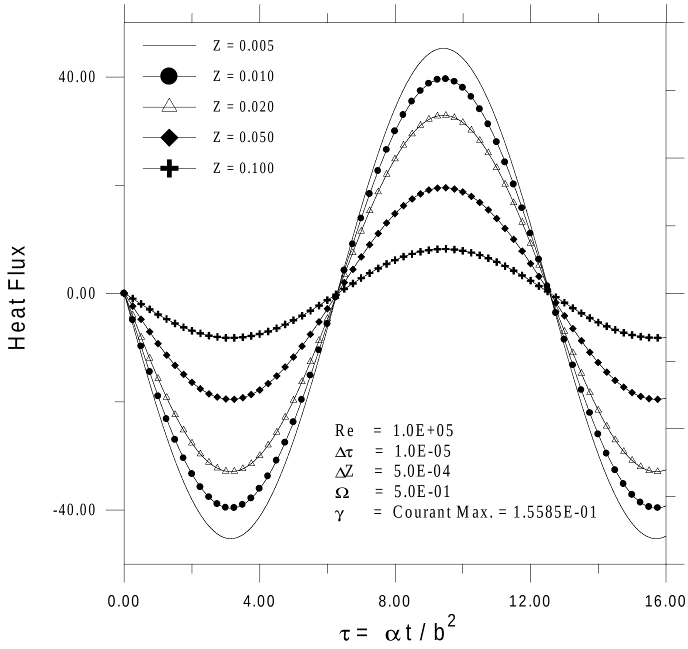

Figure 4a – Variation of the wall heat flux as a function of time for Ω = 0.5.

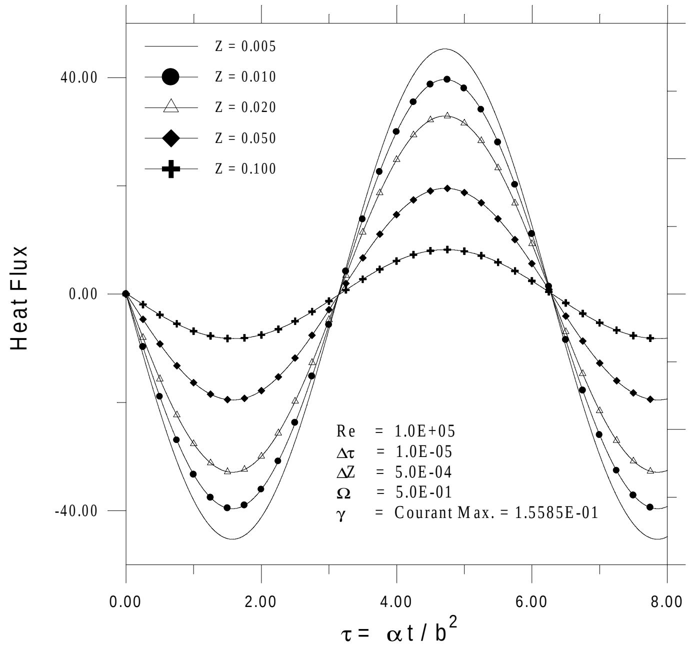

Figure 4b – Variation of the wall heat flux as a function of time for Ω = 1.0.

5. CONCLUSIONS

A hybrid integral transform/finite difference approach was advanced to accurately handle transient turbulent forced convection in channels. The diffusion operator is eliminated from the original partial differential equation through integral transformation, while the resulting hyperbolic system is numerically handled through an explicit second-order accurate modified upwind scheme. This strategy allows for a more reliable stability control of the numerical solution process, avoiding the necessity for a domain discretization in the normal direction, also improving the overall accuracy of the computation.

The procedure is first validated against the laminar flow situation and the code is employed to study more closely transient turbulent forced convection due to a sinusoidal time variation of the fluid inlet temperature, when the expected physical trends are observed.

Future work shall include the extension of the analysis and code to deal with rough channel walls and thermal capacitance effects at the duct walls.

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