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A NOVEL FINITE ELEMENT PROCEDURE FOR HYDRODYNAMIC THIN GAS FILM LUBRICATION

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Abstract. *An efficient and accurate finite element procedure, based on the Galerkin weighted residual method and a novel family of high-order shape functions, is devised to model high speed thin film gas flows. The novel procedure performs as efficiently as upwind finite element methods developed for high speed gas bearings without introducing numerical diffusion into the solution. The computational efficiency and accuracy of the novel high-order finite element procedure are evaluated for plane slider and Rayleigh step gas bearings.*

Keywords: *Gas Bearing, High Speed, Upwind Methods*

1. INTRODUCTION

Gas bearings are largely used in machinery applications that require low friction or an oil free environment, such as in high-precision instruments, computer magnetic storage devices, gyroscopes, microbearings, and many others. Advances in gas bearing technology have been accompanied by the development of accurate numerical tools to predict the bearing static and dynamic behavior under stringent operating conditions.

The finite element method (FEM) has been widely used in both thin and ultra-thin gas film lubrication problems due to its flexibility to represent complex bearing geometries and associated boundary conditions. Thin gas film flows are described by the classical Reynolds equation for compressible fluids. Some problems of interest are grooved gas bearings and seals used in auxiliary turbomachinery. Reddi & Chu (1970) are pioneers in the application of the FEM in thin gas film lubrication. They developed a FEM incremental scheme based on the Galerkin weighted residual method to analyze plane slider and grooved gas bearings. Hernandez & Boudet (1995) and Zirkelback & San Andrés (1998) also carried out analyses of grooved gas bearings by using FEM schemes based on the Galerkin method.

In ultra-thin gas film lubrication, the gas flow is described by the modified Reynolds equation, which accounts for the molecular gas rarefaction effects. Computer flying heads and microbearings are some problems of interest. Mitsuya & Ohkubo (1987), Kubo *et al.* (1988), and Peng & Hardie (1995) performed FEM analyses of computer flying heads by using the Galerkin method. Most of the recent advances in gas bearing technology have been driven by the computer industry (Pan, 1990).

Higher efficiency and productivity demand increased operating speeds for gas bearings. The Reynolds equation for compressible fluids is a convective-diffusion transport equation. At high speeds, the Couette flow terms (advection transport terms) dominate the gas flow. Procedures based either on central finite difference schemes or on the Galerkin weighted residual method usually exhibit numerical oscillations in the solution of gas lubrication problems, where convective flow terms are important. In order to render numerically stable solutions for the Reynolds equation at high speeds, upwind schemes and adaptive methods have been used to build efficient FEM procedures for gas bearings.

FEM upwind schemes are generally based on the Petrov-Galerkin weighted residual method. These schemes have been implemented using either non-symmetrical weighting functions (Heinrich *et al*., 1977) or special numerical integration of the advection terms (Hughes, 1978). Garcia-Suarez *et al*. (1984) presented one of the first FEM analyses of gas slider bearings for computer magnetic storage devices using an upwind scheme, based on the selective reduced integration proposed by Hughes (1978). Wahl *et al*. (1996) developed an efficient FEM upwind scheme, based on the streamline-upwind/Petrov-Galerkin formulation devised by Brooks & Hughes (1982), to study slider bearings for hard disk drives. Bonneau *et al.* (1993) performed a FEM analysis of grooved gas bearings employing an upwind scheme based on Heinrich *et al*. (1977). FEM procedures using principles of adaptive methods have been developed by Hendriks (1988) and Nguyen (1994) to analyze slider bearings in computer peripheral devices.

Upwind methods generally require special schemes to evaluate the advection flow terms of the Reynolds equation. In this paper, a novel and efficient FEM procedure is developed to analyze high speed hydrodynamic gas bearings. This novel procedure founded on the Galerkin weighted residual method employs a novel class of high-order shape functions different from the polynomials widely used in the FEM. The high-order shape functions are analytically determined from an approximate solution to the non-linear Reynolds equation within an element. The advection flow terms are evaluated without resort to special numerical schemes. Steady-state analyses of one-dimensional gas-lubricated plane slider and Rayleigh step gas bearings are carried out to validate the novel procedure. The novel high-order FEM procedure performs as efficiently as procedures based on the Petrov-Galerkin method without introducing numerical diffusion into the solution.

2. GAS BEARING MODELING

Two simple gas bearing geometries, shown in Fig. 1, are studied to test the novel finite element procedure.

The dimensionless form of the steady-state one-dimensional Reynolds equation for isothermal ideal gas flows (DiPrima, 1968) is expressed as

$$\frac{\partial}{\partial X}\left(PH^{3}\frac{\partial P}{\partial X}\right)=\Lambda\frac{\partial}{\partial X}(PH) \tag{1}$$

where P is the dimensionless hydrodynamic pressure along the bearing. The bearing leading and trailing edges are at atmospheric pressure ($P(0)=P(L)=1$). H is the dimensionless fluid film thickness ($H=h/h_2$). X is the dimensionless axial coordinate ($X=x/L$) and L is the bearing length. Λ is the bearing number or compressibility number given by

$$\Lambda=\frac{6\mu uL}{P_a h_2^2} \tag{2}$$

where u is the velocity of the runner surface and P_a is the atmospheric pressure. The fluid viscosity is given by μ. The bearing number is an important parameter in gas lubrication problems, since it describes the level of the fluid compressibility as function of the sliding speed. For low bearing numbers, Eq. (1) is an elliptic-type differential equation, while for high bearing numbers ($\Lambda\rightarrow\infty$) this equation becomes a parabolic-type differential equation.

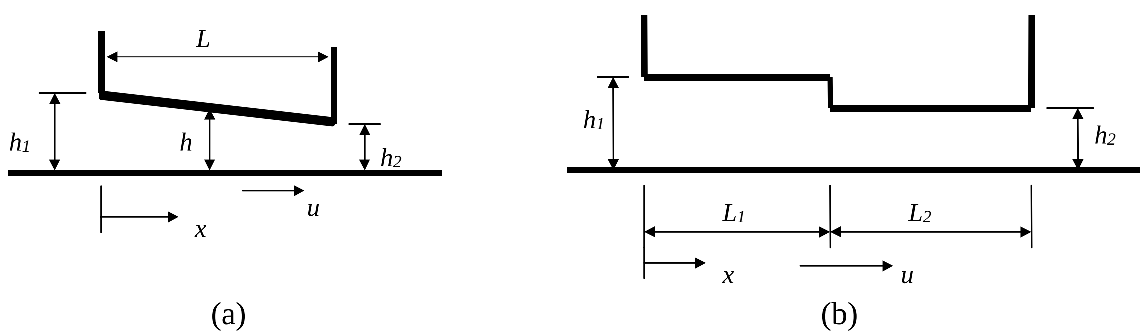

Figure 1 – Schematic views of a plane slider bearing (a) and a Rayleigh step bearing (b).

3. FINITE ELEMENT FORMULATIONS

Two finite element procedures are employed to solve the steady-state Reynolds equation for gas slider bearings. An upwind FEM scheme, based on the Petrov-Galerkin weighted residual method using non-symmetrical weighting functions, and the novel high-order FEM scheme based on the Galerkin method, are implemented into the same generic algorithm to permit comparisons of computational efficiency and accuracy.

3.1 A Petrov-Galerkin FEM Scheme

The flow domain is divided into two-node finite elements. The steady-state Reynolds equation for an arbitrary finite element *(e)* is given by

$$\frac{d}{dX_e}\left(\frac{P_eH_e^3}{l_e}\frac{dP_e}{dX_e}-\Lambda P_eH_e\right)=\frac{d(m_e)}{dX_e}=0 \tag{3}$$

where X_e ($0\leq X_e\leq 1$) represents the element local axial coordinate and l_e is the element dimensionless length ($l_e=L_e/L$). The dimensionless flow rate within the finite element domain is represented by m_e. Linear polynomial interpolation functions $\{N_i^e\}_{i=1,2}$ for pressure and quadratic polynomial weighting functions $\{\psi_i^e\}_{i=1,2}$ introduced by Heinrich *et al*. (1977) are used in the formulation.

$$\psi_1^e = 1 - X_e + 3\alpha(X_e^2 - X_e) \quad , \quad \psi_2^e = 1 - X_e - 3\alpha(X_e^2 - X_e) \; ; \tag{4.a}$$

$$N_1^e = 1 - X_e \quad , \quad N_2^e = X_e \tag{4.b}$$

α is an upwind parameter ($0\leq\alpha\leq 1$) controlling the level of upwinding in the solution. The proper selection of α is crucial to avoid numerical artificial diffusion in the computations. Full upwinding effects are obtained for $\alpha=1$, while for $\alpha=0$ the procedure becomes the classical Galerkin or Bubnov-Galerkin weighted residual method.

The following element equation for pressure is derived using the polynomials given in Eq. (4)

$$K_{ji}^e P_i^e = Q_j^e \qquad (i=1,2;j=1,2) \tag{5}$$

where

P_i^e are the nodal pressures for element (e),

$Q_j^e = (\psi_j^e m_e)\Big|_{X_e=1} - (\psi_j^e m_e)\Big|_{X_e=0}$ is the internal flux balance over element *(e)*, and

$$K_{ji}^e = \int_0^1 \Big(\frac{P_o^e H_e^3}{l_e}\frac{dN_i^e}{dX_e}\frac{d\psi_j^e}{dX_e} - \Lambda H_e N_i^e \frac{d\psi_j^e}{dX_e}\Big).dX_e \text{ is the element fluidity matrix.} \tag{6}$$

In the Petrov-Galerkin method, the weighting functions are different from the shape functions only in the evaluation of the advection terms, given in the second right-hand side term of Eq. (6). The element fluidity matrix takes the following form

$$\left[K^e\right] = \frac{P_o^e H_e^3}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{\Lambda H_e}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} + \frac{\alpha\Lambda H_e}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} . \tag{7}$$

The global fluidity matrix obtained by assembling the element matrices given in Eq. (7) is an asymmetric tri-diagonal matrix. When Λ increases the global fluidity matrix tends to become ill conditioned if $\alpha=0$. The proper selection of α makes the fluidity matrix remain diagonally dominant for any bearing number. The method of successive substitutions (Stewart, 1996) is used to solve the finite element system of equations. Ambient pressure is used as the initial value of pressure field in the iterative process that ends when $\left|P_{n+1} - P_n\right| < 10^{-6}$, where P_{n+1} and P_n are the dimensionless pressures computed at iterations ($n+1$) and (n), respectively. The dimensionless bearing load capacity F_z is computed by integrating the pressure field (P-1) over the bearing domain.

3.2 The High-Order Galerkin FEM Scheme

A novel family of shape functions is analytically obtained from the approximate solution of the dimensionless one-dimensional Reynolds equation for an element (e).

$$\frac{d}{dX_e}\left(\frac{P_e H_e^{\,3}}{l_e}\frac{dP_e}{dX}\right) = \Lambda \frac{d}{dX_e}(P_e H_e) \tag{8}$$

Over an element domain, $\left(P_e H_e^3\right)$ and $\left(\Lambda H_e\right)$ are computed from averaged values of pressure and film thickness. Equation (8) is re-written in linear form as

$$\frac{d^2 P_e}{dX_e^{\,2}} = \lambda_e \frac{dP_e}{dX_e} \tag{9}$$

where $\lambda_e = \dfrac{\Lambda l_e}{P_{av} H_{av}^2}$ is a local bearing parameter computed for meaningful averaged values of pressure (P_{av}) and film thickness (H_{av}) within (e). Solution of Eq. (9) renders two exponential shape functions hereby called “exact” shape functions,

$$N_1^e = \frac{e^{\lambda_e X_e} - e^{\lambda_e}}{1 - e^{\lambda_e}} \quad , \quad N_2^e = \frac{1 - e^{\lambda_e X_e}}{1 - e^{\lambda_e}} \;. \tag{10}$$

The “exact” shape functions are used as both the interpolation and weighting functions in the computation of the element fluidity matrix. These functions are of higher order than the order of polynomials widely used in the FEM. The fluidity matrix for an element (e) is given by

$$\left[K^e\right] = \frac{P_o^e H_e^3}{2l_e}\lambda_e \frac{(e^{\lambda_e}+1)}{(e^{\lambda_e}-1)}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{\Lambda H_e}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}. \tag{11}$$

For the limit case $\Lambda \to 0$, the “exact” shape functions produce the same fluidity matrix as that given by the classical Galerkin method using linear shape functions. For $\Lambda \to \infty$, the fluidity matrix has the same structure as that obtained by the Petrov-Galerkin method with full upwind. The upwinding effect is intrinsically contained in the “exact” functions without resort to special schemes for the advection terms. No artificial viscosity is therefore introduced into the solution.

4. NUMERICAL RESULTS

The validation of the finite element procedures herein developed and an analysis of accuracy and efficiency of the high-order Galerkin FEM scheme follow. A full upwind parameter, $\alpha=1$, is used in all computations performed with the Petrov-Galerkin scheme.

4.1 Validation

A first example for validation of the FEM procedures shows the predictions of the dimensionless pressure distribution of a plane slider bearing with film thickness ratio $h_1/h_2=3$ and $\Lambda=10$ as computed by Reddi & Chu (1970). Figure 2 depicts comparative results for the dimensionless pressure predicted by the Petrov-Galerkin and high-order Galerkin schemes for a domain with 40 elements. Both FEM schemes render satisfactory results for this low bearing number example. However the full upwind Petrov-Galerkin scheme presents some loss in accuracy due to the numerical artificial diffusion, which is caused by the use of full upwind parameter in a diffusion-dominated problem. No numerical diffusion is introduced into the solution computed by the high-order Galerkin scheme.

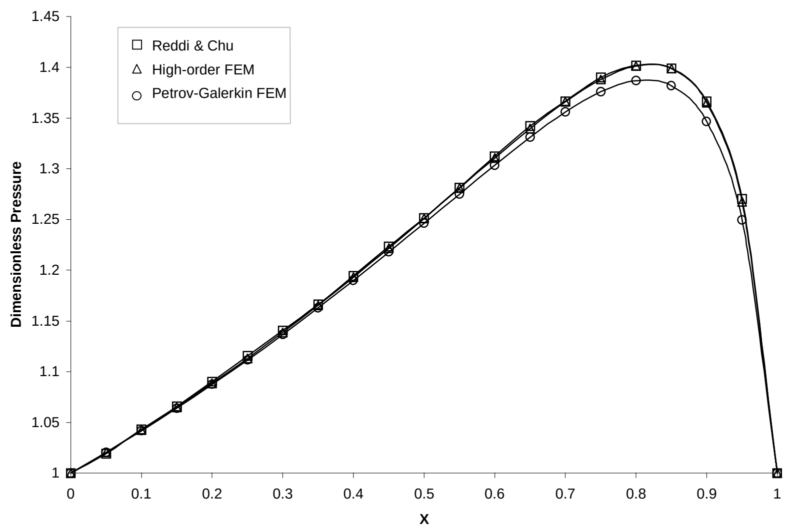

Figure 2 – Comparative results for dimensionless pressure on a plane slider bearing.

The computation of the dimensionless bearing load capacity of Rayleigh step bearings is the second example selected to validate the FEM procedures. Predictions obtained by both the Petrov-Galerkin and High-order Galerkin schemes for domains with 100 elements are compared with the linearized solution presented by Hamrock (1994). Figure 3 shows the comparative results for three cases of Rayleigh step bearings ($L_1/L_2=1$). As the film thickness ratio (h_1/h_2) increases, the difference between the linearized and the FEM solutions increases. The linearized solution generally overpredicts the load for low bearing numbers ($\Lambda<50$) and under-predicts it for moderate and high Λ. All solutions tend to the same asymptotic values as the bearing number increases.

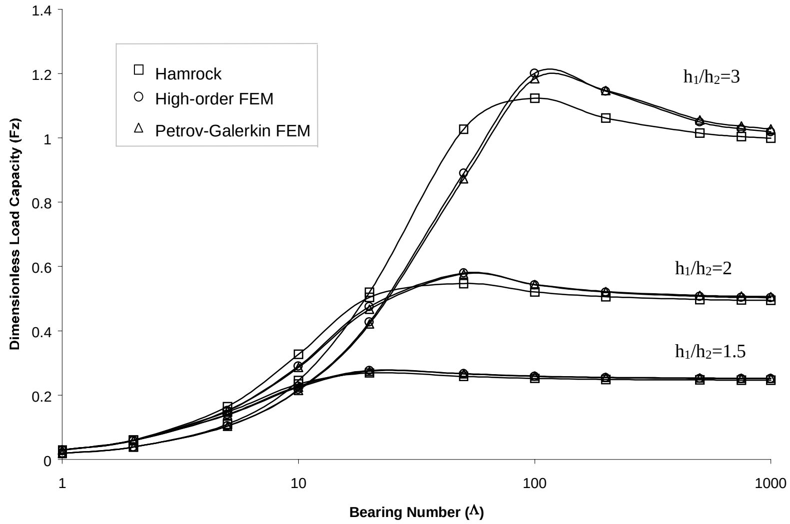

Figure 3 – Dimensionless bearing load capacity for Rayleigh step bearings ($L_1/L_2=1$).

4.2 Efficiency of the High-order Scheme

The computational efficiency of the high-order Galerkin scheme is evaluated against the Petrov-Galerkin scheme. Figure 4 depicts the normalized computational times, obtained by both the Petrov-Galerkin (dotted line) and high-order Galerkin (solid line) schemes to compute the load capacity of gas plane slider bearing with $h_1/h_2=3$ for low, moderate and high bearing numbers (Λ), versus the number of elements. The normalization is performed in relation to the lowest computational time taken by the high-order scheme. Both schemes perform similarly in this example. For high bearing numbers and large number of elements the Petrov-Galerkin scheme seems to present a performance slightly superior. Predictions for load capacity computed by both schemes present relative deviation of 1.3 % for Λ=10 and meshes with 100 elements. For all other cases, the relative deviations are smaller than 1 %.

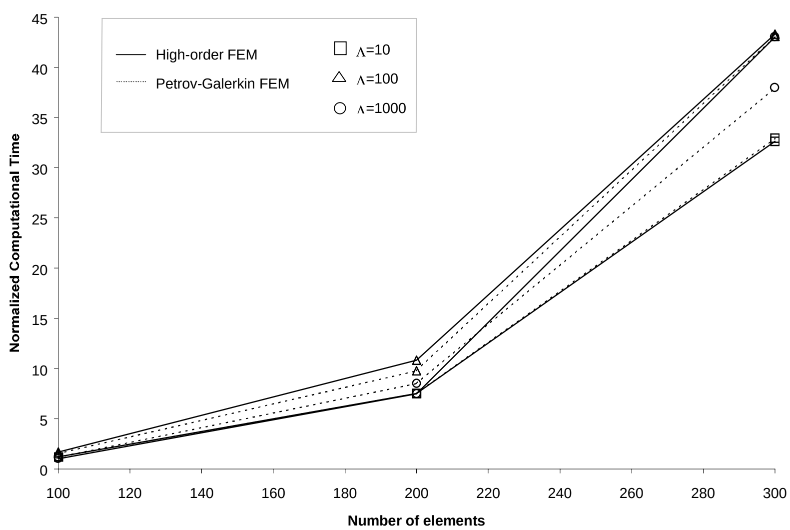

Figure 4 – Efficiency of the FEM procedures for a plane slider bearing ($h_1/h_2=3$).

The computational efficiency of the FEM procedures is also evaluated for a case of Rayleigh step bearing ($h_1/h_2=2$, $L_1/L_2=1$) at three bearing numbers (Λ). Figure 5 depicts the normalized computational times taken by both schemes to compute the load capacity of a Rayleigh step bearing versus the number of elements. Again both schemes present similar performance. However, for high bearing numbers and large meshes the high-order Galerkin scheme tends to perform more efficiently than the Petrov-Galerkin scheme. For Λ=10 and meshes with 100 elements, predictions obtained by both schemes present relative deviation of 1.40 %. For all other cases, the relative deviation is smaller than 0.71%.

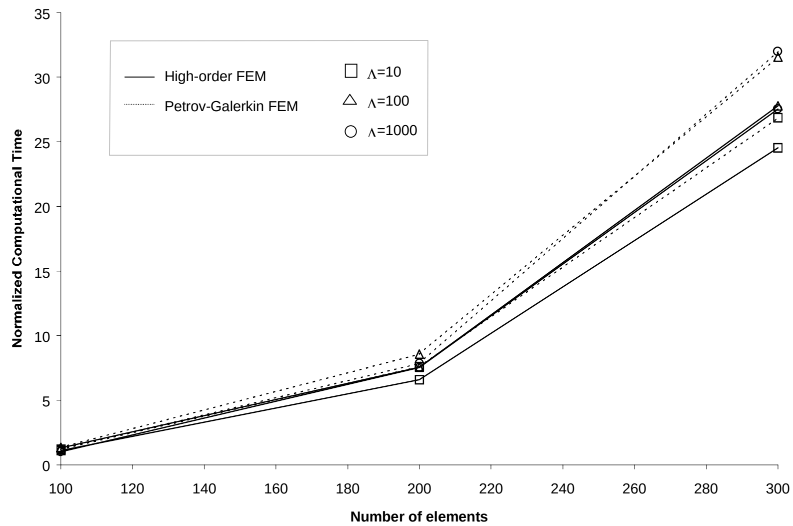

Figure 5 – Comparative computational efficiency for a Rayleigh step bearing ($h_1/h_2=2$, $L_1/L_2=1$).

To illustrate the limitations of conventional numerical procedures in dealing with high speed gas bearing problems, Fig. 6 shows the pressure distribution computed by the classical Galerkin (dotted line) and high-order Galerkin (solid line) FEM schemes for two cases of Rayleigh step bearings ($L_1/L_2=1$), at bearing number $\Lambda=1000$. The number of elements is 100 in all cases. Numerical oscillations usually arise in the solution provided by the classical Galerkin method in the regions of large pressure gradients (trailing edge of the bearing). A remedy for the classical scheme would be to use very fine meshes, with about 350 elements, to render results as accurate as those provided by the high-order scheme. The classical Galerkin FEM scheme shows poor computational efficiency for high speed gas bearings.

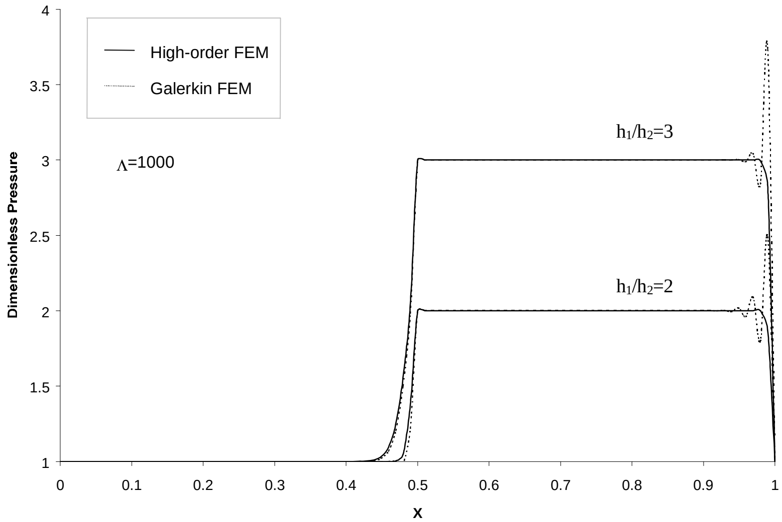

Figure 6 – Numerical oscillations at the trailing edge of Rayleigh step bearings ($L_1/L_2=1$).

CONCLUSIONS

Demands for more efficiency and higher productivity have prompted the development of efficient and accurate engineering tools to analyze bearings with compressible fluids operating at high speeds. A novel finite element procedure, based on a high-order formulation of the Galerkin weighted residual method, is shown to be as efficient as upwind finite element procedures in the analysis of high speed gas bearings, without resort to any special scheme for evaluation of the advection flow terms of the Reynolds equation. The accuracy and efficiency of the high-order scheme are successfully tested for one-dimensional gas-lubricated plane slider and Rayleigh step bearings.

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