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A REFINED MULTI-LAYER SHELL ELEMENT USING A MINDLIN´S THEORY EVOLUTION

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Abstract. *A multi-layer finite element adapted to thick shell structures is presented. The displacement components, which are linear by layer, allow a better representation of the shear transverse effects. The validation of the model, based on the comparison with un first order theory element, is made on an orthotropic square plate with different ply orientations and different boundary conditions*

Keywords: *Shell, Multi-layer, Composite Materials, Finite elements.*

1. INTRODUCTION

The theory in which Kirchoff propose the null transverse shear, provide the mechanical behaviour of isotropic thin shells. Nevertheless, when the structures are thick and/or multi-layer in composite materials, the effect of the transverse shear become important and in this case this classical theory is not adequate.

The transverse shear effect is taken into account in first order theories, Ahmad *et al.*, (1970), Vlachoutsis, (1987) and in higher order theories, Reddy, (1984), Phan *et al*, (1985). In fisrt order theories, the distribution of the transverse shear is constant through the thickness and need a correction factor, whereas in higher order theories, the transverse shear is parabolic, nevertheless it needs the introduction of additionals coefficients.

The purpose of this work is to propose a multi-layer shell element placed between the first order teories and the higher order teories. The displacement fields in this case is linear by layer and defined by the extension of Mindlin's theory over all layers away the neutral surface. A model to prevent the damping defined from an energetic aproximation, Adams *et al*, (1973) adapted to this multi-layer shell element is presented.

The contribution of this new formulation is shown in the comparison with the classical theory of laminates where we search the frequencies and the associated modal damping. In this study we use square plates of carbon/epoxy in two differents stacking of plies and two different boundary conditions.

2. THE ELEMENT FORMULATION

Here we propose an 8 nodes quadrilateral element with a quadratic approximation, Fig.1. The geometry of the element is defined as:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum_{i=1}^{8} N_i(\xi,\eta) \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix} + \sum_{i=1}^{8} N_i(\xi,\eta) \frac{\zeta}{2}\, h\, \vec{V}_{3_i} \tag{1}$$

The index *i* represents the points on the neutral surface. The interpolation functions $N_i(\xi,\eta)$ defined in the domain ξ *[-1,1]* and η*[-1,1]* are of the Serendip type. The variable ζ is the linear coordinate in the thickness direction defined in the domain *[-1,1]*. The thickness of the shell is *h* and the normal vector to the neutral surface $\vec{V}_{3_i}$ is defined as follows:

$$\vec{V}_{3_i} = \begin{Bmatrix} \left(\sum_j \frac{\partial x}{\partial \xi}\right)_i & \left(\sum_j \frac{\partial x}{\partial \eta}\right)_i \\ \left(\sum_j \frac{\partial y}{\partial \xi}\right)_i & \left(\sum_j \frac{\partial x}{\partial \eta}\right)_i \\ \left(\sum_j \frac{\partial z}{\partial \xi}\right)_i & \left(\sum_j \frac{\partial x}{\partial \eta}\right)_i \end{Bmatrix} \tag{2}$$

The cinematic of the element is associated with Mindlin, (1951), which defines the displacement field by 3 translations u_i, v_i and w_i in the global coordinates system $(o,\bar{x},\bar{y},\bar{z})$ and 2 rotations α_i and β_i, around the unitary vectors $\vec{V}_{1i}$ and $\vec{V}_{2i}$, orthogonals and tangent to the neutral surface of the element, associated with node *i*, Fig. 1.

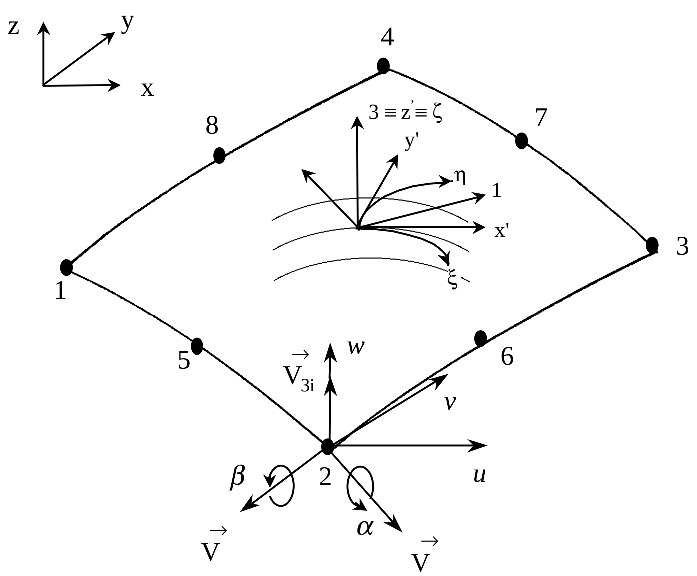


Figure 1- Multi-layer shell element

In the case of multi-layer shell, the cinematic formulation is generalised by a pair of symetric layers localised at the k^a position in relation to the neutral surface for the addition of ks rotations α_i and β_i, Fig. 2. Thus, the displacement fields, with a $2k+3$ degree of freedom (dof) per node, is:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{8} N_i(\xi,\eta) \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \sum_{k=1}^{nl}\left[\sum_{i=1}^{8} N_i(\xi,\eta)\zeta\, h_k \left[\vec{V}_{1i}, \vec{V}_{2i}\right] \begin{Bmatrix}\alpha_i \\ \beta_i\end{Bmatrix}_k\right] \tag{3}$$

where nl is total number of layers. In the constitutive relationship of the material, the normal stress at the neutral surface σ_{33} is negligeable and the shear modulus G_{13} and G_{23} can be affected by a correction factor as proposed by Reissner (1945) and Mindlin (1951). In this work, the transverse shear is corrected by a $k = 1.2$.

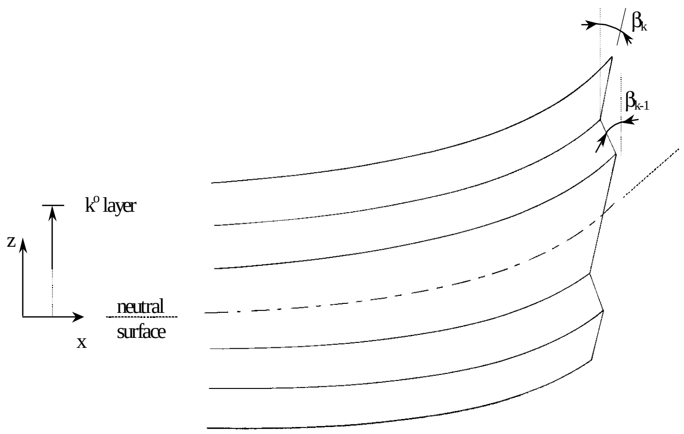


Figure 2 - The displacement fields for a symetric shell

Each layer can be arbitrarily disposed with an angle of fiber θ in relation to x' axis in the local coordinate system $(o, \vec{x}', \vec{y}', \vec{z}')$. The transformation matrix $[T_\theta]$ is used to translate the constitutive matrix [C] from the orthotropic axes (1,2,3) to the local coordinate system, Pereira (1996):

$$\left[C'\right] = \left[T_\theta\right]^t \left[C\right]\left[T_\theta\right] \tag{4}$$

The stiffiness matrix is determined from the strain energy of the layer k:

$$U_k = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \left\langle \varepsilon' \right\rangle \left[C'\right]\left\{\varepsilon'\right\} |J|\, d\xi\, d\eta\, d\zeta \tag{5}$$

where |J| is the determinant of the jacobian matrix. The strain vectors $\left\langle \varepsilon' \right\rangle$ measured in the local coordinate system $(o, \vec{x}', \vec{y}', \vec{z}')$ is:

$$\left\langle \varepsilon' \right\rangle = \left\langle \varepsilon'_{xx}, \varepsilon'_{yy}, \varepsilon'_{xy}, \varepsilon'_{xz}, \varepsilon'_{yz} \right\rangle \tag{6}$$

The $[B']$ matrix shown in [3] relates the displacements to the strains by Eq. (7):

$$\{\varepsilon'\} = [B']\{\delta\} \tag{7}$$

where $\{\delta\}$ is the nodal displacement vector of the shell element with 8 nodes and $2k+3$ dofs per node:

$$\langle\delta\rangle = \langle u_1, v_1, w_1, \cdots, \alpha_8, \beta_8, \cdots, \alpha_{1_k}, \beta_{1_k}, \cdots, \alpha_{8_k}, \beta_{8_k}\rangle \tag{8}$$

Finally, the stiffness matrix of the layer k is:

$$[K]_k = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} [B']^t [C'][B']|J|\, d\xi\, d\eta\, d\zeta \tag{9}$$

The expression of the cinetic energy of the layer k is given by Eq. (10):

$$T_k = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \rho \langle \overset{\circ}{u} \rangle \{\overset{\circ}{u}\} |J|\, d\xi\, d\eta\, d\zeta \tag{10}$$

where ρ is the volumetric mass of the material of the layer k. The $[L_i]$ matrix relates the global displacements to the nodal displacements as follows:

$$\{u\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = [[L_i]] \langle u_1, v_1, w_1, \cdots, \alpha_8, \beta_8, \cdots, \alpha_{1_k}, \beta_{1_k}, \cdots, \alpha_{8_k}, \beta_{8_k}\rangle^t \tag{11}$$

Thus, the mass matrix of the layer k is:

$$[M]_k = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \rho\, [L]^t [L] |J|\, d\xi\, d\eta\, d\zeta \tag{12}$$

The natural frequencies ω_i and the associated modes ϕ_i are the solutions of the eigenvalue problem of the conservative system:

$$[[K] - \omega^2 [M]]\{\delta\} = \{0\} \tag{13}$$

where [M] is the global mass matrix, [K] is the global stiffness matrix and $\{\delta\}$ is the nodal displacement vector.

The integration of the matrices in the space (ξ, η, ζ) are done numerically with a 2 x 2 points of Gauss in the plane and 2 point of Gauss in the thickness direction for each layer.

3. THE DAMPED ELEMENT MODEL FOR MULTI-LAYER SHELL

In the prediction of the multi-layer damping shell structure the model proposed by Adams (1973) is used, in which just the in-plane dissipated energy is considered**.** In this model the materials are supposed to be linear viscoelastic and the loading of the structures are in the harmonic state. Thus, the specific damping capacity is defined as:

$$\psi = \frac{\Delta U}{U} \tag{14}$$

Using Eq. (5) and from the definition of specific damping capacity in Eq. (14), the dissipated energy of the layer *k* is:

$$\Delta U_k = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\langle \varepsilon' \rangle [T_\theta]^t [\psi][C][T_\theta]\{\varepsilon'\}|J|\, d\xi\, d\eta\, d\zeta \tag{15}$$

Using the same procedure to obtain the stiffness matrix earlier, the damped matrix of the layer *k* calculated in the local coordinate system is:

$$[K_a]_k = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B']^t [T_\theta]^t [\psi][C][T_\theta][B']|J|\, d\xi\, d\eta\, d\varsigma \tag{16}$$

Finaly, the damping capacity ψ_i for the mode ϕ_i is obtained by the expression:

$$\psi_i = \sum_{i}^{ne} \frac{\langle \phi \rangle_i \left(\sum_{k}^{nl} [K_a] \right) \{\phi\}_i}{\langle \phi \rangle_i \left(\sum_{k}^{nl} [K] \right) \{\phi\}_i} \tag{17}$$

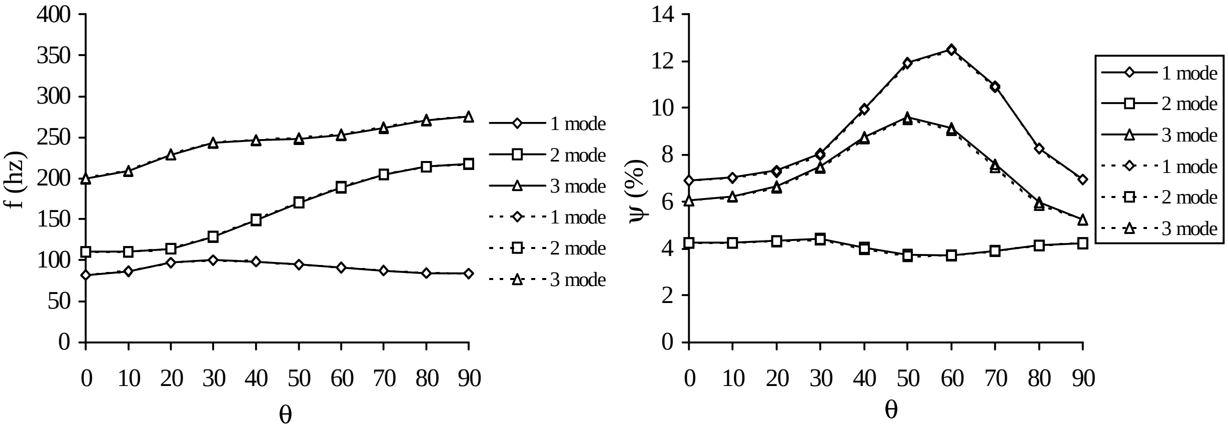
where *ne* is the total number of elements.

4. COMPARISION AND CONCLUSION

In this section, we show the evolution of the damping in multi-layer plates of carbon/epoxy on two differents boundaries conditions: free on all sides and clamped on all sides; with the following stacking plies orders: [0°,+θ°,+θ°,0°] and [0°,+θ°,0°,0°,+θ°,0°]. The parameter θ, the orientation of the fiber, range from 0° to 90°. The first three natural frequencies and their associated modal damping, are examinated. The dimensions of the plates are 200 mm x 200 mm and the thickness is 2 mm for the first stacking ply and 3 mm for the second stacking plies. The mechanical properties of the carbon/epoxy are: E_1 = 172.7 GPa, E_2 = 7.2 GPa, G_{12} = 3.76 GPa, G_{23} = 2.77 GPa, $\nu_{12} = 0.3$, ρ = 1566 kg/m^3 and their specific damping capacity are: ψ_{11} = 0.45, ψ_{22} = 4.22 et ψ_{44} = 7.05. In the comparison with this new formulation, it is used a first order element proposed by Pereira (1999). In figures below the continus line representes the new formulation and the dotted lines representes the element proposed by Pereira (1999).

Case 1: Free on all sides.

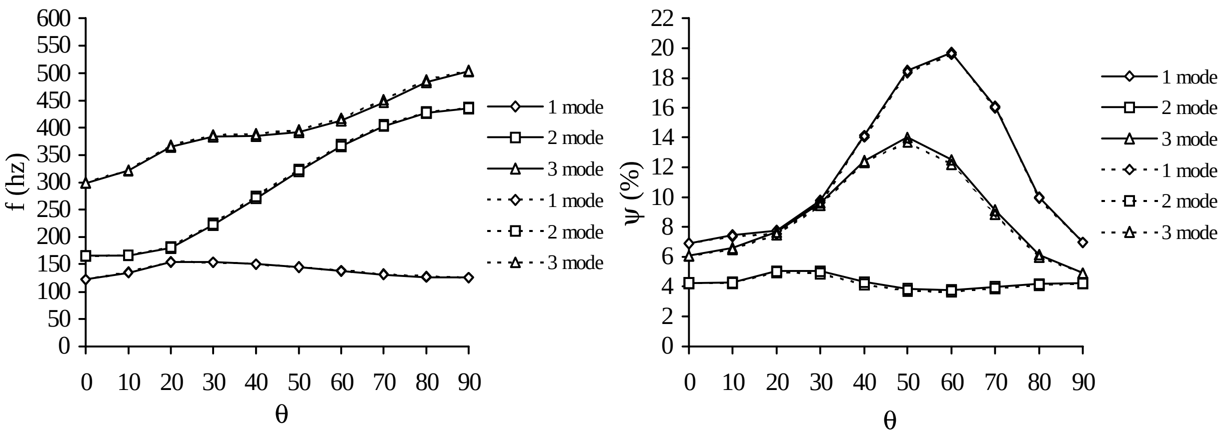
a- Laminate orientation [0°, +θ°, +θ°, 0°]



(a) (b)

Figure 3 - Natural frequencies (a) and their modal damping associated (b)

b- Laminate orientation [0°, +θ°, 0°, 0°, +θ°, 0°]

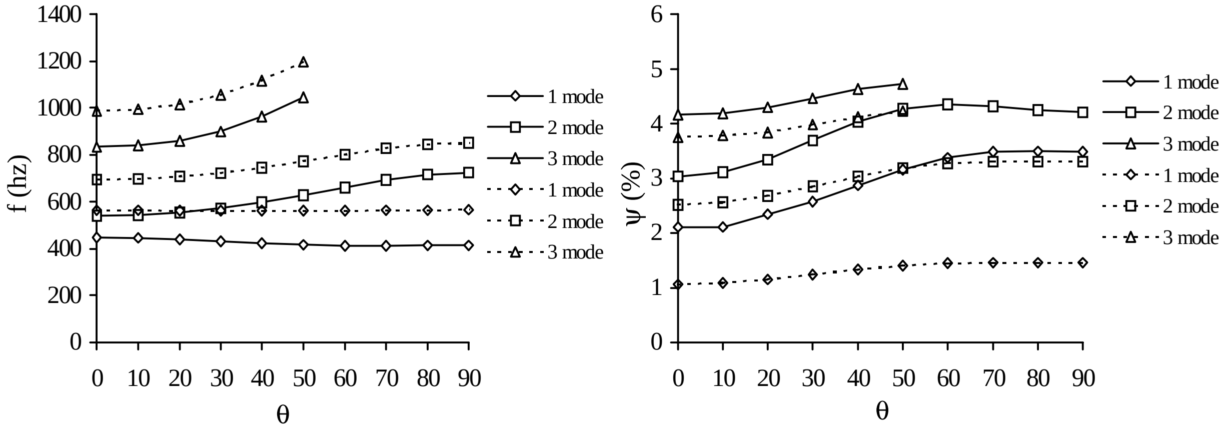


(a) (b)

Figure 4 - Natural frequencies (a) and their modal damping associated (b)

Case 2: Clamped on all sides.

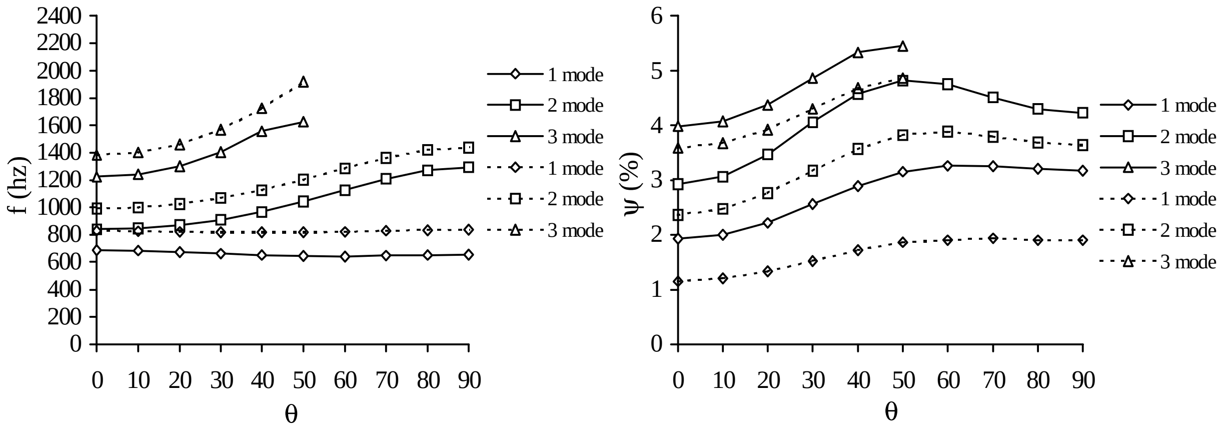
c- Laminate orientation [0°, +θ°, +θ°, 0°]



(a) (b)

Figure 5 - Natural frequencies (a) and their modal damping associated (b)

d- Laminate orientation [0°, +θ°, 0°, 0°, +θ°, 0°]



(a) (b)

Figure 6 - Natural frequencies (a) and their modal damping associated (b)

The comparition between the classical theory laminates and the multi-layer finite element presented underline the transverse shear effect in differents stacking plies and differents boudary conditions. With the first boundary condition, this effect is not so appreciable, but with the second boundary condition the transverse shear effect is more pronounced than the first one. This can be noted in the frequencies and their associated modal damping. In certain stacking plies the error can be up to 35 % on the frequencies and 20 % on the modal damping.

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