COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

LAMINAR PROFILE DESIGN THROUGH OPTIMIZATION TECHNIQUES

Pedro Paglione
Roberto M. Girardi
Instituto tecnológico de Aeronáutica (ITA)
Praça Mal. eduardo Gomes, 50
12.228-900 São josé dos Campos, SP, Brasil

Abstract. *The objective of the present work is to report a method for designing laminar profiles using optimization techniques. The potential flow over an airfoil is calculated by using a modified Hess & Smith method, where a variable vortex distribution along the airfoil surface is imposed through a weight function. The boundary layer flow is obtained with a computational code, based on the integral equations. This code is capable to calculate laminar and turbulent flows and the Michael transition criterium is used to determine the transition point. The potential and boundary layer flows are coupled through a transpiration procedure, which simulates the boundary-layer growth on the airfoil surface. The optimization code searches for the airfoil shape, for which the distance of the transition point, from the airfoil leading edge, is maximum. This optimization procedure can be subjected to constraints, as the specification of a certain value for the airfoil internal volume.*

Key words: *Laminar profile, Optimization technique, Panel method, Viscous-non-viscous interaction*

1. INTRODUCTION

Laminar profiles are intensively used in gliders, mainly in the high performance ones, due to high values reached for Cl/Cd, which are associated to high values of maximum range. For such kind of aircraft, the Reynolds number is relatively low due to the low values of the cruise velocity and of the wing chord. This situation is favorable to a laminar profile design. On the other hand, for higher values of Reynolds number, more sophisticated methods are required for such design, because the transition point tends toward the stagnation point, near the leading edge.

The first attempts to laminar profile design were performed with the so-called indirect method, where a pressure distribution on the profile surface is specified and the profile shape is obtained. The above pressure distribution is obtained from the boundary layer theory in order to maximize the transition point position.

In the method proposed in the present work, the profile shape is modified up to the maximum positions of the transition points are reached at the upper and lower surfaces of the airfoil. This objective is accomplished by using an optimization technique, where the objective is obtained subjected to some constraints, as for example, a minimum value for the profile internal volume or a profile with a certain value for the moment coefficient.

The flow over the airfoil is calculated by a procedure, where a potential flow code is coupled to a boundary-layer code, in order to obtain estimate values for the drag coefficient, for the laminar-turbulent transition point position and for the separation point location

One of the most used procedures for calculating potential flow over isolated airfoils and cascades is the panel method based on the source and vortex singularities (Hess & Smith, 1966, method). This method provides very good solutions for airfoils with sharp trailing edge, but some problems are observed for the pressure distribution to the case of a cusped trailing edge. In the Hess & Smith (1966) method the source panel strength is variable along the airfoil surface but the vortex panels have the same strength, which is determined through a unique equation, which imposes the Kutta condition. The problems mentioned above can be solved through a vortex panel strength variation, along the airfoil surface, which is performed by a weight function, as shown by Girardi & Bizarro (1997). In such case, the vortex panels near the trailing edge region have small strength and in the leading edge region the vortex strength reaches its maximum value. It is worth to mention that although the vortex panel strengths vary along the discretization, only one variable is necessary to determine the vortex distribution and such variable continues to be calculated by the Kutta condition.

The boundary layer flow is obtained by using the integral equation method, developed by Rotta (1971). The calculation is started at the stagnation point, whose position is determined from the pressure distribution previously obtained with the potential flow code. Then, the boundary layer code is used twice, for the upper and lower surface. The transition point is estimated by the Michael (see Schetz, 1984) criterium and the displacement thickness is calculated along the airfoil surface. The viscous-non-viscous coupling is performed by a transpiration procedure, where non zero values for the normal velocity are used to simulated the boundary-layer in the potential flow calculation. These new values are obtained from the displacement thickness distribution, calculated in the boundary layer code.

2. NUMERICAL METHOD

2.1 Panel Method

The Hess & Smith (1966) procedure is a panel method based on source and vortex singularities, where the boundary condition is satisfied by prescribing normal velocity along the profile surface and Kutta condition at the trailing edge region. The profile outline is discretized in panels (see Fig. 1) and the above singularities are distributed along each panel with a constant strength.

In a general way, the conjugate complex velocity at the control point of a panel "k" (Zc_k), induced by a panel "j" can be written by

$$^{*}(Zc_k) = W^s_{kj}\sigma_j \quad (1a)$$

$$^{*}(Zc_k) = W^v_{kj}\gamma_j \quad (1b)$$

where W^{s}_{kj} and W^{v}_{kj} are, respectively, the conjugate complex velocities at the control point (located at the middle of the panel) of panel "k" induced by a panel "j", with a unit strength of source and vortex. The parameters σ_j and γ_j are source and vortex panel strengths, which are the unknowns of the problem.

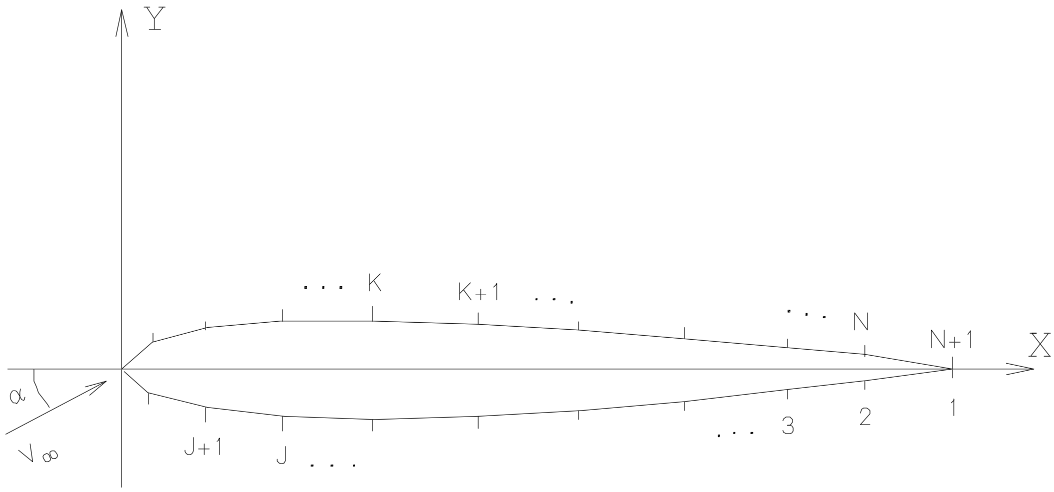

FIGURE -1: Discretization of a profile with N panels.

For the case of an isolated profile, the conjugate complex velocity can be expressed by:

$$W^{s}_{kj} = \frac{t_j^*}{2\pi} Log\left(\frac{Zc_k - Z_{j+1}}{Zc_k - Z_j}\right) \qquad \text{for } k \neq j \qquad (2a)$$

$${}^{s}_{kj} = i\frac{t_j^*}{2} \qquad \text{for } k = j \qquad (2b)$$

$$W^{v}_{kj} = i\frac{t_j^*}{2\pi} Log\left(\frac{Zc_k - Z_{j+1}}{Zc_k - Z_j}\right) \qquad \text{for } k \neq j \qquad (3a)$$

$${}^{v}_{kj} = -\frac{t_j^*}{2} \qquad \text{for } k = j \qquad (3b)$$

where "*i*" is the complex unit, Z_j and Z_{j+1} are the extremities of the panel "j" and t^*_j is the conjugate complex of the unit vector tangent to the panel and it is expressed by

$$t_j = \frac{Z_{j+1} - Z_j}{\left|Z_{j+1} - Z_j\right|} \qquad (4)$$

Equations (2b) and (3b) are valid only for a discretization in the clockwise direction as in the Fig. 1. For a counter clockwise discretization, the signals of the above expressions have to be changed.

The source and vortex strengths are obtained by imposing the boundary conditions and the Kutta condition. Considering the normal velocity equal to zero at the control point of a panel "k", the following system of equations can be obtained:

$$\sum_{j=1}^{N} \mathrm{Re}\,al\left(W_{kj}^{s}\, it_k\right)\sigma_j + \bar{\gamma}\sum_{j=1}^{N} \mathrm{Re}\,al\left(W_{kj}^{v}\, it_k\right) = -\mathrm{Re}\,al\left(V_\infty e^{-i\alpha}\, it_k\right) \quad \text{for k=1,2, ..., N} \quad (6)$$

where N is the number of panels, it_k is the normal unit vector of panel "k", α is the angle of attack and V_∞ is the undisturbed flow velocity. In the classical Hess & Smith method, all vortex panels have the same strength (constant vortex distribution along the profile surface, with a panel strength $\bar{\gamma}$), which is determined with the Kutta condition. There are some ways to consider such condition, but in the present work the Kutta condition is satisfied prescribing equal tangent velocities for the panels adjacent to the trailing edge.

In order to solve the problems observed for the case of profiles with cusped trailing edge, other distribution is prescribed in the present work, where the vortex strength tends smoothly to zero for the panels at the trailing edge (panel numbers 1 and N) and reach its maximum value to a panel near to the leading edge. A cubic distribution was chosen and the following weight function (WF) was proposed to represent such distribution.

$$\mathrm{WF_j} = \mathrm{a\,X_j^3} + \mathrm{b\,X_j^2} + \mathrm{c\,X_j} + \mathrm{d} \quad (7)$$

where the constants a, b, c and d are determined by the following conditions:

$$\begin{aligned} &\mathrm{X_j} = 0 \;\Rightarrow\; \mathrm{WF_j} = 1 \\ &\mathrm{X_j} = 1 \;\Rightarrow\; \mathrm{WF_j} = 0 \\ &\mathrm{X_j} = 1 \;\Rightarrow\; \frac{\mathrm{d(WF_j)}}{\mathrm{d(X_j)}} = 0 \\ &\mathrm{X_j} = \bar{\mathrm{X}} \Rightarrow\; \mathrm{WF_j} = 1/2 \end{aligned} \quad (8)$$

The parameter $\bar{\mathrm{X}}$ can be used to change the shape of the cubic distribution but in the present work it is fixed with a value equal to 1/2. The variable X_j is equal to 1 at the discretization ends (for j=1 and j=N) and it is equal to 0 near the profile leading edge. This variable is connected to the panel index (j=1,2,...,N) by the following expression

$$\mathrm{X_j} = \frac{\|\mathrm{J - N/2}\|}{\mathrm{N/2}} \quad (9)$$

The weight function is used to define a circulation distribution along the profile surface and the circulation density for each panel (γ_j) is given by

$$\gamma_{\mathrm{j}} = \mathrm{WF_j}\,.\,\bar{\gamma} \quad (10)$$

where $\bar{\gamma}$ is a constant and, in fact, it is the only unknown of the circulation distribution. With this new distribution, the boundary condition satisfied at panel k (given by eq. 6) has to be rewritten in the following form

$$\sum_{j=1}^{N} \mathrm{Re}\,al\left(W_{kj}^{s} it_k\right)\sigma_j + \bar{\gamma} \sum_{j=1}^{N} \mathrm{Re}\,al\left(W_{kj}^{v} it_k\right) WF_j = -\mathrm{Re}\,al\left(V_\infty e^{-i\alpha} it_k\right)$$

$$\text{for } k=1,2,\ldots,N \qquad (11)$$

Finally, it is important to note that the Kutta condition also must be changed in order to consider the weight function.

2.2 Boundary Layer and potential flow Coupling

As it is well known, the pressure coefficient obtained from the potential flow solution (panel method) is used for the boundary layer calculation. Meanwhile, the results obtained with the potential flow code have to be adapted, in order to be read by the boundary layer code. Initially, the stagnation point location has to be determined on the profile surface, because the boundary layer calculation is started at this point. Then, the boundary layer code is called twice for calculating the upper and lower surface.

Two boundary layer code results are used in the present work: (i) the friction coefficient, for determining the friction drag through an integration along the profile surface and (ii) the displacement thickness, which is used to perform the coupling with the potential flow. Such coupling can be accomplished: (a) by changing the profile shape, in order to consider the space occupied by the boundary layer (that is, the new shape is obtained by summing the old one with the boundary layer displacement thickness) or (b) by implementing new values for the normal velocity (potential flow boundary condition), which models the boundary layer growth, through a transpiration effect. In the present paper, the transpiration procedure was adopted because the potential flow code can not treat open discretization at the trailing edge, which would appear if a profile surface rediscretization has to be performed, in order to redefine the potential flow frontier, outside the boundary layer.

In the transpiration procedure, the normal velocity at the control point of each panel is a function of the displacement thickness growth, as shown in Bizarro (1998). In the same reference is discussed a problem occurring at the trailing edge region, which is caused by an explosive growth of the displacement thickness. This problem can be minimized by a procedure presented in Bizarro (1998).

After the new boundary condition enforcement, the potential flow code gives new pressure coefficient distribution, which is used to calculate the lift coefficient and the pressure drag. The total drag coefficient is then obtained considering the friction and the pressure items.

When flow separation is detected by the boundary layer code, such information is transferred to the main program, in order to be used by the optimization codes.

2.3 Optimization Method

The laminar profile design through optimization techniques can be mathematically stated as the following optimization problem:

$$J = \max[F(x)]$$
$$\textit{subject to } g_i(x), \quad i = 1,2,\ldots,q \qquad (12)$$

where x is a vector containing the design variables and J, figure of merit of this problem, is the boundary-layer transition from laminar to turbulent . The term $g_i(x)$ defines all the q-constraints pertinent to the problem as described bellow.

For obtaining the optimal profile shape according to a specific aerodynamic figure of merit (maximal lift coefficient, maximal boundary layer transition, minimal drag coefficient, etc) serving as the objective function, it is used in this work the numerical optimization procedure Globex, implemented by Jacob (1982). It employs a robust local minimization algorithm of a real valued function of several variables that converge quickly to the nearest relative extreme point insensitive to curved valleys and sharp ridges in the variables-criterion space and can handle any type of constraint with no need of gradients evaluation since each time a constraint violation is detected, the optimization algorithm is signaled to provide a new set of variables until a set is obtained that violates no boundary.

For aerodynamic shape optimization this is a advantageous strategy, besides its simplicity, since the gradients evaluation could have a computational cost prohibitive due to iterative procedures or even impossible in some particular cases. The global extreme of the function is reached, with great likelihood, through the use of a three step procedure based on normally distributed random number. In the first step, the initial values of the variables are estimated. The vectorial mean value of these normally distributed points as well the mean quadratic deviation is from the user initial points given. In each one of these points, a local extremization procedure is started. In the second step, around the variables that resulted in the best function value once more normally distributed random numbers are generated and in each one of the these points a new local optimization is calculated. Once a better function value is found, this point is used as the new mean value for another random searching and the mean quadratic deviations are multiplied by 0.9 (localization of the global extreme). The best of all in these both step found point is stored and used as the initial value for a third optimization step. Although the global extreme can not be determined with absolute security, the probability it is found increases with the number of random estimated values.

Design Methodology. The airfoil upper and lower surface Y-coordinates are given by two different methods for shape definition. In the first one, the airfoil is defined as a linear combination of others given airfoils, known as basis airfoils, by the relationship (Vanderplaats, 1979):

$$Y = a_1 Y_1 + a_2 Y_2 + ... + a_n Y_n \qquad (13)$$

where Y is a vector of the upper- and lower-surfaces coordinates y_{us} and y_{ls} respectively, and Y_i, i = 1, …, n, contains the surface coordinates of the basis airfoil i. The design variables are the a_1, …, a_n coefficients of the linear combination in Eq. (13).

In the second method, the analytical distributions suggested by Raymond and Vanderplaats (1975) were used and, they are given by:

$$y_{us} = a_1\sqrt{x/c} + a_2(P_2+1) + a_3(P_3-1) + a_4(P_4+1) + a_5(P_5-1) + a_6(P_6+1)$$
$$y_{ls} = b_1\sqrt{x/c} + b_2(P_2+1) + b_3(P_3-1) + b_4(P_4+1) + b_5(P_5-1) + b_6(P_6+1) \qquad (14)$$

where y_{us} and y_{ls} are the upper- and lower surface coordinates, respectively, and P_2, ..., P_6 are Legendre polynomials given by

$$P_2 = 2(x/c) - 1$$
$$P_3 = 6(x/c)^2 - 6(x/c) + 1$$
$$P_4 = 20(x/c)^3 - 30(x/c)^2 + 12(x/c) - 1$$
$$P_5 = 70(x/c)^4 - 140(x/c)^3 + 90(x/c)^2 - 20(x/c) + 1$$
$$P_6 = 252(x/c)^5 - 630(x/c)^4 + 560(x/c)^3 - 210(x/c)^2 + 30(x/c) - 1 \quad (15)$$

The coefficients a_1, ..., a_6 and b_1, ..., b_6 are the design variables determined by the optimization program Globex to achieve the optimum design figure of merit, namely, the boundary-layer transition point.

The profiles are constrained to have no negative thickness, a trailing-edge thickness no great than 0.5 percent of the chord, its volume above 0.075 and below 0.09. Besides theses geometrical constraints, the Cp-distribution on the lower surface can not be greater than the upper one, no boundary-layer separation is allowed and the moment coefficient, Cm, has to be greater than –0.075.

3. DESIGN RESULTS

The numerical optimization problem here is the determination of the maximal boundary-layer transition point, so that the profile can be considered as a laminar one, subjected to geometric and aerodynamic constraints, mentioned above.

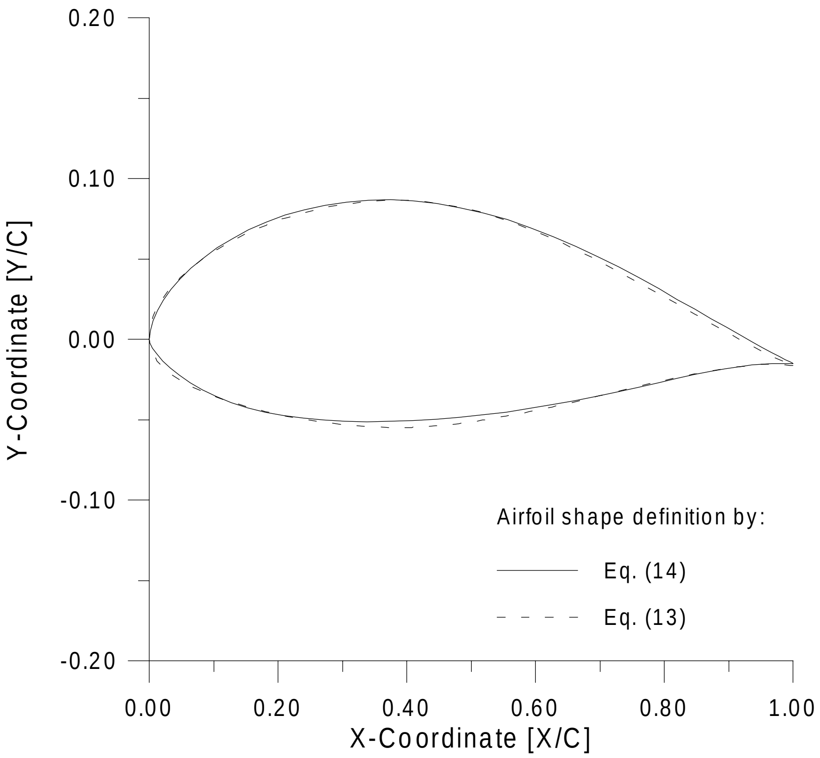

Figure-2 Optimal profile shapes obtained with the two methods for airfoil shape definition for 1 degree of angle of attack and Re = 1.10^7.

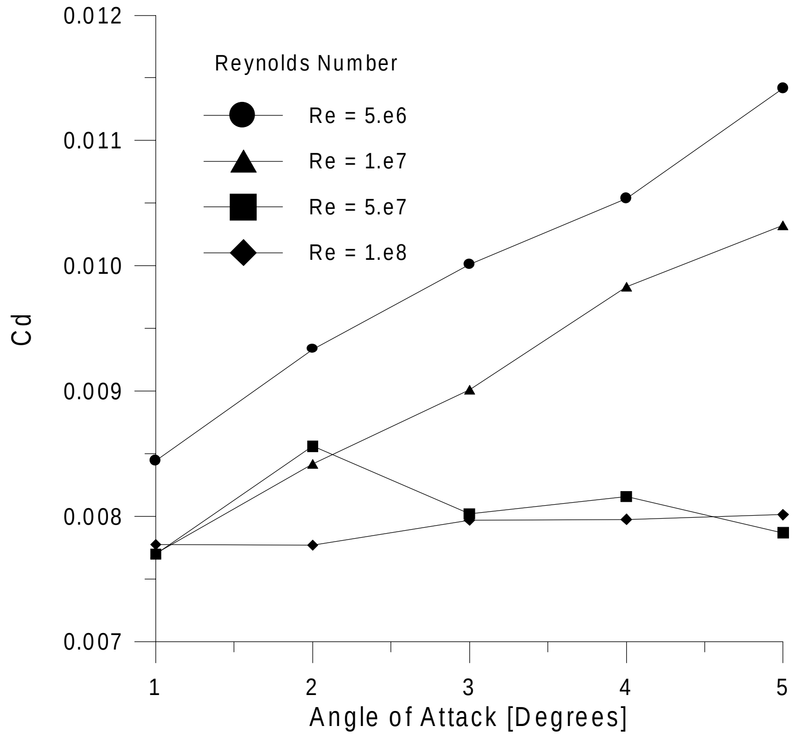

Figure-3 Drag coefficient as a function of angle of attack for different Reynolds number, for the second method.

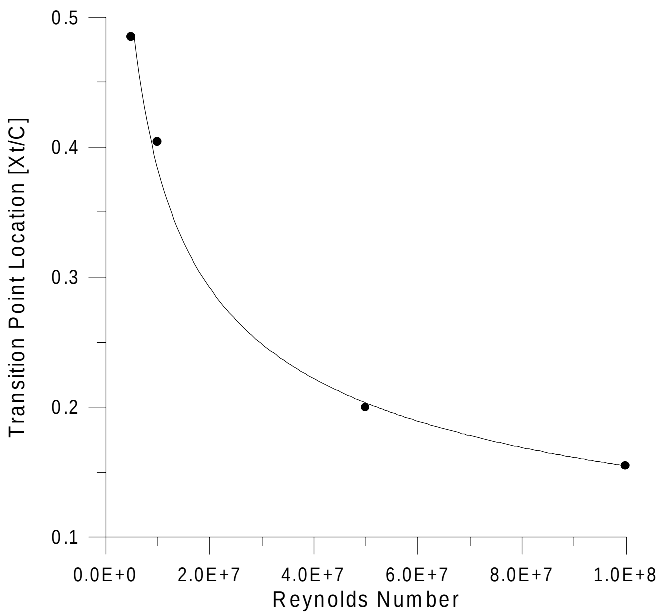

Figure-4 Transition point position as a function of Reynolds Number, for the second method

Figure 2 shows the result obtained for both methods used for the airfoil shape definition. In such case, the angle of attack is 1 degree and the Reynolds number is 1.10^7 . So that the comparasion of these different methods for the upper and lower surface coordinates can be seen, the y-coordinate is in a favorable scale in the Fig. 2. As one sees, almost the same optimal profile was found with both methods. The differences occur because the optimal shape is also constrained to the possibilities allowed by the method used for the airfoil shape definition.

As can be seen in the Fig. 3, the drag coefficient increases with the angle of attack for the two lower Reynolds number considered in the present work. For the two greater ones, the drag coefficient is almost constant. This kind of behavior is typical for laminar profiles, because transition point moves to the leading edge on the upper surface and in the opposite direction on the lower one as the angle of attack is increased and, such changes are responsible for the drag coefficient increment. For the greater values of Reynolds number, the transition point is already very near to the leading edge and then, its movement is limited in such case, resulting a nearly constant value as the angle of attack is varied.

For the same angle of attack, the drag coefficient decreases when the Reynolds number is incremented (see Fig. 3). On the other hand, for the greater values of Reynolds number, its influence on the drag coefficient is almost negligible. These two behaviors can be observed in practice and shows that the aerodynamic computational method works well.

As mentioned above, the transition point position moves toward the airfoil leading edge as the Reynolds number (Re) is incremented (see Fig. 4). For the two lower values of Re, the profiles can be considered as laminar ones, because the transition point is located approximately at the middle of the profile chord. For the two greater Re values the transition point is located near the leading edge and this configuration can not be characterized as a laminar profile.

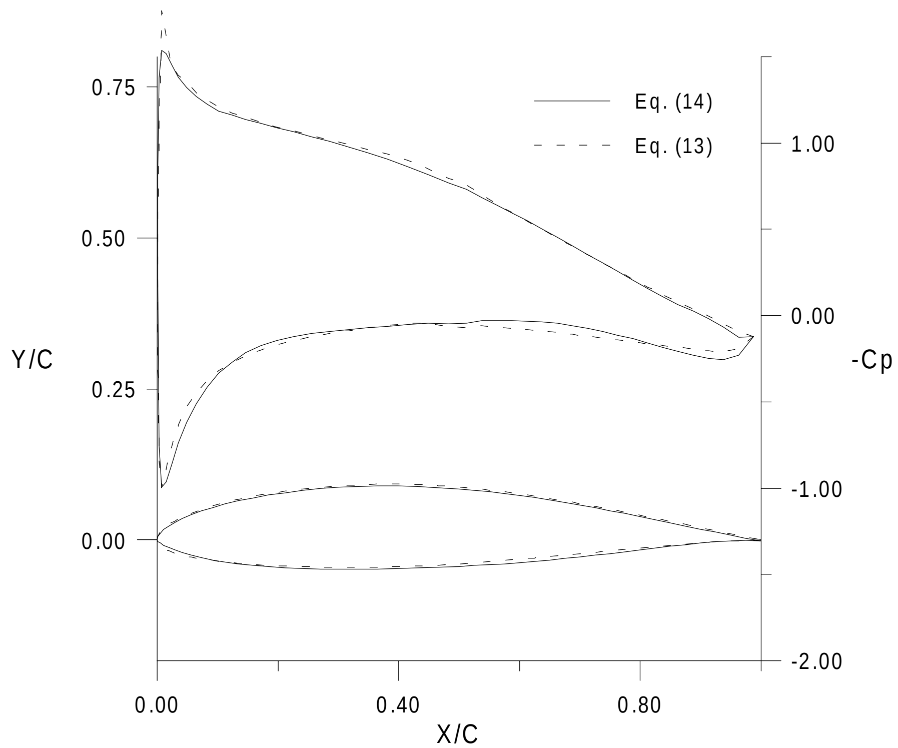

Figure-5 Laminar profile shape and pressure distribution for 5 degrees of angle of attack and $Re = 1.10^7$. Comparison between both methods for airfoil shape definition.

Figure 5 show the pressure distributions for the laminar profiles obtained using both methods of airfoil shape definition. The distributions are very similar and present a small adverse pressure gradient, which is fundamental for delaying the laminar-turbulent transition. The profile shapes are also similar, as shown in the Fig. 5, and are defined by the following coefficients for the Eqs. (14) and (13).

Table 1 – Coefficients for Eq. (14)

a_1	a_2	a_3	a_4	a_5	a_6
.156896199	-.086212181	-.036004583	.006322314	.002087801	.000648862
b_1	b_2	b_3	b_4	b_5	b_6
-.063617409	.042430002	.020016314	-.007286685	.000178736	-.003944022

Table 2 – Coefficients for Eq. (13)

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
.273932646	.078865868	.315816373	.385242115	-.39645759	.143564921	.000002635	.000000376

4. FINAL REMMARKS

A technique for the laminar profile design of low-speed airfoil with optimization was presented.

The numerical method used for the aerodynamic calculation furnishes very good results for the drag coefficient and for the transition point location. The Reynolds number influence, observed in practice, was also obtained in the present work.

The geometric and aerodynamic constraints were defined for obtaining optimal profile shapes that could be used in practice, as for example, the moment coefficient limitation to low values, which are very important to minimize the induced drag produced by na aircraft horizontal tail. Other important constraint is the internal volume limitations, because weight can be reduced due to structural optimization.

The methods used to define the airfoil shape furnish equivalent results.

The optimization procedure, GLOBEX, is very robust and gives, with great probability, the global optimum for the figure of merit used, in the present work, the maximum transition point location.

5. REFERENCES

Bizarro, A.F. "Interação Viscosa - não Viscosa na Análise do Escoamento em Grades Lineares de Máquinas de Fluxo". Tese de Mestrado, Instituto Tecnológico de Aeronáutica (ITA), S J Campos, SP, 1998.

Girardi, R.M. & Bizarro, A.F., 1995, Modification of the Hess& Smith Method for calculating cascades and airfoils with cusped trailing edge, XIII Congresso Brasileiro de Engenharia Mecânica, 12 a 15 de Dezembro, Belo Horizonte, MG, Brasil.

Hess, J.L. & Smith, A.M.O., 1966, Calculation of potential flow about arbitrary bodies. Progress in Aeronautical Sciences, Vol. 8, Pergamon Press, 1966.

Jacob, H., G., Rechnergestuetzte Optimierung sticher and dynamischer Systeme – Beispiele mit FORTRAN-Programmen, Springer-Verlag, Berlin, B. R. Deutschland.

Raymond, M., H., and Vanderplaats, G., N., 1975, Application of Numerical Optimization to the Design of Low-Speed Airfoils, NASA – TM X – 3213.

Rotta, J.C., 1971, Fortran IV, Rechenprogramm für Grenzschichten bei kompressiblen Ebenen und Achsenssymmetrishen Strömungen, DLR FB 71-51, Göttingen, Germany.

Schetz, J.A., 1984, Foundations of Boudary Layer Theory for Momentum, heat and Mass Transfer, Prentice-Hall Inc., Englewood Cliffs, New jersey, USA.

Vanderplaats, G., N., 1979, Efficient Algorithm for Numerical Airfoil Optimization, Journal of Aircraft, Vol.16, No. 12, pp.842-847.