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THE EFFECT OF THE FIBER ORIENTATION ON THE DYNAMIC BEHAVIOUR OF MULTI-LAYER PLATES

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Abstract. *The effect of fibre orientation on the damping of multi-layer plates using a first order theory element is investigated. The validation of the model, is based on the comparison with un first order theory element found in the literature. The optimal damping for orthotropic square plates in different ply orientations for square plates in carbon/epoxy and in glass/epoxy is presented.*

Keywords: *Multi-layers shell, Composite materials, Damping, Finite elements.*

1. INTRODUCTION

The composite materials allow us to obtain industrial structures combining high stiffness with low mass. In dynamics these interesting properties in principle emphasize the vibration problems. The study of the dissipative capacity of the structures is in this context is usefull. The orthotropic aspect can be used to optimize the global damping using parameters of the material such as the thickness, the angle and the stacking sequence. Two approachs are used to provide the damping in composite structures. Hashin examined the concept of the complex elastic module supposing the linear viscoelasticity of the materials in an harmonic state. The structural damping η is defined as the ratio between the imaginary part and the real part of the elastic module Hashin, (1970-I), Hashin, (1970-II). The complex eigenvalue problem yields to a time calculating penalisation, but this problem can be parcially solved by the modal method, Noor *et al.* (1989). One global method based on the deformation energy was proposed by Adams *et al.* (1973). He introduces the specific damping capacity ψ_{ij} of the materials in the case of thin plates. The values of the ψ_{ij} are obtained in an experimental way which are highly conditioned to the quality of the model to prevent the global damping. Lin *et al.* (1984) has extended Adams' method by introducing the transverse shear effect in order to prevent the damping in thick plates. Chapuis (1992) by an accurate definition of the elastic energy of a linear viscoelastic material under cyclic solicitation, shows that the method proposed by Adams is a direct consequence of the complex elasticity modules theory, expressed in an energetic way.

In this work, a method to prevent the damping in composite structures proposed by Adams *et al.* (1973) and Lin *et al.* (1984) is presented. It leads to a global method of a energetic type adapted to the finite element proposed in this work. A comparison between the model proposed by Lin *et al.* (1984) and this new formulation is made. The optimal damping is searched in carbon/epoxy and glass/epoxy plates.

2. THE ELEMENT FORMULATION

Here we propose a 8 nodes quadrilateral element with a quadratic approximation, Fig. 1. The geometry of the element is defined as:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum_{i=1}^{8} N_i(\xi,\eta) \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix} + \sum_{i=1}^{8} N_i(\xi,\eta) \frac{\zeta}{2}\, h\, \vec{V}_{3_i} \tag{1}$$

The index *i* represents points on the neutral surface. The interpolation functions $N_i(\xi,\eta)$ defined in the domain $\xi\ [-1,1]$ and $\eta[-1,1]$ are of the Serendip type, Pereira (1996). The variable ζ is the linear coordinate in the thickness direction defined in the domain $[-1,1]$. The thickness of the shell is h and the normal vector to the neutral surface $\vec{V}_{3_i}$ is defined as follows:

$$\vec{V}_{3_i} = \begin{Bmatrix} \left(\sum_j \frac{\partial x}{\partial \xi}\right)_i & \left(\sum_j \frac{\partial x}{\partial \eta}\right)_i \\ \left(\sum_j \frac{\partial y}{\partial \xi}\right)_i & \left(\sum_j \frac{\partial x}{\partial \eta}\right)_i \\ \left(\sum_j \frac{\partial z}{\partial \xi}\right)_i & \left(\sum_j \frac{\partial x}{\partial \eta}\right)_i \end{Bmatrix} \tag{2}$$

The cinematic of the element is associated with [1] which introduces in the displacement field 3 translations u_i, v_i and w_i in the global coordinates system $(o,\bar{x},\bar{y},\bar{z})$ and 2 rotations α_i and β_i, around the unitary vectors $\vec{V}_{1i}$ and $\vec{V}_{2i}$, orthogonals and tangents to the neutral surface of the element, associated to node *i*, Fig. 1.

In the case of multi-layer shells, the cinematic formulation is generelised by a pair of symmetric layers localised at the $k^{\underline{a}}$ position in relation the neutral surface for the rotations α_i and β_i.

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{8} N_i(\xi,\eta) \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \sum_{k=1}^{nl} \left[\sum_{i=1}^{8} N_i(\xi,\eta)\zeta \frac{h_k}{2} \left[\vec{V}_{1i}, \vec{V}_{2i} \right] \begin{Bmatrix} \alpha_i \\ \beta_i \end{Bmatrix} \right] \tag{3}$$

where *nl* is total number of layers in the given element and the variable ζ is computed in function of the position *k* of the layer. In the constitutive relation of the material, the normal stress at the neutral surface σ_{33} is negligeable and the shear modulus G_{13} and G_{23} can be affected by a correction factor as proposed by Reissner (1945) and Mindlin (1951). In this work, the transverse shear is corrected by k = 1.2.

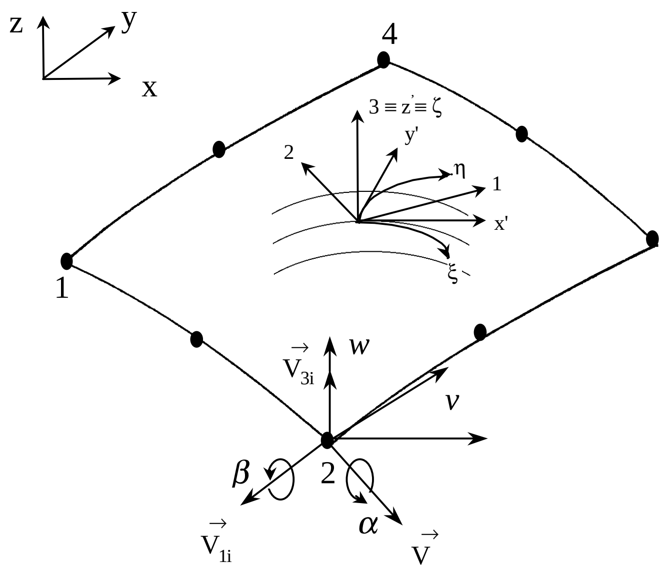


Figure 1- Multi-layer shell element

Each layer can be arbitrarily disposed with an angle of fiber θ in relation to x' axe in the local coordinate system $(o, \vec{x}', \vec{y}', \vec{z}')$. The transformation matrix $[T_\theta]$ is used to translate the constitutive matrix [C] in the orthotropic axis (1,2,3) to the local coordinate system, Pereira (1996):

$$[C'] = [T_\theta]^t [C][T_\theta] \tag{4}$$

The stiffiness matrix is determined from the strain energy of the layer *k*:

$$U_k = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \langle \varepsilon' \rangle [C'] \{\varepsilon'\} |J| \, d\xi \, d\eta \, d\zeta \tag{5}$$

where |J| is the determinant of the jacobian matrix. The strain vectors $\langle \varepsilon' \rangle$ measured in the local coordinate system $(o, \vec{x}', \vec{y}', \vec{z}')$ is:

$$\langle \varepsilon' \rangle = \langle \varepsilon'_{xx}, \varepsilon'_{yy}, \varepsilon'_{xy}, \varepsilon'_{xz}, \varepsilon'_{yz} \rangle \tag{6}$$

The [B'] matrix shown in Pereira (1996) relates the displacements to the strains by Eq. (7):

$$\{\varepsilon'\} = [B'] \{\delta\} \tag{7}$$

where {δ} is the nodal displacement vector of the shell element with 8 nodes and 5 dofs per node:

$$\langle \delta \rangle = \langle u_1, v_1, w_1, \cdots, \alpha_8, \beta_8 \rangle \tag{8}$$

Finally, the stiffiness matrix of the layer *k* is:

$$[K]_k = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B']^t\,[C'][B']|J|\,d\xi\,d\eta\,d\zeta \tag{9}$$

The expression of the cinetic energy of the layer *k* is given by Eq. (10):

$$T_k = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\rho\left\langle \overset{\circ}{u}\right\rangle \left\{\overset{\circ}{u}\right\}|J|\,d\xi\,d\eta\,d\zeta \tag{10}$$

where ρ is the volumetric mass of the material of the layer *k*. The $[L_i]$ matrix lies the global displacements to the nodal displacements as following:

$$\{u\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = [[L_i]]\langle u_1, v_1, w_1, \cdots, \alpha_8, \beta_8\rangle^t \tag{11}$$

Thus, the mass matrix of the layer *k* is:

$$[M]_k = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\rho\,[L]^t[L]|J|\,d\xi\,d\eta\,d\zeta \tag{12}$$

The natural frequencies ω_i and the associated modes ϕ_i are the solutions of the eigenvalue problem of the conservative system:

$$[[K] - \omega^2[M]]\{\delta\} = \{0\} \tag{13}$$

where [M] is the global mass matrix, [K] is the global stiffness matrix and {δ} is the nodal displacement vector.

The integration of the matrices in the space (ξ, η, ζ) are done numerically with a 2 x 2 points of Gauss in the plane and 2 point of Gauss in the thickness direction for each layer.

3. THE MODAL DAMPING CAPACITY

In the prediction of the damping of multi-layer shell structure the model proposed by Adams *et al.* (1973) is used, in which only the in-plane dissipated energy is considered**.** In this model the materials are supposed to be linear viscoelastic and the loading of the structure are in the harmonic state. Thus, the specific damping capacity is defined as:

$$\psi = \frac{\Delta U}{U} \tag{14}$$

Using Eq. (5) and from the definition of specific damping capacity in Eq. (14), the dissipated energy of the layer *k* is:

$$\Delta U_k = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\langle \varepsilon' \rangle [T_\theta]^t [\psi][C][T_\theta]\{\varepsilon'\}|J|\, d\xi\, d\eta\, d\zeta \tag{15}$$

Using the same procedure to obtain the stiffness matrix earlier, the damped matrix of the layer *k* may be calculated in the local coordinate system as:

$$[K_a]_k = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B']^t [T_\theta]^t [\psi][C][T_\theta][B']|J|\, d\xi\, d\eta\, d\varsigma \tag{16}$$

Finaly, the damping capacity ψ_i for mode ϕ_i is obtained by the expression:

$$\psi_i = \sum_{i}^{ne} \frac{\langle \phi \rangle_i \left(\sum_{k}^{nl} [K_a] \right) \{\phi\}_i}{\langle \phi \rangle_i \left(\sum_{k}^{nl} [K] \right) \{\phi\}_i} \tag{17}$$

where *ne* is the total number of elements. The modal damping η_i measured experimentally is related to the damping capacity ψ_i by:

$$\psi_i = 2\pi\eta_i \tag{18}$$

4. VALIDATION OF THE MODEL

In the validation of the model we used a square multi-layer plate of carbon/epoxy and glass/epoxy supported on a soft foam rubber, Lin *et al.* (1984), whose laminate orientation of the plates was (0°, 90°, 0°, 90°, 90°, 0°, 90°, 0°). The characteristics of the plates are on Tables 1 and 2. The results obtained with the present model, shown in the Table 3 and 4, were compared with the numerical and the experimental results obtained by Lin *et al.* (1984).

Table 1. Characteristics of the plates

Material	ρ (kg/m^3)	h (mm)	l (mm)	Fibre volume
carbon/epoxy	1446.2	2.12	234.5	0.342
glass/epoxy	1813.9	2.05	227	0.451

Table 2. Mechanical properties of the plates

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ψ_{11} (%)	ψ_{22} (%)	ψ_{44} (%)	ν_{12}
carbon/epoxy	172.7	7.20	3.76	0.45	4.22	7.05	0.3
glass/epoxy	37.78	10.90	4.91	0.87	5.05	6.91	0.3

Table 3. Natural frequency and modal damping (ψ) - carbon/epoxy

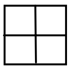
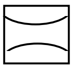
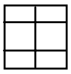
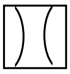
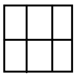
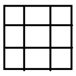
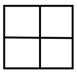
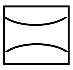
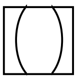
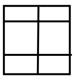
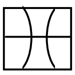
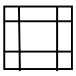
fibre direction →	Lin *et al.* (1984)		Present model	
Mode	f (hz)	ψ (%)	f (hz)	ψ (%)
	58.10 (68.88)	7.80 (6.65)	65.59	5.94
	213.31 (218.9)	0.91 (1.05)	229.85	1.84
	243.47 (251.2)	2.50 (2.6)	265.24	2.84
	302.51 (305.4)	0.60 (0.92)	329.40	0.87
	324.16 (323.5)	1.51 (1.7)	354.56	1.64
	441.62 (467.9)	2.74 (3.0)	483.29	2.61

Table 4. Natural frequency and modal damping (ψ) - glass/epoxy

Fibre direction →	Lin *et al.* (1984)		Present model	
Mode	f (hz)	ψ (%)	f (hz)	ψ (%)
	66.42 (62.2)	7.16 (6.7)	67.33	6.64
	131.62 (131.4)	2.47 (2.8)	129.48	3.10
	164.46 (159.2)	1.62 (1.9)	163.11	1.12
	189.79 (180.5)	4.87 (4.9)	188.42	4.88
	208.87 (200.05)	3.73 (3.2)	210.33	3.64
	347.16 (326.7)	5.09 (5.8)	345.62	4.71

The Tables above show the first six modes of the predicted results obtained with the present model and the prediction and the experimental results obtained by Lin et al. On the whole, there is a good agreement between the two predictions and the experimental results. The higher differences between the present model and the experimental results, in Table 3, is because the mechanical properties in Table 2 are from a standard set given for a 50% fiber volume which have to be corrected to the real fibre volume. In the Table 4, this problem is not so appreciable.

5. THE OPTIMAL DAMPING OF MULTI-LAYERS PLATES

In this section, we search the optimal damping in multi-layer plates on the free-free configuration with the following laminate orientation: [0°,+θ°,+θ°,0°] and [0°,+θ°,0°,0°,+θ°,0°]. The parameter θ, the orientation of the fiber, ranges from 0° to 90° for the square plates of carbon/epoxy and glass/epoxy. The first three natural frequencies and their associated modal damping, as shown in Fig. 3, are examinated. The properties of the plates are in Table 2 and 5.

mode 1: torsion (T) mode 2: bending (F) mode 3: torsion+bending (T+F)

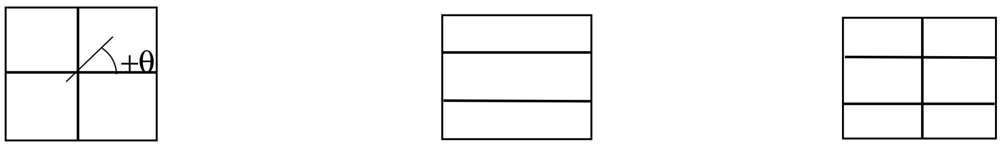
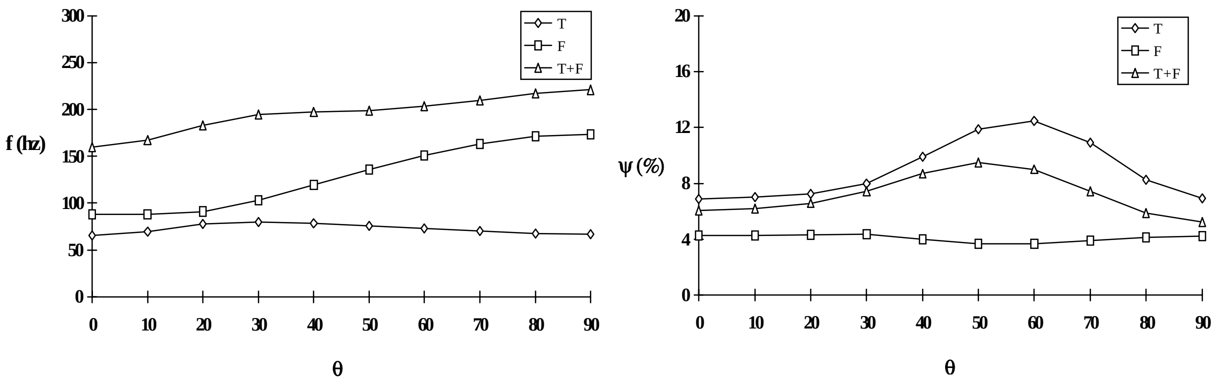


Figure 3 – Nodal lines for the first three modes of the plates

Table 5. Plates characteristics

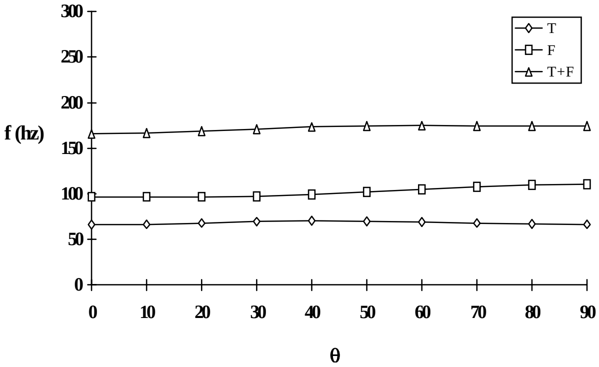
material	ρ (kg/m^3)	h (mm)	l (mm)	Fibre volume
carbon/epoxy	1566	1.60	200	0.516
glass/epoxy	1971	1.60	200	0.568

a- Laminate orientation [0°, +θ°, +θ°, 0°]

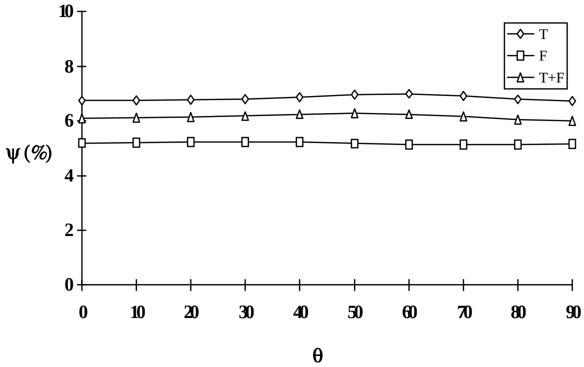


(a) (b)

Figure 5 - Natural frequencies (a) and their associated modal damping (b) - carbon/epoxy



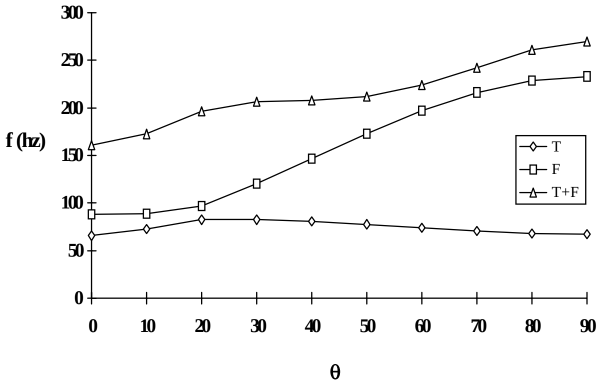
(a)



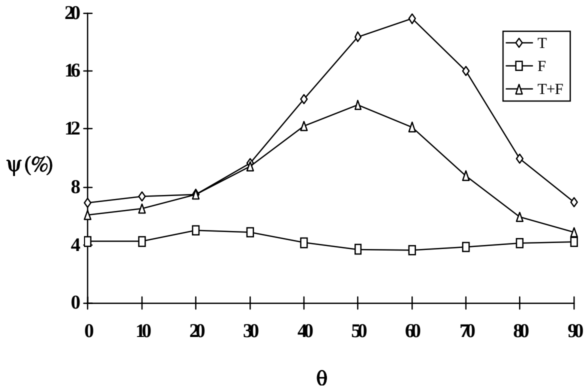
(b)

Figure 6 - Natural frequencies (a) and their associated modal damping (b) - glass/epoxy

b - Laminate orientation [0°, +θ°, 0°, 0°, +θ°, 0°]

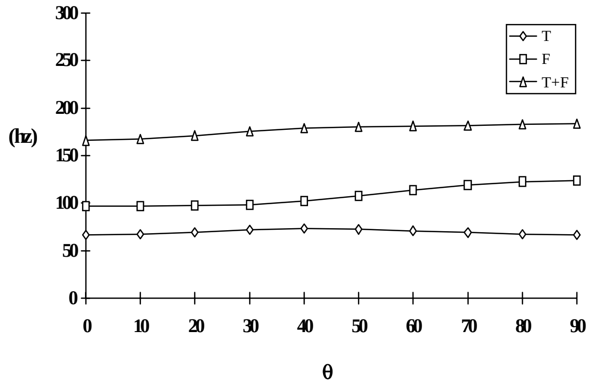


(a)

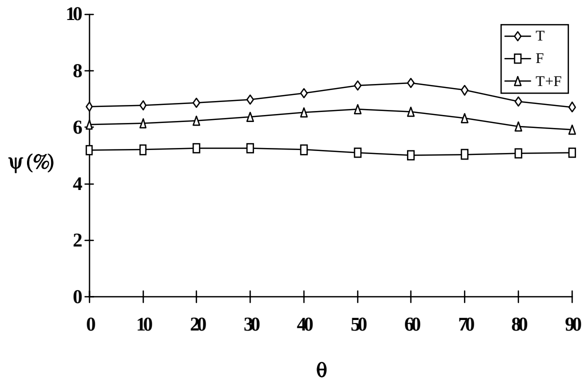


(b)

Figure 7 - Natural frequencies (a) and their associated modal damping (b) - carbon/epoxy



(a)



(b)

Figure 8 - Natural frequencies (a) and their associated modal damping (b) - glass/epoxy

4. COMMENTS AND CONCLUSIONS

This study shows the maximun damping for θ ranging from 50° to 60°. This phenomenon is more accentuated in the case of carbon/epoxy plates, because of its strong non isotropic characteristic ($E_1 >> E_2$). Even though the specific damping capacity of the glass/epoxy plates is higher than those one of the carbon/epoxy plates, the globals damping of the carbon/epoxy plates are higher than those of the glass/epoxy plates.

The comparison of the two configurations shown puts in evidence the second configuration [0°, +θ°, 0°, 0°, +θ°, 0°]. This configuration allows without to much disturb the frequencies in relation the configuration [0°, +θ°, +θ°, 0°] to increase in a significant way the global damping (near 60 % in the case of carbon/epoxy plates and 10 % in the case of glass/epoxy plates).

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