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EXACT DETERMINATION OF THE THREE PARAMETERS OF A HERSCHEL-BULKLEY FLUID

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Abstract. *This work presents an algorithm for the determination of the three parameters of a Herschel-Bulkley fluid, namely, τ_o, n and K, for a given set of Fann readings. This problem is not trivial, since the shear rate applied to the fluid in between the bob and rotor sleeve walls is a function of the behavior index. The whole problem can be decomposed into two other problems, which must be solved in alternation. The first problem consists of the determination of the shear rate at the bob wall, for each value of the rotational speed, as a function of the three parameters in the Herschel-Bulkley model. This is done through the use of Newton's method. The second problem consists of the determination of the three Herschel-Bulkley parameters as a function of the shear stresses and calculated shear rates at the bob wall by the use of both regression and Newton's method. To start the iterative process, the fluid is assumed initially as Newtonian. The whole problem converges quickly. The related problem of calculating the expected Fann readings as a function of the Herschel-Bulkley parameters is also considered. Numerical applications are presented and the error associated to the usual way of adopting Newtonian shear rates is evaluated.*

Key-words: *Rheology, Herschel, Bulkley, Viscometer, Regression*

1. INTRODUCTION

This study concerns the calculation of the three parameters of a Herschel-Bulkley fluid from readings of Fann or Couette type viscometers. These viscometers comprise a stationary inner cylinder and a rotational outer sleeve (Fig. 1). The inner cylinder is called the bob. As the outer sleeve rotates, torque is transmitted to the bob by the fluid contained in the annular space between the bob and the sleeve. The bob actuates a spring, bringing about a deflection of a needle by some angle, which can be read on the dial reading. Figures 2 and 3 presents a more schematic representation of this viscometer, and are going to be used in the developments of the theory.

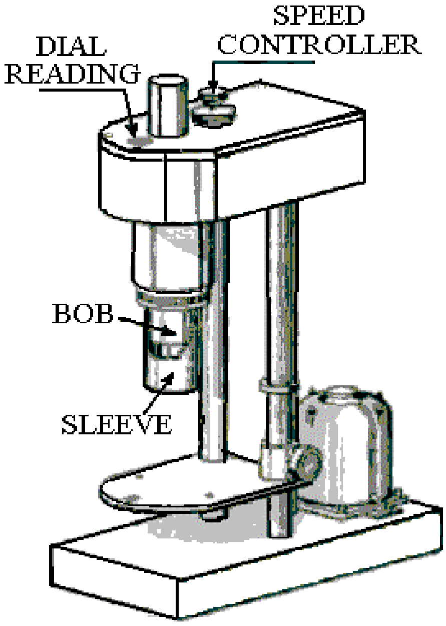

Figure 1 – Couette type viscometer. The bob is kept stationary while the sleeve rotates. Torque is transmitted to the bob wall by the fluid contained in the annulus.

Although this study could as well be extended to Searle type viscometers, in which the bob rotates, rather than the outer sleeve, this will not be performed here, and is left for future works.

The fluid placed in the annular space, in between the bob and the sleeve, is assumed to follow the Herschel-Bulkley model, also known as the yield-power-law model

$$\tau = \tau_0 + K\dot{\gamma}^n \qquad (1)$$

where, τ is shear stress, $\dot{\gamma}$ is the shear rate and τ_0, K and n are the three parameters of the Herschel-Bulkley model which are called the yield point, the consistency index and the behavior index, respectively.

Other rheological models can are used to relate τ to $\dot{\gamma}$, e.g., the Bingham

$$\tau = \tau_0 + \mu_p\dot{\gamma} \qquad (2)$$

where τ_0 is the yield point and μ_p is the plastic viscosity; the Ostwald-de Waele or power-law

$$\tau = K\dot{\gamma}^n \qquad (3)$$

where K is the consistency index and n is the behavior index.

It should be emphasized that, although the rheological parameters share different names in different models, their numerical values are not the same. This way, the yield point in the

Herschel-Bulkley model and the yield point in the Bingham model have different values for the same fluid.

The Herschel-Bulkley model, the Bingham and the Ostwald-de Waele models, among others, are fitted to the viscometer data through statistical regression. Hemphill et al. (1993) considered the Herschel-Bulkley model more accurate than both the Bingham and the Power law for several different fluid systems. That conclusion looks reasonable, since both the Bingham and the Ostwald-de Waele models can be obtained from the Herschel-Bulkley model by setting n=1 or τ_0=0, respectively, in Eq. (1). The Bingham and the power law are two parameter models whereas the Herschel-Bulkley is a three parameter model. There are other three parameter models, as well, a very promising one being that presented by Robertson and Stiff (1976)

$$\tau = A(\dot{\gamma} + \dot{\gamma}_0)^n \tag{4}$$

where A is a parameter related to consistency index, and $\dot{\gamma}_0$ is a corrective parameter added to the shear rate. The advantage of the Robertson-Stiff model lies in that it is more mathematically tractable than the Herschel-Bulkley. The disadvantage in that the parameters have a more cumbersome physical interpretation than the parameters in the Herschel-Bulkley model..

One problem posed for the Herschel-Bulkley model is how to calculate its three parameters, the yield point, the consistency index and the behavior index, from readings of a rotational viscometer. Other problem is how to calculate pressure drop through pipe and annular flows, in both the laminar and the turbulent regimes, with these fluids. Another problem is how to calculate heat transfer with these fluids. In this work only the first of these problems is considered.

The literature contains a number of studies related with all the three problems referred above. Hakki et al. (1992) have derived and expression for the axial laminar flow of a Robertson-Stiff fluid in concentric annuli while stating that a similar expression could not be developed for a Herschel-Bulkley fluid. Hemphill et al. (1993) used the Herschel-Bulkley model to study some new drilling fluids, called mixed metal hydroxide, and found this model to be superior to both the Bingham and Ostwald-de Waele fluids. Vinod et al. (1994) have conducted a very interesting study of the flow of a Herschel-Bulkley fluid through an eccentric annuli, with co-existing regions of laminar and turbulent regions. Bing et al. (1995) have used the Herschel-Bulkley model to calculate surge and swab pressures during pipe trips in inclined and horizontal oil wells. Merlo et al. (1995) have developed analytical expressions and a computer program for calculating pressure drop in pipes and annulus using the Herschel-Bulkley model. Maglione et al. (1996) attempted to use the standing pipe pressures to determine the Herschel-Bulkley parameters at the rig site while drilling and oil well. The problem of finding the correct values of the parameters of a Herschel-Bulkley fluid, nevertheless, have not been properly addressed in the literature. The proper method for doing this is described in the sections ahead.

2. BACKGROUND

In this section , the fundamental equations and concepts related to the problem of finding the three parameters of a Herschel-Bulkley fluid are presented. An implicit equation for the shear rate and expressions for statistically fitting the Herschel-Bulkley model are developed. Both these developments will need numerical techniques in their applications.

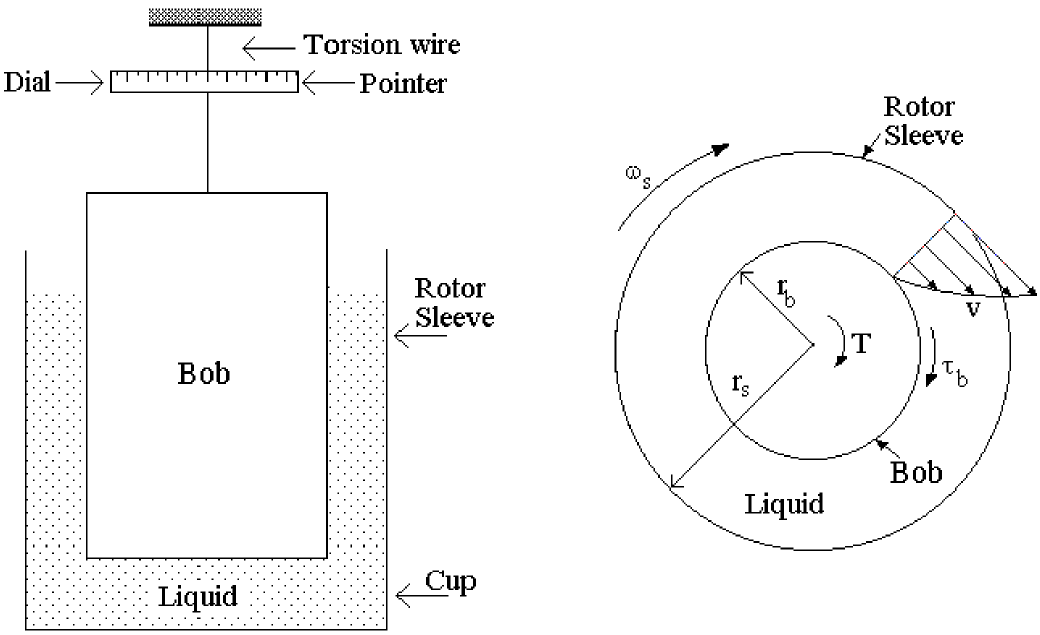

Figure 2 – Lateral and top views of a Couette-type viscometer.

2.1 Shear stress and shear rate equation

The shear stress can be determined from the viscometer reading if the geometry of the system and the value of the spring constant are known. The torque at a point in the annular space between the bob and the sleeve is given by

$$T = \tau_{r\varphi}(2\pi rL)r \tag{5}$$

where $\tau_{r\phi}$ is the shear stress, L is the bob length and r is the distance to the bob axis. The torque is also related to the angle of deflection of the needle, θ, by

$$T = c\theta \tag{6}$$

where c is the spring constant. The spring constant c is chosen in a way as to make the viscometer reading θ numerically equivalent to the viscosity of a Newtonian fluid, expressed in centipoise, when the rotor sleeve is turned at a rotational speed of 300 rpm.

The combination of Eqs. (5) and (6) gives

$$\tau_{r\varphi} = \frac{c\theta}{2\pi r^2 L} \tag{7}$$

which shows that the shear stress varies inversely with the radius. It is possible to write

$$\tau_{r\varphi} = \frac{\tau_b r_b^{\;2}}{r^2} \tag{8}$$

where r_b is the bob radius and τ_b is the shear stress at the bob wall surface (see Fig. 2).

The velocity at a point in between the bob and the sleeve is given by

$$v = \omega r \tag{9}$$

where ω is the angular velocity and r is the distance to the bob axis. By taking the derivative

$$\frac{dv}{dr} = \frac{d\omega}{dr} r + \omega \tag{10}$$

it can be seem that the second term in the second hand side is related to translation, while the first term in the second hand side is related to deformation, i.e., to the shear rate. Hence

$$\dot{\gamma}_{r\varphi} = \frac{d\omega}{dr} r \tag{11}$$

Now, combining Eqs. (11) and (1) a differential equation is obtained

$$\tau_{r\varphi} = \tau_0 + K\left(r\frac{d\omega}{dr} \right)^n \tag{12}$$

Integrating from the bob wall to the sleeve wall, using Eq. (8) and

$$\omega_s = \frac{2\pi N}{60} \tag{13}$$

where N is the rotational speed in rpm, the following expression ensues

$$\frac{2\pi N K^{1/n}}{60} = \int_{r_b}^{r_s} \frac{1}{r}\left(\frac{\tau_b r_b^{\;2}}{r^2} - \tau_0 \right)^{1/n} dr \tag{14}$$

An alternate expression is obtained by substituting Eq. (7) into (14)

$$\frac{2\pi N K^{1/n}}{60} = \int_{r_b}^{r_s} \frac{1}{r}\left(\frac{c\theta}{2\pi r^2 L} - \tau_0 \right)^{1/n} dr \tag{15}$$

and even then an algorithm could be devised from this expression, the authors have taken a different route. Equation ((15) is therefore, presented only for the sake of completeness.

It is convenient to change variables in Eq. (14)

$$y = \frac{r}{r_b} \tag{16}$$

so that it could be written

$$\frac{2\pi N K^{1/n}}{60} = \int_1^{y_s} \frac{1}{y}\left(\frac{\tau_b}{y^2} - \tau_0\right)^{1/n} dy \tag{17}$$

Finally, substituting for the bob wall shear stress, the expression used in this work for the calculation of the shear rate is derived

$$f(\dot{\gamma}_b) = \frac{60}{2\pi N K^{1/n}} \int_1^{y_s} \frac{1}{y}\left(\frac{\tau_0 + K\dot{\gamma}_b^{\,n}}{y^2} - \tau_0\right)^{1/n} dy - 1 = 0 \tag{18}$$

This equation must be solved numerically, since the integral has no algebraic solution. In addition, the expression inside the parenthesis cannot become negative, and therefore the following condition must be fulfilled:

$$\Pi = \frac{\tau_0 (y^2 - 1)}{K\dot{\gamma}_b^{\,n}} \le 1 \tag{19}$$

2.1 Regression equations

Expressions to fit the Herschel-Bulkley model to the viscometer data are developed in this section. The viscometer data is a set of ordered pairs (N_i, θ_i), $i=1...n$, summarizing the viscometer reading for each value of the rotational speed. Normally, $n=6$, with N taking values of 2, 3, 6, 100, 200, 300 and 600 rpm. In order to fit the Herschel-Bulkley model to the viscometer data, the set (N_i, θ_i), $i=1...n$ must be transformed into the set $(\dot{\gamma}_i, \tau_{bi})$, $i=1...n$, with the help of Eqs. (7) and (18). Evidently, to perform this transformation, guessed values of τ_0, K and n must be chosen. The fitting of the Herschel-Bulkley equation requires the solution of the following minimization problem

$$\text{Min } S(\tau_0, K, n) = \sum_i \left(\tau_0 + K\dot{\gamma}_i^{\,n} - \tau_{b_i}\right)^2 \tag{20}$$

where the index i is a label for each data point.

Taking advantage of the linear dependency of S in relation to τ_0 and K, these parameters can be written explicitly as

$$\tau_0 = \frac{\sum_i \tau_{b_i} \sum_i \dot{\gamma}_{b_i}^{\;2n} - \sum_i \tau_{b_i} \dot{\gamma}_{b_i} \sum_i \dot{\gamma}_{b_i}^{\;n}}{n \sum_i \dot{\gamma}_{b_i}^{\;2n} - \left(\sum_i \dot{\gamma}_{b_i}^{\;n} \right)^2} \tag{21}$$

$$K = \frac{n \sum_i \tau_{b_i} \dot{\gamma}_{b_i}^{\;n} - \sum \gamma_{b_i}^{\;n} \sum \tau_{b_i}}{n \sum_i \dot{\gamma}_{b_i}^{\;2n} - \left(\sum_i \dot{\gamma}_{b_i}^{\;n} \right)^2} \tag{22}$$

As for the behavior index, an implicit equation is obtained

$$g(n) = \tau_0 \sum_i \dot{\gamma}_i \ln \dot{\gamma} + K \sum_i \dot{\gamma}_i^{\;2n} \ln \dot{\gamma}_i - \sum_i \tau_{b_i} \dot{\gamma}_i^{\;n} \ln \dot{\gamma}_i = 0 \tag{23}$$

4. NUMERICAL PROCEDURE

Given a data set consisting of viscometer readings θ_i at different rotational speed, N_i, i.e., (N_i,θ_i), i = 1 to n, an algorithm can be formulated as a series of steps:

(a) Calculate the bob wall shear stresses, τ_{bi}, for each θ_i, using Eq. (7);
(b) Assume arbitrary values for τ_0, K and n. Suggestion: $n=1$; $\tau_0=0\ N/m^2$; $K=1\ N\ s^n/m^2$;
(c) Solve Eq. (18) using Gauss quadrature and Newton's method: get $\dot{\gamma}_i$ for each N_i;
(d) Solve equations (21) to (23) using Newton's method: get τ_0, K and n;
(e) Compare this new values of τ_0, K and n with the ones guessed in step (b);
(f) If the difference is too large, substitute the guessed values at step (b) by the calculated values of step (d) and iterate from step (c);
(g) If the difference is sufficiently small, accept the values of τ_0, K and n as the solution.

It may be convenient also to check the residue of Eqs. (18) and (23) after achieving convergence in step (g). This algorithm should converge quickly if the data is legitimate. Of course, it is always dangerous to input arbitrary data to any algorithm.

The authors have implemented this algorithm in two programming languages, FORTRAN and PASCAL. They have also implemented the algorithm in Microsoft EXCEL, with the help of the SOLVER capability.

5. RESULTS

The theory developed in the previous sections are now used to study a practical case. The viscometer parameters are given in Table 1. The case examined consists in the calculation of the Herschel-Bulkley parameters for an oil base mud studied by Hemphill et al. (1993). Table 2 and Fig. 3 summarize the results. The residues of the functions f($\dot{\gamma}_i$), Eq. (18), and g(n), Eq. (23) are shown close to zero as demanded by the theory. In the process of finding the zeros of these functions, the condition for Π, expressed by Eq. (19), has been violated at the 3 rpm rotational speed. A possible explanation is for the existence of a plug flow, or a region of no-shear in the annular space, located close to the sleeve wall, where the shear stress is lower. This condition would invalidate the theory developed previously.

Table 1 – Viscometer data.

Parameter	Value
Spring constant, c	$3,62615\text{x}10^{-5}$ N m/degree
Bob radius, r_b	$1,7245\text{x}10^{-2}$ m
Rotor sleeve radius, r_s	$1,8415\text{x}10^{-2}$ m
Bob length, L	$3,8\text{x}10^{-2}$ m

Table 2 – Rheological parameters for the oil-base mud through the present method of analysis.

Rotational Speed N_i, rpm	Dial Reading q_i, degree	Bob Wall Shear Rate $\dot{\gamma}_i$, sec^{-1}	Bob Wall Shear Stress τ_{bi}, Pa	Function Eq. (18) $f(\dot{\gamma}_i)$	Parameter Eq. (19) Π_i	Herschel Bulkley Parameters
3	7	5,107	3,57	-	1,7429	n = 0.883
6	8	15,593	4,09	$4,1\text{x}10^{-07}$	0,6504	K = 0.0628 Pa s^n
100	18	178,942	9,19	$1,5\text{x}10^{-05}$	0,0754	τ_0 = 3.292 Pa
200	28	351,230	14,30	$6,2\text{x}10^{-06}$	0,0416	$g(n)$ = $-5.2\text{x}10^{-7}$
300	38	523,172	19,41	$-2,3\text{x}10^{-04}$	0,0292	
600	63	1039,449	32,17	$4,1\text{x}10^{-04}$	0,0159	

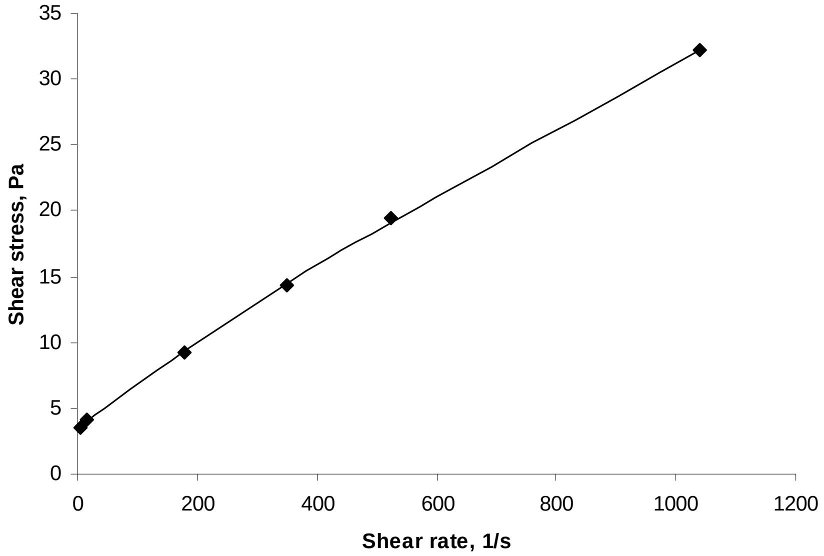

Figure 3 - Rheogram for the oil-based mud analyzed.

Table 3 – Rheological parameters for the oil-base mud through the classical method of analysis.

Rotational Speed N_i, rpm	Dial Reading q_i, degree	Bob Wall Shear Rate $\dot{\gamma}_i$, sec^{-1}	Bob Wall Shear Stress τ_{bi}, Pa	Function Eq. (18) $f(\dot{\gamma}_i)$	Parameter Eq. (19) Π_i	Herschel Bulkley Parameters
3	7	5,107	3,57	-	1,7091	n = 0.876 K = 0.0666 Pa s^n τ_0 = 3.388 Pa $g(n)$ = -1.3x10^{-8}
6	8	10,214	4,09	2,3E-01	0,9311	
100	18	170,229	9,19	2,7E-03	0,0791	
200	28	340,459	14,30	1,0E-03	0,0431	
300	38	510,688	19,41	6,4E-04	0,0302	
600	63	1021,377	32,17	3,2E-04	0,0165	

Consequently, the data for 3 rpm has been disregarded and the calculation proceeded for the remaining five readings. The values of n, K and τ_0 resulted somewhat different from those obtained by Hemphill et al. (1993). This happened because Hemphill et al. (1993) have given up the task of zeroing the two functions f and g, and stopped the iterations as soon as they found the problem of evaluating the non-integer power of a negative value in Eq. (18).

Table 3 summarizes the results of the classical way of performing rheological calculations, when the values of shear rate for a Newtonian fluid is used to fit the data of a non-Newtonian fluid. It is known that, for Newtonian fluids, the shear rate do not depend on the absolute viscosity. As can be shown in Table 3, the differences are not overwhelming, even though this difference becomes lower and lower as the value of n tends to unity. In the case examined, the relative errors are about 0,8%, 6% and 3% for the values of n, K and τ_0 , respectively. These errors are perfectly acceptable for many engineering applications. Nevertheless, it would be interesting to investigate these same relative errors when the n values get farther from unity. As Figure 3 demonstrate, the oil-base mud herein analyzed behaves almost as a Newtonian fluid.

5. CONCLUSIONS

A theory for determining the three parameter of a Herschel-Bulkley fluid has been developed and an algorithm described to apply the theory. The authors have implemented the algorithm in two programming languages, FORTRAN and PASCAL, and also in a spreadsheet, Microsoft EXCEL. In this last case, the SOLVER capability has been used.

It has been shown through a case of the literature how to calculate the three parameters with the equations herein developed. It is suggested that a plug flow ensues in the annular space close to the sleeve wall at small shear rates. This violates the condition of validity of the theory. The way around the problem was to disregard the data for small values of rotational speed and use the remaining data to perform the calculations.

The comparison with the classical method of using Newtonian shear rates gave acceptable errors. It remains to study the case when the value of n gets too far away from unity.

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