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TIME DOMAIN-BASED IDENTIFICATION OF MODAL PARAMETERS AND EXCITATION FORCES USING ORTHOGONAL FUNCTIONS

Ricardo P. Pacheco *(rpacheco@mecanica.ufu.br)*
Valder Steffen Jr. *(vsteffen@mecanica.ufu.br)*
Federal University of Uberlândia, Department of Mechanical Engineering
38400-902, Uberlândia-MG, Brazil
Domingos A. Rade *(domingos@ufu.br)*
Federal University of Uberlândia, Department of Physical Sciences

Abstract. *A set of orthogonal functions can be integrated using a so-called operational matrix. This property can be used to transform linear differential equations into algebraic equations, which can be easily solved. When dealing with mechanical systems, structural and/or modal models can be determined from these algebraic equations. In this paper, this procedure is investigated by using different types of orthogonal functions: Fourier series, Legendre polynomials, Jacobi polynomials, Chebyshev series, Block-Pulse functions and Walsh functions. A feasibility study is performed on this technique when applied to the problems of modal parameter identification and input force reconstruction, using the time responses of the structure. The main features and capabilities of the method are demonstrated through applications to both numerically simulated and experimentally tested mechanical systems.*

Key words: *Parameter identification, Force identification, Orthogonal functions, Inverse problems*

1. INTRODUCTION

Modal parameters – natural frequencies, modal damping factors and mode shapes – have long been used in various applications in the realm of Mechanical Engineering, such as: dynamic analysis of complex structures, finite element model updating, evaluation of dynamic loads, prediction of dynamic response, control, damage detection, etc.

Some time domain modal identification techniques became very popular the last two decades. Ibrahim Time Domain Method (Ibrahim and Mikulcik, 1977), complex exponential method (Brown et al., 1979), Polyreference Time Domain Method (Vold et al., 1982), Eigensystem Realization Algorithm (Juang and Pappa, 1985) and Autoregressive-Moving Average Model (Smail et al., 1993) have been successfully applied to different kinds of mechanical systems.

Another problem that has received much attention from researchers and engineers is the identification of excitation forces from the dynamic responses. The main motivation for this is that in a number of practical situations the direct measurement of forces, using force gauges, proves to be ineffective or even impossible. Such is the case, for example, when the forces are applied at inaccessible locations of the structure or when the introduction of force transducers is likely to significantly change the dynamic characteristics of the mechanical system. In these cases, the indirect identification of input forces from the dynamic responses of the structure - which can generally be easily acquired - appears as a valuable alternative.

Several techniques for force identification, operating either in the time domain or in the frequency domain, have been proposed (Stevens, 1987). As for the time domain methods, the most widely known is that named SWAT (Sum of Weighted Accelerations Technique) (Bateman et al., 1992). This method is based on the modal decomposition of the acceleration time responses and utilizes the modal equilibrium equations for the rigid body modes. Recently, Genaro and Rade (1998) studied a method which enables to extend the range of application of SWAT method, by taking into account both rigid body and elastic modes.

Various types of orthogonal functions have been used for analysis, identification and control purposes since the mid seventies, such as: Walsh functions (Chen and Hsiao, 1975), Laguerre polynomials (Shih et al., 1986), Block-Pulse functions (Wang and Marleau, 1987), Legendre polynomials (Chou, 1987), Chebyshev series (Mohan and Datta, 1988), Jacobi polynomials (Horng and Chou, 1987), Fourier series (Chung and Sun, 1987) and Hermite polynomials (Kekkeris and Paraskevopoulos, 1988).

The main purpose of this paper is to present an unified approach for the use of various types of orthogonal functions for the identification of modal parameters and excitation forces based on time domain responses of mechanical systems.

In the remainder, a brief description of the orthogonal functions used and the basic formulation of the identification method are first presented. Then, numerical applications to both numerically simulated and experimentally tested mechanical systems are shown, aiming at illustrating the main features of the method.

2. ORTHOGONAL FUNCTIONS

2.1 Definitions

A set of functions $\{\phi_i(t)\}$, i = 1, 2, 3, ... is said to be *orthogonal* in the interval [a,b] if:

$$\int_a^b \phi_m(t)\phi_n(t)dt = K_{mn}, \qquad \text{where: } \begin{cases} K_{mn} = 0 \ \ if \ m \neq n \\ K_{mn} \neq 0 \ \ if \ m = n \end{cases}$$

If K_{mn} is the Kronecker's delta, the set of functions $\{\phi_i(t)\}$ is said *orthonormal*. The following integral property holds for a set of r orthogonal functions:

$$\underbrace{\int_0^t \cdots \int_0^t}_{n\ times} \{\phi(\tau)\}(d\tau)^n \cong [P]^n \{\phi(t)\} \tag{1}$$

where: $[P] \in \Re^{r,r}$ is a square matrix with constant elements, called *operational matrix*

$\{\phi(t)\} = \{\phi_0(t) \quad \phi_1(t) \quad \ldots \quad \phi_{r-1}(t)\}^T$ is the vectorial basis of the orthogonal series

In the following sections, the vectorial basis and operational matrix related to each type of orthogonal function considered in this paper are briefly reviewed.

2.2 Fourier series

Vectorial basis in the interval [0,T]	Operational matrix
$\{\varphi(t)\}=\{\varphi_0(t)\ \ \varphi_1(t)\ \ \ldots\ \ \varphi_h(t)\ \ \varphi_1^*(t)\ \ \ldots\ \ \varphi_h^*(t)\}^T$ $\varphi_n(t)=cos\dfrac{2n\pi t}{T}$, n = 0, 1, ..., h+1 $\varphi_n^*(t)=sin\dfrac{2n\pi t}{T}$, n = 1, 2, ..., h $r=2h+1$	$[P]=\left[\begin{array}{c\|c\|c} \dfrac{T}{2} & \{0\}_{1\times h} & -\dfrac{T}{\pi}\{\tilde{e}\}_h^T \\ \hline \{0\}_{h\times 1} & [0]_{h\times h} & \dfrac{T}{2\pi}[\tilde{I}]_{h\times h} \\ \hline \dfrac{T}{2\pi}\{\tilde{e}\}_h & -\dfrac{T}{2\pi}[\tilde{I}]_{h\times h} & [0]_{h\times h} \end{array}\right]$ $\{\tilde{e}\}_h=[1\ \ 1/2\ \ 1/3\ \ \cdots\ \ 1/h]^T$ $[\tilde{I}]_{h\times h}=diag\{1\ \ 1/2\ \ 1/3\ \ \cdots\ \ 1/h\}$

2.3 Shifted Legendre polynomials

Recursive formula in the interval $t\in[0,\ t_f]$	Operational matrix
$(n+1)\,p_{n+1}(t)=(2n+1)\left(\dfrac{2t}{t_f}-1\right)p_n(t)-n\,p_{n-1}(t)$, n = 1, 2, 3, ..., r-1 $p_0(t)=1$ $p_1(t)=2t/t_f-1$	$[P]=\dfrac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ -\dfrac{1}{3} & 0 & \dfrac{1}{3} & 0 & \cdots & 0 & 0 \\ 0 & -\dfrac{1}{5} & 0 & \dfrac{1}{5} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & \dfrac{1}{2r-3} \\ 0 & 0 & 0 & 0 & \cdots & \dfrac{-1}{2r-1} & 0 \end{bmatrix}$

2.4 Shifted Jacobi polynomials

Recurrence formula in the interval $t\in[0,\ t_f]$	Operational matrix
$2i(i+\alpha+\beta)(2i+\alpha+\beta-2)J_i(t)=(2i+\alpha+\beta-1)$ $[(2i+\alpha+\beta)(2i+\alpha+\beta-2)(2t/t_f-1)+\alpha^2-\beta^2]J_{i-1}(t)-$ $-2(i+\alpha-1)(i+\beta-1)(2i+\alpha+\beta)J_{i-2}(t)$, i = 2, 3, ..., r-1 $\alpha>-1$ $\beta>-1$ $J_0(t)=1$ $J_1(t)=-(\beta+1)+(\lambda+1)t/t_f$ $\lambda=\alpha+\beta+1$	$[P]=t_f\begin{bmatrix} b_0 & c_1 & 0 & 0 & \cdots & 0 & 0 \\ b_1 & d_1 & c_2 & 0 & \cdots & 0 & 0 \\ b_2 & e_1 & d_2 & c_3 & \cdots & 0 & 0 \\ b_3 & 0 & e_2 & d_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ b_{r-2} & 0 & 0 & 0 & e_{r-3} & d_{r-2} & c_{r-1} \\ b_{r-1} & 0 & 0 & 0 & 0 & e_{r-2} & d_{r-1} \end{bmatrix}$ $b_0=(\beta+1)/(\lambda+1)$ $b_1=[-(\alpha+1)(\beta+1)/\lambda(\lambda+1)(\lambda+2)]-\Gamma(\beta+2)/2!\,\lambda\Gamma(\beta)$ $b_i=(-1)^i\,\Gamma(i+\beta+1)/(i+\lambda-1)(i+1)!\,\Gamma(\beta)$ (i = 2, 3, ..., r-1) $c_i=(i+\lambda-1)/(2i+\lambda-1)(2i+\lambda-2)$ (i = 1, 2, ..., r-1) $d_i=(\alpha-\beta)/(2i+\lambda+1)(2i+\lambda-1)$ (i = 1, 2, ..., r-1) $e_i=-(i+\alpha+1)(i+\beta+1)/(2i+\lambda+2)(2i+\lambda+1)(i+\lambda)$ (i = 1, 2, ..., r-2)

2.5 Shifted Chebyshev polynomials

Recurrence formula in the interval $t \in [0,\ t_f]$	Operational matrix
$T_{i+1}(t) = 2\left(\frac{2t}{t_f} - 1\right) T_i(t) - T_{i-1}(t)$ $i = 1, 2, ..., r\text{-}1$ $T_0(t) = 1$ $T_1(t) = \frac{2t}{t_f} - 1$	$[P] = \frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -1/4 & 0 & 1/4 & 0 & \cdots & 0 & 0 & 0 \\ -1/3 & -1/2 & 0 & 1/6 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{(-1)^{r-1}}{(r-1)(r-3)} & 0 & 0 & 0 & \cdots & \frac{-1}{2(r-3)} & 0 & \frac{1}{2(r-1)} \\ \frac{(-1)^r}{r(r-2)} & 0 & 0 & 0 & \cdots & 0 & \frac{-1}{2(r-2)} & 0 \end{bmatrix}$

2.6 Block-Pulse functions

Vectorial basis in the interval $[0, rT]$	Operational matrix
$\psi_i(t) = \begin{cases} 1 & for\,(i-1)T \le t \le iT \quad , i = 1,2,...,r \\ 0 & if\ not \end{cases}$	$[P] = T^{k+1}\frac{1}{(k+2)!}\begin{bmatrix} f_1 & f_2 & f_3 & \cdots & f_r \\ 0 & f_1 & f_2 & \cdots & f_{r-1} \\ 0 & 0 & f_1 & \cdots & f_{r-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & f_1 \end{bmatrix}$ k: number of successive integrations - 1 $f_1 = 1$ $f_i = i^{k+2} - 2(i-1)^{k+2} + (i-2)^{k+2} \quad , i = 2, 3, ..., r$

2.7 Walsh functions

Vectorial basis in the interval $t \in [0,\ t_f]$	Operational matrix
$\phi_n(z) = \{r_q(z)\}^{b_q}\{r_{q-1}(z)\}^{b_{q-1}}\{r_{q-2}(z)\}^{b_{q-2}}\cdots$ $, n = 0, 1, ..., r\text{-}1$ $z = t/t_f$ $\phi(z)$: Walsh function $r(z)$: Rademacher function $q = [log_2 n] + 1$ $[\bullet]$: “the largest integer of” $b_q b_{q-1} \cdots b_1$: binary expression of n	$[P] = t_f\left[\begin{array}{c:c} \begin{array}{c:c} \begin{array}{c:c} \frac{1}{2}\ \ddots & -\frac{2}{r}[I]\left(\frac{r}{8}\right) \\ \hdashline \frac{2}{r}[I]\left(\frac{r}{8}\right) & [0]\left(\frac{r}{8}\right) \end{array} & -\frac{1}{r}[I]\left(\frac{r}{4}\right) \\ \hdashline \frac{1}{r}[I]\left(\frac{r}{4}\right) & [0]\left(\frac{r}{4}\right) \end{array} & -\frac{1}{2r}[I]\left(\frac{r}{2}\right) \\ \hdashline \frac{1}{2r}[I]\left(\frac{r}{2}\right) & [0]\left(\frac{r}{2}\right) \end{array}\right]$

3. FORMULATION OF THE TIME DOMAIN IDENTIFICATION TECHNIQUE

The proposed identification method can exploit either free or forced time responses, in terms of either displacements, velocities or accelerations. Since the formulations for these three types of responses are quite similar, only the formulation for forced and free systems, in terms of displacements, will be presented in the following.

The equation of motion for a N d.o.f. system is given by:

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \tag{2}$$

where $[M]$, $[C]$ and $[K] \in \Re^{N,N}$ are, respectively, the inertia, damping and stiffness matrices, $\{x(t)\} \in \Re^{N,1}$ is the vector of displacement time responses and $\{f(t)\} \in \Re^{N,1}$ is the vector of excitation forces.

Integrating Eq. (2) twice in the interval $[0,t]$, one obtains:

$$\begin{aligned}&[M](\{x(t)\} - \{x(0)\} - \{\dot{x}(0)\}t) + [C]\left(\int_0^t \{x(\tau)\}d\tau - \{x(0)\}t\right) + \\ &+ [K]\int_0^t\int_0^t \{x(\tau)\}d\tau^2 = \int_0^t\int_0^t \{f(\tau)\}d\tau^2\end{aligned} \quad (3)$$

where $\{x(0)\}$ and $\{\dot{x}(0)\}$ are the vectors of initial displacements and velocities, respectively.

The signals $\{x(t)\}$ and $\{f(t)\}$ can be expanded in truncated series of r orthogonal functions as follows:

$$\{x(t)\} = [X]\{\phi(t)\} \qquad \{f(t)\} = [F]\{\phi(t)\} \quad (4)$$

where: $[X] \in \Re^{N,r}$ is the matrix of the coefficients of the expansion of $\{x(t)\}$
$[F] \in \Re^{N,r}$ is the matrix of the coefficients of the expansion of $\{f(t)\}$

Substituting Eq. (4) in Eq. (3) and applying the integral property given by Eq. (1), the following system of algebraic equations is obtained:

$$[H][J] = [E] \quad (5)$$

where: $[H] = \left[\ [M] \ \vdots\ -[M]\{x(0)\} \ \vdots\ -[M]\{\dot{x}(0)\} - [C]\{x(0)\} \ \vdots\ [C] \ \vdots\ [K]\ \right]$
$[J] = \left[\ [X]^T \ \vdots\ \{e\} \ \vdots\ [P]^T\{e\} \ \vdots\ [P]^T[X]^T \ \vdots\ [P]^{T\,2}[X]^T\ \right]^T$
$[E] = [F][P]^2$

In this equation, $\{e\} \in \Re^{r,1}$ is a constant vector whose form depends on the particular choice of the orthogonal series: for the Block-Pulse functions, $\{e\} = \{1 \quad 1 \quad \cdots \quad 1\}^T$; for Fourier, Chebyshev, Legendre, Jacobi and Walsh series, $\{e\} = \{1 \quad 0 \quad \cdots \quad 0\}^T$.

Solving system (5) for matrix $[H]$ one obtains the structural model of the system, represented by matrices $[M]$, $[C]$ and $[K]$ and the set of initial conditions. A computationally stable solution to (5) can be achieved by using the least square method combined with the singular value decomposition technique.

If the free responses are used, a system of equations similar to (5) is obtained, with:

$$[H] = \left[\ \{x(0)\} \ \vdots\ \{\dot{x}(0)\} + [M]^{-1}[C]\{x(0)\} \ \vdots\ -[M]^{-1}[C] \ \vdots\ -[M]^{-1}[K]\ \right]$$
$$[J] = \left[\ \{e\} \ \vdots\ [P]^T\{e\} \ \vdots\ [P]^T[X]^T \ \vdots\ [P]^{T\,2}[X]^T\ \right]^T$$
$$[E] = [X]$$

As can be seen in the equations above, it is not possible to identify, separately, matrices $[M]$, $[C]$ and $[K]$ when the free responses are used. However, regardless of the nature of the responses, it is always possible to form the following state matrix, whose eigensolutions provide the natural frequencies, modal damping factors and complex vibration modes of the system:

$$[A]=\begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \in \Re^{2N,2N}$$

Due to practical constraints it is generally impossible to use the same number of sensors as the vibration modes contributing in the response. Thus, in order to create oversized mathematical models with a reduced amount of instrumentation, a technique named *Transformed Stations Technique* has been used, together with the *Modal Confidence Factor (MCF).* The MCF is used to separate the structural modes from computational ones. Details are given in Pacheco and Steffen (1999).

4. FORCE IDENTIFICATION USING ORTHOGONAL FUNCTIONS

The approach for force identification is similar to that presented in the previous section for the identification of modal parameters. Based on the assumption that matrices $[M]$, $[C]$ and $[K]$ are known, it is only needed to rearrange Eq. (5) for estimating matrix $[F]$ which contains the coefficients of the excitation forces. In this case, the matrices in (5) are given by:

$$[H]=\left[\ [F] \;\vdots\; [M]\{x(0)\} \;\vdots\; [M]\{\dot{x}(0)\}+[C]\{x(0)\}\ \right]$$

$$[J]=\left[\ [P]^{T\,2} \;\vdots\; \{e\} \;\vdots\; [P]^{T}\{e\}\ \right]^{T}$$

$$[E]=[M][X]+[C][X][P]+[K][X][P]^{2}$$

Solving system (5) for the matrix $[H]$, it is then possible to identify the initial conditions and the coefficients of the excitation forces.

5. APPLICATIONS

5.1 Modal parameter identification

The method was used for the identification of the modal parameters of the three-story frame, shown in Fig. 1. In the low frequency domain, this structure behaves like a 3 d.o.f. system.

For the purpose of comparison, the frequency response functions of the test-structure were obtained using a spectral analyzer and impact excitation. These FRFs were then used for estimating the natural frequencies and modal damping factors by using the half power bandwidth method (HPBW).

Besides the three real measurement stations, six assumed stations were used. The modal parameters obtained, compared to the values provided by the HPBW method, are presented in Table 1. As can be seen, the large majority of the values of the identified modal parameters are very close to those provided by the HPBW.

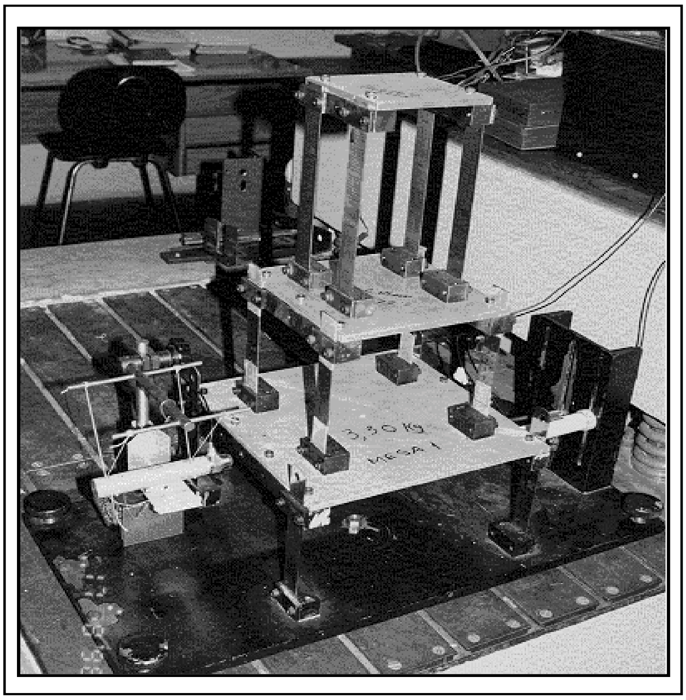


Figure 1 – Experimentally tested mechanical system

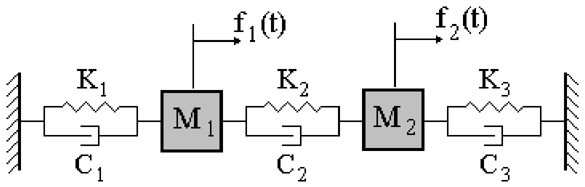
Table 1 – Identified modal parameters of the test-structure

	f_{n_1} [Hz]	f_{n_2} [Hz]	f_{n_3} [Hz]	ξ_1 (%)	ξ_2 (%)	ξ_3 (%)
HPBW	6.0254	11.751	18.124	0.847	0.672	0.469
Fourier (r=55)	6.0481 (0.38%)	11.669 (0.70 %)	18.071 (0.29 %)	0.854 (0.86 %)	0.689 (2.53 %)	0.484 (3.24 %)
Chebyshev (r=70)	5.8902 (2.24 %)	11.648 (0.88 %)	18.043 (0.44 %)	0.704 (16.87 %)	0.552 (17.90 %)	0.432 (7.90 %)
Legendre (r=60)	5.9237 (1.69 %)	11.678 (0.62 %)	18.087 (0.20 %)	0.819 (3.29 %)	0.646 (3.89 %)	0.507 (8.10 %)
Jacobi (r=73)	5.9145 (1.84 %)	11.718 (0.28 %)	18.072 (0.29 %)	0.838 (1.04 %)	0.704 (4.74 %)	0.482 (2.71 %)
Block-Pulse (r=250)	5.9058 (1.98 %)	11.767 (0.13 %)	18.391 (1.47 %)	0.785 (7.27 %)	0.590 (12.14 %)	0.504 (7.45 %)
Walsh (r=128)	6.0361 (0.18 %)	11.775 (0.21 %)	18.412 (1.59 %)	0.638 (24.69 %)	0.552 (17.80 %)	0.488 (4.15 %)
Fourier * (r=45)	5.9546 (1.17 %)	11.697 (0.46 %)	18.053 (0.39 %)	0.843 (0.42 %)	0.744 (10.65 %)	0.393 (16.20 %)

Note: Percentual values in brackets indicate the relative errors w.r.t. the HPBW method
* Results using the response of the lower mass and four assumed stations

5.2 Force identification

To illustrate the use of the method for the identification of excitation forces, it was applied to the 2 d.o.f. system shown in Fig. 2. The identification was performed in two steps. In the first one, the structural model of the system (matrices $[M]$, $[C]$ and $[K]$) was determined using two supposedly known harmonic forces applied to the two masses. Then, in the second step, this structural model was used for identifying different types of forces (assumed to be unknown) applied to the system. In all the identification computations it was assumed that the responses at coordinates 1 and 2 were available.



$M_1 = 1\,Kg$ $\quad K_1 = 3000\,N/m \quad C_1 = 5\,Ns/m$

$M_2 = 2\,Kg$ $\quad K_2 = 2000\,N/m \quad C_2 = 10\,Ns/m$

$K_3 = 1000\,N/m \quad C_3 = 15\,Ns/m$

Figure 2 – 2 d.o.f. mechanical system

First step: identification of the structural model. Two harmonic forces were simultaneously applied to masses 1 and 2:

$$f_1(t) = sin(2\pi\, t)\ [N] \qquad f_2(t) = sin(4\pi\, t)\ [N], \qquad 0 \le t \le 0.5\, s$$

with the following initial conditions:

$$\{x(0)\} = \left\{0.5\times10^{-3} \quad 0\right\}^T\ [m] \qquad \{\dot{x}(0)\} = \left\{0 \quad 0.5\times10^{-2}\right\}^T\ [m/s]$$

The exact and the identified matrices, obtained by using the Fourier series with r = 31, are the following (very little differences were noticed when using other orthogonal functions):

$$[M]_{ex} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} [kg]; \qquad [C]_{ex} = \begin{bmatrix} 15 & -10 \\ -10 & 25 \end{bmatrix} [N.s/m]; \quad [K]_{ex} = \begin{bmatrix} 5000 & -2000 \\ -2000 & 3000 \end{bmatrix} [N/m]$$

$$[M]_{id} = \begin{bmatrix} 0.9983 & -0.0094 \\ -0.0041 & 2.0032 \end{bmatrix}; \quad [C]_{id} = \begin{bmatrix} 14.9043 & -9.9970 \\ -10.0552 & 24.3054 \end{bmatrix}; \quad [K]_{id} = \begin{bmatrix} 5005.2 & -2007.8 \\ -1999.2 & 3002.9 \end{bmatrix}$$

Second step: identification of excitation forces. Two kinds of force were identified: periodic and transient.

First case: periodic forces. The applied forces were:

$$f_1(t) = 2\,sin(2\pi\, t) + 5\,cos(8\pi\, t) + 3\,sin(20\pi\, t)\ [N]$$

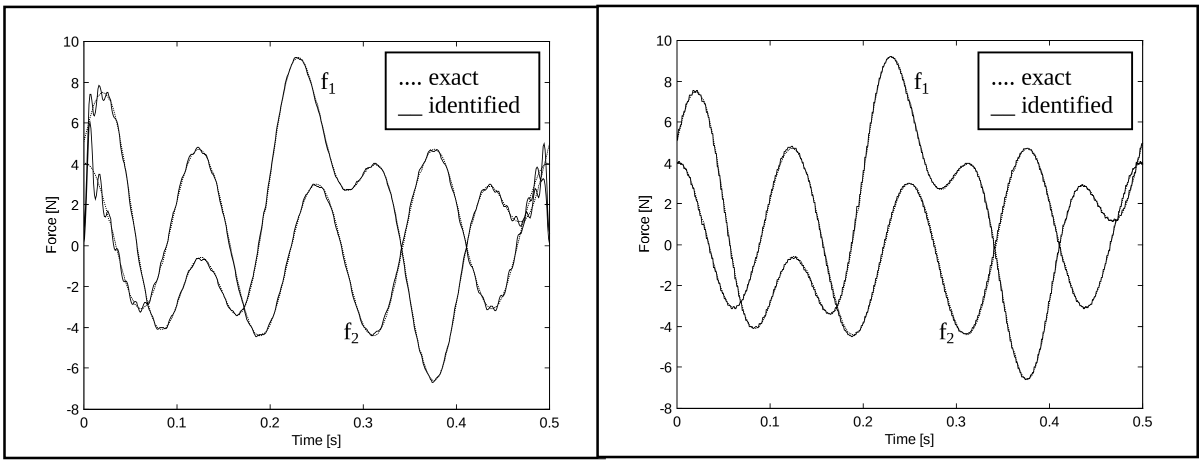
$$f_2(t) = sin(4\pi\, t) + 4\,cos(16\pi\, t)\ [N]$$

The exact and identified forces, obtained by using Fourier series (r = 105) and Block-Pulse functions (r = 256), are presented in Fig. 3.

The values of the relative RMS identification errors are the following:

Fourier: $f_1(t): 10{,}3\%$, $f_2(t): 15{,}7\%$. Block-Pulse: $f_1(t): 2{,}9\%$, $f_2(t): 4{,}6\%$.

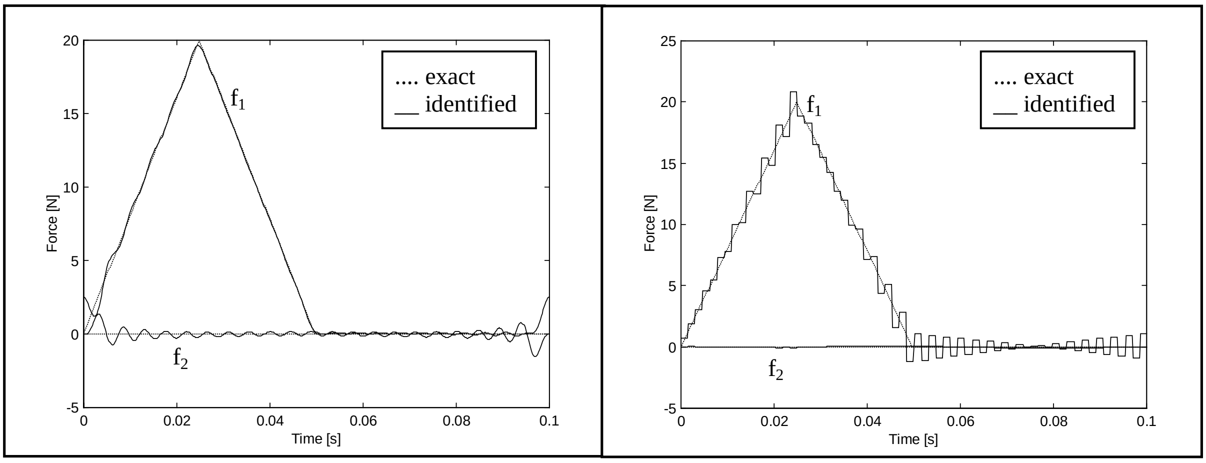
As can be seen in Fig. 3, the forces identified by using Fourier series exhibit the Gibbs phenomenon which causes distortions at the extremities of the time window.



(a) (b)

Figure 3 – Exact and identified forces using Fourier (a) and Block-Pulse series (b)

Second case: transient forces. In this example, a short duration force, simulating an impact force was applied to the mass number 1. The identification procedure was carried out without prior knowledge of the actual location of the excitation, aiming at verifying the capability of the method in identifying the input location. The exact and identified forces, obtained by using Fourier series (r = 45) and Block-Pulse functions (r = 64) are presented in Fig. 4. The values of the relative RMS identification errors for $f_1(t)$ are: Fourier: 4.8%, Block-Pulse: 9.1%. As can be seen, the method was able to correctly identify the location of the excitation force, as indicated by the comparative little force levels found for $f_2(t)$.



(a) (b)

Figure 4 – Exact and identified forces using Fourier (a) and Block-Pulse series (b)

6. CONCLUSIONS

It has been shown how different types of orthogonal functions can be conveniently employed for the identification of modal parameters and excitation forces in mechanical systems, using straightforward computational procedures.

Combination of the transformed station technique and the orthogonal function identification method provides a more suitable identification scheme for practical implementation, since a reduced amount of instrumentation is required.

All sets of orthogonal functions used have demonstrated the capability of providing fairly accurate results, though Legendre, Jacobi and Chebyshev polynomials provided less accurate force identification results, as compared to other orthogonal functions.

In the sequence of this work, it is intended to investigate the performance of the method for force reconstruction when incomplete data are concerned, that is to say, when the number of sensors are smaller than the order of the mathematical model.

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