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THE EFFECT OF THE FIBER ORIENTATION ON THE DYNAMIC BEHAVIOUR OF ROTORS IN WOUNDING-SHAFT

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Abstract. *The purpose of this work is to analyse the dynamic behaviour of simple-supported rotors in which its shaft are made of fibre/resin in a wounding process. The orientation of the wounding angle is an important parameter on determing the properties of the section such as the equivalent bending stiffness <EI> and the equivalent torsional stiffness <GJ>, which can modify the strain energy of the shaft. The Campbell Diagram, in which the bending and torsion modes of the rotor is included, can be appreciably changed with the evolution of the orientation of the wounding angle. This analysis can be used in an optimisation process maximizing the critical velocities distance from the operating rotation of the rotor.*

Keywords: *Rotor, Composite, Homogeneity, Campbell Diagram.*

1. INTRODUCTION

In rotordynamics prediction, we currentily employ the finite element method to analyse this type of structure in searching the undamped natural frequencies (Campbell Diagram), the steady-state response to unbalance, the transient response to unbalance and external driving forces as shown in Rossi *et al.* (1989). Nelson *et al.* (1976) and Özgüven *et al.* (1984) using the finite element method introduced different effects as the rotatory inertia, gyroscopic moments, axial load, etc. Steffen *et al.* (1987) linked a finite element package with an optimization program in order to perform the optimal structural design with the purpose of to maximizing the distance of the critical speeds from it-selves. In these studies seen earlier the material of the shaft is considered isotropic.

In this work it is introduced a new parameter in rotordynamics analysis given by the fiber orientation in the case of rotors in which the shaft is made of fiber/resin in a wounding process. In this case, the homogeneity properties of the cross section of the shaft <EI> and <GJ> in determining the strain energy in bending and torsion are used. As a first approach, the Rayleigh-Ritz method is used and only the first mode in bending and in torsion are observed

The introdution of the parameter wounding angle of the shaft can modify the behaviour in bending and torsion of the rotor and this effect can be included in optimization techniques in searching the optimal design of the rotor machine.

2. THE EQUATIONS OF ENERGY OF THE ROTOR

2.1 The kinetic energy of the disc

From the Fig. 1, we can deduce the instantaneous vector of rotation in the reference coordinate system, Lalanne *et al.* (1986):

$$\vec{\omega} = \dot{\psi}\ \vec{z} + \dot{\theta}\ \vec{x}_1 + \dot{\phi}\ \vec{y} \tag{1}$$

where $\vec{z}$, $\vec{x}_1$, $\vec{y}$ are unitary vecteurs.

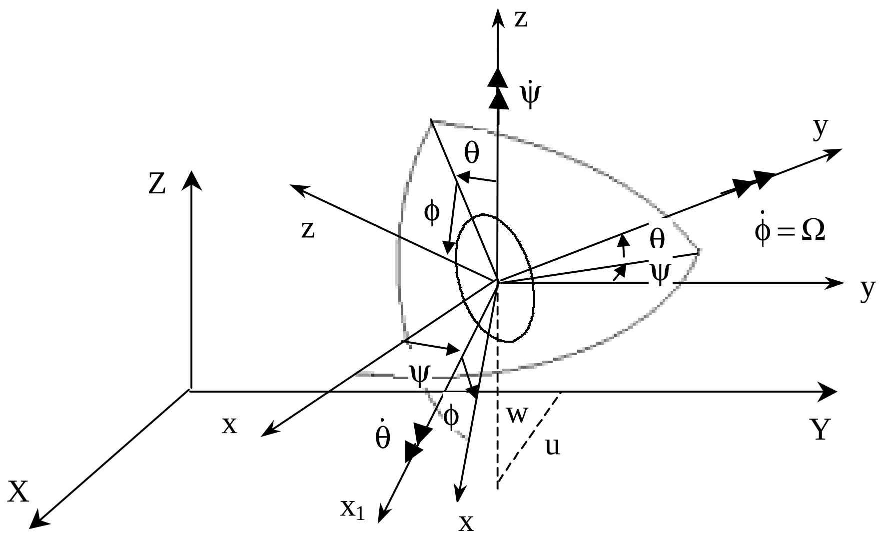

Figure 1- Reference coordinate system (x, y, z)

The angular velocity of the disc is $\dot{\phi}$ and the compounds of $\vec{\omega}$ in the reference coordinate system is:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -\dot{\psi}\cos\theta\,\text{sen}\,\phi + \dot{\theta}\cos\phi \\ \dot{\phi} + \psi\,\text{sen}\,\theta \\ \dot{\psi}\cos\theta\cos\phi + \dot{\theta}\,\text{sen}\,\phi \end{bmatrix} \tag{2}$$

The kinetic energy of the disc can be expressed by:

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}\left(I_{Dz}\ \omega_z^{\ 2} + I_{Dy}\ \omega_y^{\ 2} + I_{Dz}\ \omega_z^{\ 2}\right) \tag{3}$$

where u and w are the coordinates of the center of inertia of the disc and I_{Dx}, I_{Dy} and I_{Dz} are the inertia moments of the disc in the reference coordinate system. Taking in account that the angles θ and ψ are small, the velocity of rotation is $\dot{\phi} = \Omega$ and the symetry of the disc implies $I_{Dx} = I_{Dz}$, we may obtain from Eq. (3) that:

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}\ I_{Dx}\ \left(\dot{\theta}^2\ \ \dot{\psi}^2\right) + I_{Dy}\ \ \Omega\ \dot{\psi}\ \theta + \frac{1}{2}\ I_{Dy}\ \ \Omega^2 \tag{4}$$

From the application of the Eq. (4) in the Lagrange's equation, in which the generalised coordinates are u, w, θ, ψ , we can identify the inertia effect and the giroscopic effects of the disc.

2.2 The strain energy of the shaft in bending

The general expression for the strain energy is:

$$U = \frac{1}{2} \int_\tau \varepsilon^t\ \sigma\, d\tau \tag{5}$$

where $\sigma = E\ \varepsilon$:

Let be u* and w* be components of the displacement of a point P in the cross section in the reference coordinate system, Fig. 2. If we consider only the linear effect of the longitudinal strain:

$$\varepsilon_1 = -\,x\,\frac{\partial^2 u^*}{\partial y^2} - z\,\frac{\partial^2 w^*}{\partial y^2} \tag{6}$$

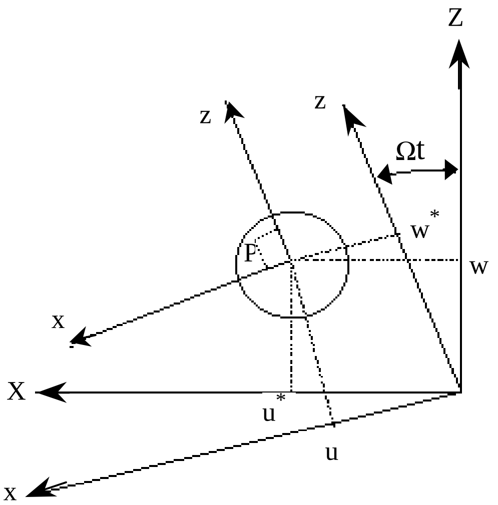

Figure 2 – The reference coordinate system of the shaft

The final expression of the strain energy is:

$$U = \frac{1}{2} \; E \int_{\tau} \left[- z \frac{\partial^2 w^*}{\partial y^2} - x \frac{\partial^2 u^*}{\partial y^2} \right]^2 d\tau \tag{7}$$

The displacements u* and w* in the global coordinate system are:

$$\begin{aligned} u^* &= - w \;\text{sen}\; \Omega\, t + u \;\cos \Omega\, t \\ w^* &= \;w \cos \Omega\, t + u \;\text{sen}\; \Omega\, t \end{aligned} \tag{8}$$

The Eq. (7) in terms of u and w is:

$$U = \frac{1}{2} \; E\, I \int_0^L \left[\left(\frac{\partial^2 w}{\partial y^2} \right)^2 + \left(\frac{\partial^2 u}{\partial y^2} \right)^2 \right] dy \tag{9}$$

From the application of the Eq. (9) in to the Lagrange's equation, in which the generalised coordinates are u, w, we can identify the stiffness of the shaft in bending.

2.3 The movement in torsion of the shaft

The general expression for the kinetic energy of the shaft in torsion is, Lalanne *et al.* (1986):

$$T = \frac{1}{2} \rho\, J \int_0^L \dot{\varphi}\, dy \tag{10}$$

where φ is the torsion angle. The general expression for the strain energy of the shaft in torsion is:

$$U = \frac{1}{2} G\, J \int_0^L \left(\frac{\partial \varphi}{\partial y} \right)^2 dy \tag{11}$$

By using Eq. (10) and Eq. (11) in the Lagrange's equations, in which the generalised coordinate is φ, we can deduce the movement of torsion of the shaft.

3. RAYLEIGH-RITZ METHOD

With a reasonable aproximation of the displacement field, we can deduce the equations of the movement in bending and in torsion using the Raileigh-Ritz Method. In this work, we search the first frequencies in bending and the first frequency in torsion for a simple-supported rotor, as shown in the Fig. 3.

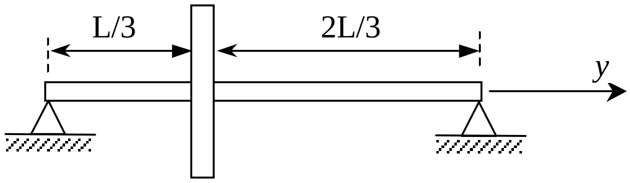

Figure. 3 – Rotor simple-supported on the ends

An approach for the displacement in bending for this configuration is:

$$u(y,t) = \sin\frac{\pi y}{L}\,p_1$$
$$w(y,t) = \sin\frac{\pi y}{L}\,p_2 \tag{12}$$

and,

$$\theta(y,t) = \frac{\partial w}{\partial y} = \frac{\pi}{L}\cos\frac{\pi y}{L}\,p_1$$
$$\psi(y,t) = -\frac{\partial u}{\partial y} = -\frac{\pi}{L}\cos\frac{\pi y}{L}\,p_2 \tag{13}$$

Using Eq. (12) and Eq. (13) in Eq. (4), the kinetic energy of the disc is given by:

$$T_D = \frac{1}{2}\left[M_D\sin^2\left(\frac{\pi y}{L}\right) + I_{Dx}\left(\frac{\pi}{L}\right)^2\cos^2\left(\frac{\pi y}{L}\right)\right]\left(\dot{p}_1{}^2 + \dot{p}_2{}^2\right) - I_{Dy}\,\Omega\left(\frac{\pi}{L}\right)^2\cos^2\left(\frac{\pi y}{L}\right)\dot{p}_2\,p_1$$

(14)

Using Eq. (12) in Eq. (9), the strain energy of the shaft may be written as:

$$U = \frac{1}{2} < EI > \left(\frac{\pi}{L}\right)^4\left[\int_0^L \sin^2\left(\frac{\pi y}{L}\right)\right] dy\left(p_1{}^2 + p_2{}^2\right) \tag{15}$$

The application of the Lagrange's equation in Eq. (14) and (15) leads to:

$$a\,\ddot{p}_1 + b\,\Omega\,\dot{p}_2 + c\,p_1 = 0$$
$$a\,\ddot{p}_2 - b\,\Omega\,\dot{p}_1 + c\,p_2 = 0 \tag{16}$$

with: $a = \left[M_D \sin^2\left(\frac{\pi}{L}\right) + I_{Dx}\left(\frac{\pi}{L}\right)^2 \cos^2\left(\frac{\pi}{L}\right)\right], b = I_{Dy} \left(\frac{\pi}{L}\right)^2 \cos^2\left(\frac{\pi}{L}\right),\ c = < EI > \left(\frac{\pi}{L}\right)^3 \left(\frac{\pi}{2}\right)$

The solution for the Eq. (16) are:

$$p_n = P_n\, e^{rt} \tag{17}$$

where $r = \pm\, j\, \omega(\Omega)$ are the natural frequencies in bending on each velocity of rotation of the rotor.

Using Eq. (17) in Eq. (16) we obtain the characteristic equation for the rotor in which the roots are the natural frequencies:

$$r^4 + \left(\frac{2c}{a} + \frac{b^2}{a^2}\Omega^2\right) r^2 + \frac{c^2}{a^2} = 0 \tag{18}$$

An approach for the displacement in torsion for this configuration is:

$$\varphi(y,t) = (ay).\, q \tag{19}$$

By replacing Eq. (19) in Eq. (10), the kinetic energy of the shaft is given as:

$$T = \frac{1}{2}\left(\frac{1}{3} J\rho L\right)\dot{q}^2 \tag{20}$$

By Using Eq. (20) in Eq. (13), the strain energy of the shaft is given as:

$$U = \frac{1}{2}\ \frac{< GJ >}{L} q^2 \tag{21}$$

The application of the Lagrange’s equation in Eq. (20) and (21) leads to:

$$\omega = \frac{1}{2\pi L}\sqrt{\frac{3 < GJ >}{J\,\rho}} \tag{22}$$

where ω is the natural frequency in torsion of the rotor.

4. HOMOGENEITY PROPERTIES OF THE SHAFT

In Eq. (15) and Eq. (22) we can identify the properties of the cross section of the shaft as the equivalent bending stiffness <EI> and the equivalent torsional stiffness <GJ>. In the case of rotors in which the shaft are made of fibre/resin in a wounding process, the orientation of the wounding angle can modify the strain energy of the shaft. So, the Campbell Diagram, can be appreciably changed with the evolution of the orientation of the wounding angle.

In this work we use the equivalent stiffness <EI> and <GJ> as a function of the wounding angle for a hollow shaft in kevlar/epoxy and glass/epoxy as shown in Pereira (1999). In this case the external diameter and the internal diameter of the wounding-shaft are 40 mm and 32 mm, and the lenght is 0,8 m. The configuration of the layers in the thickness direction of the shaft is $[\theta,-\theta]_4$.

As demonstred by Pereira (1999), the equivalent bending stiffness <EI> and equivalent torsional stiffness <GJ> can be seen in Fig. 4 for a kevlar/epoxy wounding-shaft and in Fig. 5 for a glass/epoxy wounding-shaft.

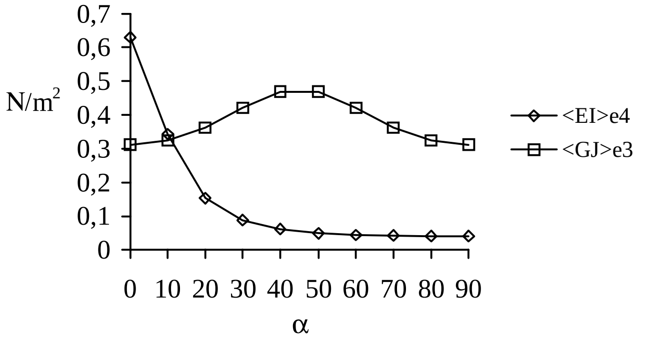

Figure 4 - Evolution of <EI> and <GJ> as function of the wounding angle α – kevlar/epoxy

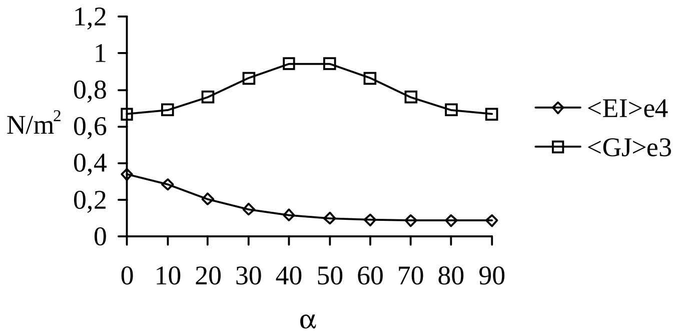

Figure 5 - Evolution of <EI> and <GJ> as function of the wounding angle α – glass/epoxy

5. APPLICATION AND CONCLUSIONS

In this section we plotted the Campbell Diagram for the simple-supported rotor shown in Fig. 3 for different wounding angles. The equivalent stiffness <EI> and <GJ> are from Fig. 4 and from Fig. 5. The properties of the disc are: I_{Dx} = 0.1225 kg.m^2, I_{Dy} = 0.245 kg.m^2 and M_D = 7,85 kg. The bending modes are in full lines and the torsional modes are in doted lines.

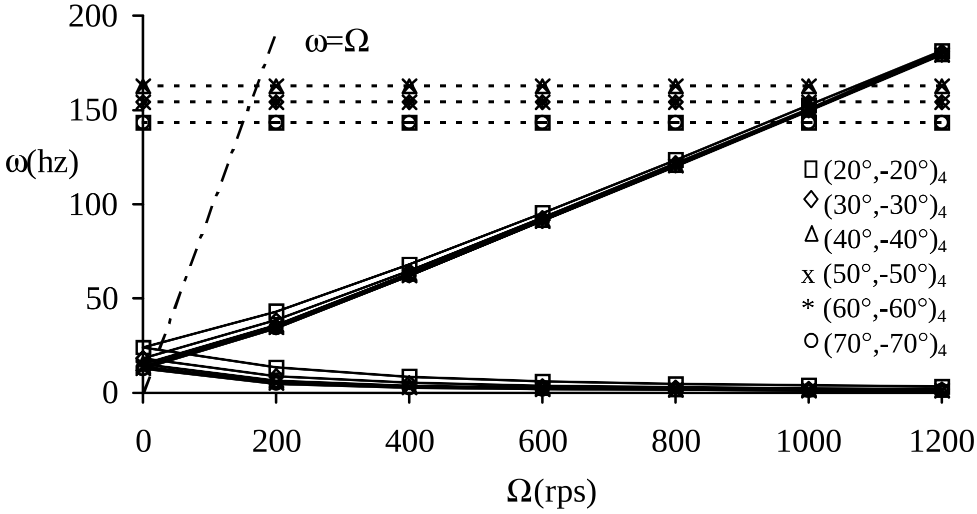

Figure 6 – Campbell Diagram - kevlar/epoxy wounding shaft

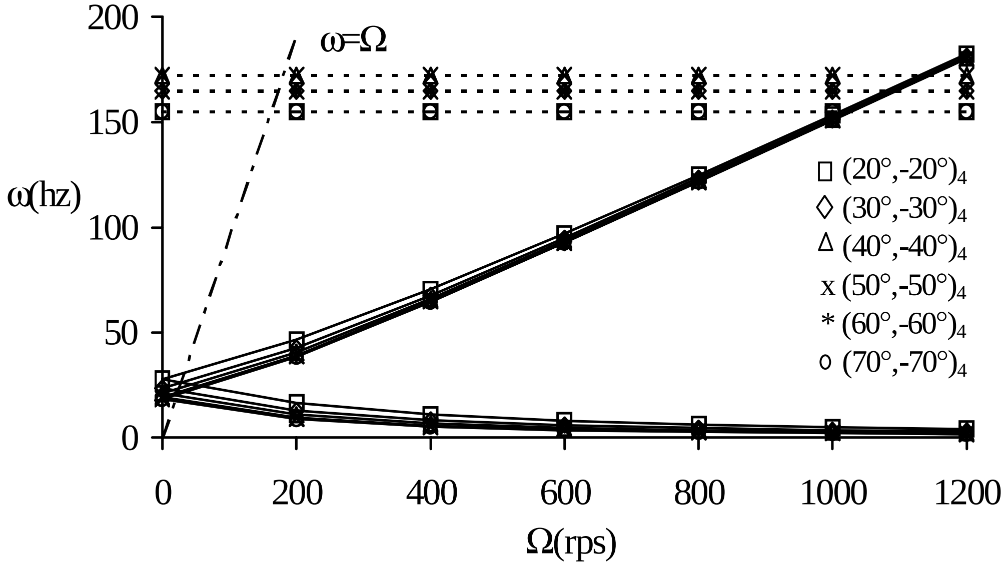

Figure 7 – Campbell Diagram - glass/epoxy wounding shaft

This work shows the application of composite materials in rotor dynamics. As we can see in the Fig. 6 and Fig. 7, the Campbell Diagram changes with the orientation of the wounding angle. This effect are more appreciably on low velocity of rotation Ω in bending modes. We can also identify a wounding angle in which we have the highest distance between the bending modes and the torsional modes on the same velocity of rotation Ω, let be 45°.

Using composite materials in rotor dynamic analysis can introduce additional design variables which may be searched for the optimal performance, as the stiffness and the damping of the material, the wounding angle, the number of layers and the weight.

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