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INFLUENCE OF THE PERTURBATION SUN-MOON IN THE TIME OF LIFE OF A SATELLITE

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Summary: the present work was carried out based on the unexpected fact of the lengthening of the time of life of the satellite µSat - thrown Victor August 29 1996. Unexpected fact is said because the predictions based on the classic graphics and the literature on the topic didn't allow suppose a remarkable lengthening of the life of this satellite.

Keywords :*Microsatelite,Time life,Perturbation*

1. **INTRODUCTION:** in the first orbit predictions that we use for propagation of the passings of the studied satellite, took into account only the gravitationals terrestrial harmonic (model cuadripolar) of low order (4X4), those coefficients are presented in the following chart

Chart 1

Zonal harmonicas
J2 = 1082.6300 E-6
J3 = -2.5321531E-6
J4 = -1.6109876 E-6

Of this chart it is inferred that the value of J2 (second harmonica) it is 400 times greater than the biggest value in the following harmonica J3, reason for which is considered that the approach carried out with J2 and rejecting J3 and J4 gives a precision of calculate reasonable.

The other perturbing force considered was the aerodynamic resistance that slows the movement of the satellite and it alters the form of the orbit.

The satellite in study has a passing to low height (approximately to the beginning of its life 230 Km) reason for which was expected a braked in the perigee that made fall slowly this height.

Paradoxically the observed effect was just the opposite, an increase of the height of the perigee, and therefore a variation of the prospective time of life.

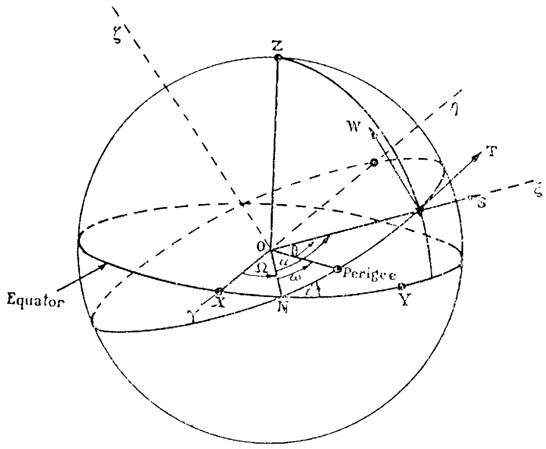


Figure 1

2. - Equation of the movement

2.1 - Problem of two bodies.

Before entering to use the equations of the perturbed movement it is convenient to review the problem of the movement of 2 bodies.

Making firstly this consideration one has the equation of the movement of a low satellite the attraction of a punctual mass that can be a planet expressed in a geocentric system as:

$$\frac{d^2 r}{dt^2} = -\mu \cdot \frac{r}{r^3} \qquad (1)$$

that which gives a system of 3 differential equations of 2do. non lineal order, therefore are 6.constants of integration The solution can be expressed so much in function of the vector of initial state as likewise in terms of the orbital parameters:

r(t) = r (x0,y0,z0, x, y, z)
r(t) = r (a, e, i, w, M) Ω

As the problem of 2 bodies it is expressed in function of these parameters, a solution of “close form” type can be obtained, therefore the new position and speed can be calculated at any instant of time.

The problem of 2 bodies can only predict the movement of the satellite for a short period of time, before the effect of the interferences becomes significant.

2.2 - Equation of the movement with interferences.

When having interferences a term it is added to the equation 1:

$$\frac{d^2 r}{dt^2} = -\mu . \frac{r}{r^3} + a_p \qquad (2)$$

Where ap is the resulting vector of the group of the interferences. Additionally the accelerations of the interferences in the solar system are 10 times minor than the accelerations due to the central body. This way, if we take the planetary equations of Lagrange expressed in function of the perturbing force components we have:

$$\frac{da}{d\theta} = \frac{2.r^2}{n^2.a^2.(1-e^2)}.[S.e.sin\theta + \frac{a}{r}(1-e^2).T]$$

$$\frac{de}{d\theta} = \frac{r^2}{n^2.a^3}.[S.sin\theta + T(\cos\theta + \cos E)]$$

$$\frac{di}{d\theta} = \frac{W.r^3.\cos u}{n^2.a^4.(1-e^2)} \qquad (3)$$

$$\frac{d\Omega}{d\theta} = \frac{W.r^3.sinu}{n^2.a^4.(1-e^2).sini}$$

$$\frac{dw}{d\theta} = \frac{r^2}{n^2.a^3.e}.[-S.\cos\theta + (1 + \frac{r}{a.(1-e^2)}).T.sin\theta - \frac{e.r}{a.(1-e^2)}.W \cot i.sinu]$$

The variations of the orbital parameters are obtained integrating the system (3) to that of a revolution of the satellite, assuming that the parameters stay constant in that interval of time.

We have this way the following interferences:

Gravitationals :

- problem of 3 bodies (sun/moon)
- non esfericity of the earth

Non gravitationals :

- aerodynamic resistance
- pressure of solar radiation
- tide effect
- outgassing

In the classic methods, most of the software of extended use has the interference due to the aerodynamic resistance and the non esfericity of the earth.

In this work the influence of a 3th.perturbing body, the sun, the moon was studied and both gravitational fields are acting on the whole.

3. -Interference due to the gravitacional attraction of the 3er. body.

The physical problem was presented like in Fig.1 according to the Ref. 1.

The problem of the calculation of the interference is presented in defining the perturbing function R, since the same one serious:

$$a_{px} = \frac{\partial R}{\partial x}$$

$$a_{py} = \frac{\partial R}{\partial y}$$

$$a_{pz} = \frac{\partial R}{\partial z}$$

therefore the system (2) it would be:

$$x + \mu.\frac{x}{r^3} = \frac{\partial R}{\partial x}$$

$$y + \mu.\frac{y}{r^3} = \frac{\partial R}{\partial y} \qquad (4)$$

$$z + \mu.\frac{z}{r^3} = \frac{\partial R}{\partial z}$$

Returning to the Fig .1 the vectors S, T, W are the perturbing forces .

In this point we can study the interference moon-sun applying 2 roads different from calculation, one determining the variation of the state vector and the other one determining the variation of the orbital parameters.

3.1 - Variation of the orbital parameters for the interference moon-sun.

In this case the expression of the function R is the classic expression of interference of celestial mechanics:

$$R = G.M_d.\left(\frac{1}{\Delta} - \frac{x.x_d + y.y_d + z.z_d}{r_d^3}\right)$$

differentiating R

$$\frac{\partial R}{\partial x} = -G.M_d.(\frac{x - x_d}{\Delta^3} + \frac{x_d}{r_d^3})$$

$$\frac{\partial R}{\partial y} = -G.M_d.(\frac{y - y_d}{\Delta^3} + \frac{y_d}{r_d^3}) \qquad (5)$$

$$\frac{\partial R}{\partial z} = -G.M_d.(\frac{z - z_d}{\Delta^3} + \frac{z_d}{r_d^3})$$

3.1.1. - Semi major axis

Replacing S,T,W in the ec. corresponding to the system (3) one has:

$$\frac{\partial a}{\partial \theta} = \frac{2.K.r^3}{n^2.a^2.(1-e^2)}.\{[-1+\frac{3}{2}.(A^2+B^2)+3.A.B.sin2u+\frac{3}{2}.(A^2+B^2).\cos 2u].e.sin\theta+$$
$$+3.[A.B.\cos 2u-\frac{1}{2}(A^2-B^2)sin2u][1+e.\cos\theta]\}$$

3.1.2. - Excentricityy

In the same way that previously

$$\frac{\partial e}{\partial \theta} = \frac{K.r^3}{n^2.a^3}.\{[-1+\frac{3}{2}.(A^2+B^2)+3.A.B.sin2u+\frac{3}{2}.(A^2+B^2)\cos 2u].sin\theta$$

$$+3.[A.B.\cos 2u-\frac{1}{2}.(A^2+B^2).sin2u].[\cos\theta+\frac{\cos\theta+e}{1+e.\cos\theta}]\}$$

integrating on 1 revolution gives:

$$\Delta e = -\frac{5.\pi.e}{n^2.a}.(\frac{1+e}{1-e})^{\frac{1}{2}}.T_p \qquad (6)$$

3.1.3. - Perigee height

The distance radial r_p = a. (1-e), it can also be expressed as:

$$r_p = a.\Delta e$$

of (6)it comes off that:

$$r_p = \frac{5.\pi.e}{n^2}.\sqrt{\frac{1+e}{1-e}}.T_p$$

3.1.4 - Ascending node

Being based on the system (3) we have that:

$$\Delta\Omega = \frac{3.\pi.K.C}{2.n^2.\sqrt{1-e^2}.sini}[5.A.e^2 sin2w + B.(2+3.e^2 - 5.e^2.\cos 2w)]$$

3.1.5 - Orbital inclination

$$\Delta i = \frac{3.\pi.K.C}{2.n^2.\sqrt{1-e^2}}.[A.(2+3.e^2.+5.e^2.\cos 2w) + 5.B.e^2.sin2w]$$

3.1.6. - Argument of perigee

$$\frac{\partial w}{\partial\theta} + \frac{\partial\Omega}{\partial\theta}.\cos i = \frac{3.\pi.K.(\sqrt{1-e^2}}{n^2}.\{5.[A.B.sin2w + \frac{1}{2}.(A^2+B^2).\cos 2w] - 1 + \frac{3}{2}.(A^2+B^2)\}$$

3.2 - Speed of change of the orbital parameters.

If we express the previous formulas in function of the time and considering that the increment of the same one is made during 1 revolution, this delta t would correspond to the period of revolution.

According to him previously expressed if the period of revolution expresses it as:

$$\frac{2.\pi}{n} \quad \text{or} \quad \frac{2.\pi.a^{\frac{3}{2}}}{\sqrt{G.M}} \qquad (7)$$

Taking the expressions of the previous point (3.2) and using the equation 7 one has :

$$e = -\frac{15.K}{2.n}.e.(1-e^2)^{\frac{1}{2}}.[A.B.\cos 2w - \frac{1}{2}.(A^2-B^2).sin2w]$$

$$r_p = \frac{15.K.a.e}{2.n}.\sqrt{1-e^2}.[A.B.\cos 2w - \frac{1}{2}.(A^2-B^2)sin2w]$$

$$\Omega = \frac{3.K.C}{4.n.\sqrt{1-e^2}.sini}.[5.A.e^2.sin2w + B.(2+3.e^2-5.e^2.\cos 2w)]$$

$$\frac{\partial i}{\partial t} = \frac{3.K.C}{4.n.\sqrt{1-e^2}}.[A.(2+3.e^2+5.e^2.\cos 2w) + 5.B.e^2.sin2w]$$

$$w + \Omega.\cos i = \frac{3.K}{2.n}.\sqrt{1-e^2}.[5.\{A.B.sin2w + \frac{1}{2}.(A^2-B^2).\cos 2w\} - 1 + \frac{3}{2}.(A^2+B^2)$$

$$\frac{5.a}{2.e.r_d}.\{1-(\frac{5}{4}A^2+B^2)\}(A\cos w + B.sinw)]$$

4. - Resonance:

As it is known in the planetary theory, it can always have resonance and if it happens the more important factor that produces changes in the height of the perigee and consequently in the time of life of a satellite.

The resonance takes place when a geometric configuration of the 3 bodies repeats periodically. To detect this phenomenon in a mathematical way should be presented that calls “conmensurability of the mean motion” that is to say the relationship n1 / n2 should consist of whole numbers as much in the numerator as in the denominator

If the conmensurability is not as exact as the previous relationships it can have a very closely to some of them and the libration takes place that is the oscillation of the geometric configuration around the resonant position.

It can have 15 cases possible of resonance, and they are when some of the following conditions are presented:

$$\beta \pm u_d \pm w = 0$$
$$2u_d \pm \beta \pm 2w = 0$$
$$\beta \pm w = 0$$

$$\beta \pm 2w = 0$$
$$w \pm u_d = 0$$
$$w = 0$$

5. - Variation of the state vector due to the moon-sun interferences.

Applying the 3ra. law of Newton the equation of the movement can be written as:

$$\frac{\partial^2 x}{\partial t^2} = -k^2 .(m_1 + m_2). \frac{x}{r_{12}^3} + k^2 .m_i .(\frac{x}{r^3} - \frac{x}{r_{12}{}^3}) \qquad (8)$$

the 2do. term indicates the interference of the 3er.body, therefore the treatment of the interference is the following one: given the perturbing body coordinates, in the case of the moon x1 y1 z1 and the coordinates of the vehicle x12 y12 z12, being x1 the distance from the satellite to the perturbing body and x12 the coordinates of the satellite to the reference system.

Therefore the 2 nd. term of 9 will give the acceleration caused by the interference.

Arrived to this point, one has the new state vector and with is calculated the new orbital parameters. Anyway, the detailed development in this topic will be object of another technical note due to its extension and complexity.

6. - Evolution of the apogee and perigee

In the Figure 2 it is compared the evolution of the acme and perigee of the satellite µSAT - Victor, according to data raised by the system NORAD (you value measured), with calculations carried out based on a software that keeps in mind the interference gravitational moon-sun.

As can it turns in this imagines it can appreciate a good correlation of measured values and calculated in the evolution of the perigee, reason for which forces to continue carrying out a deeper investigation on this topic.

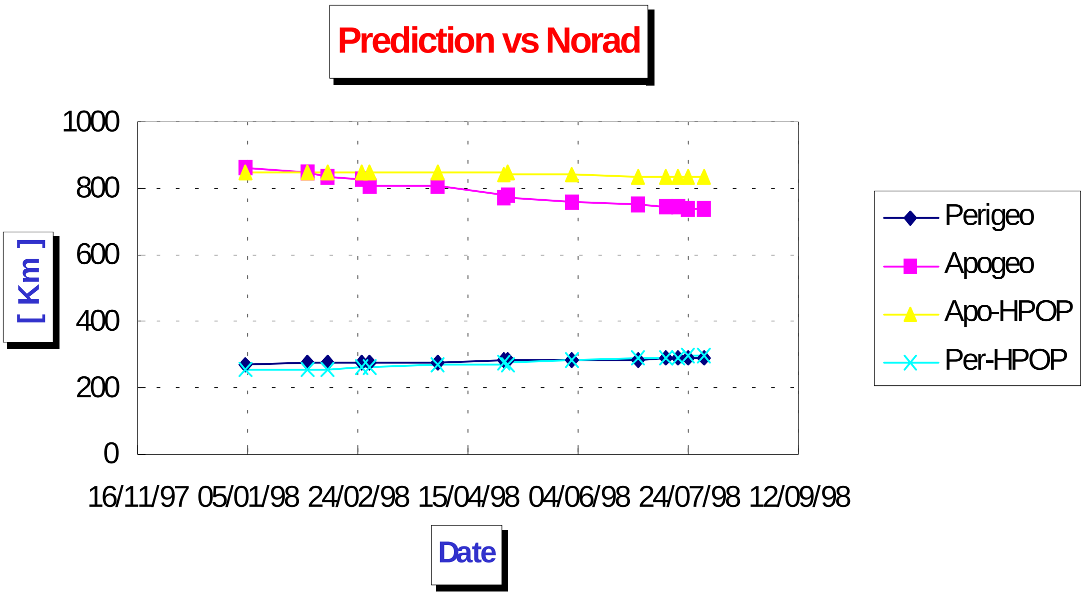
In the Figure 3, the software is also compared without interference with raised data of NORAD and the same program with the application of the interference sun-moon.

Also in this graph a fact worthy of more study is presented, since the prediction without interference accompanies the evolution perfectly in fall of the acme, but being noticed a remarkable discrepancy in the perigee like era of waiting.

7 -Conclusion:

The main object in this work was to analyze the lengthening of the time of life of the satellite µSAT Víctor, fact that is understood when the interference sun-moon is applied on the same one, causing an elevation of the perigee and an overestimation of the perigee height.

It is to investigate another orbit with inclinations far from those that give resonance, and that therefore they produce increase of perigee height, since in the studied case one has an orbit with an inclination of 62.5° very nearly to the resonant inclination of 63.4°.



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