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NUMERICAL AND EXPERIMENTAL ANALYSIS OF A PASSIVE ABSORPTION SYSTEM WITH COULOMB DAMPING

Marcus Aurellius Louroza, M.Sc.
Empresa Brasileira de Aeronáutica S/A - DTE/GAL - Posto de Correio: 0126
Av. Brigadeiro Faria Lima, 2170 - CEP: 12227-901
São José dos Campos-SP E-mail: mlouroza@embraer.com.br
Carlos Magluta, D.Sc.
Universidade Federal do Rio de Janeiro / COPPE
Centro de Tecnologia - Bloco 101 - Ilha do Fundão - Cidade Universitária - 21945-900
Rio de Janeiro-RJ E-mail: magluta@labest.coc.ufrj.br
Ney Roitman, D.Sc.
Universidade Federal do Rio de Janeiro / COPPE
Centro de Tecnologia - Bloco 101 - Ilha do Fundão - Cidade Universitária - 21945-900
Rio de Janeiro-RJ E-mail: roitman@labest.coc.ufrj.br

Summary. *The objective of this work is to understand the behavior (response) of a structure containing an absorption system with Coulomb damping and modeled with two degrees of freedom. In order to achieve this goal, a theoretical-computational model was developed and adjusted to experimental results.*

The experimental model is a cantilever beam with the absorption system placed on it. The absorption system is a spring-mass system with a friction element acting on its axis which is attached to the beam. The dynamic forces used in the experiments are: impacts, people jumping and harmonic excitation.

A parametric study was performed after the theoretical-computational model had been adjusted. The results indicate that the system is only applicable to some situations, and care should be taken in the design concept. That is linked to the non-linear behavior of the response with this kind of damping.

Keywords**:** *Passive absorption system, Coulomb damping, Computational and experimental analysis.*

1. INTRODUCTION

This work follows a research branch of COPPE Structures Laboratory with the objective of reducing vibrations in civil structures. Nowadays, the development and use of new materials and technologies turn the structures more and more slender. So, they are more exposed to dynamic loading. But vibration problems are not confined to modern structures, as the older ones may also present this kind of problem because loading may change along the time.

For example, some great old Brazilian soccer stadium have been presenting excessive vibrations during crowded games. This problem is probably due to a change in public behavior over the years.

A computational-theoretical model was developed to represent a structural system with Coulomb damping and containing two degrees of freedom. After that, many loading simulations and situations were generated to verify the structure behavior for this kind of damping. Based on simulation results, some ideas about designing an absorption system with Coulomb damping are presented.

2. THEORETICAL MODEL

The studied model is a spring-mass system with two degrees of freedom illustrated in “Fig. 1”. One degree represents the main system and the other, the absorption system. In this work, it was considered that the static friction force (fa_e) was the same as the dynamic friction force, because their values, registered in the experiments, were very similar.

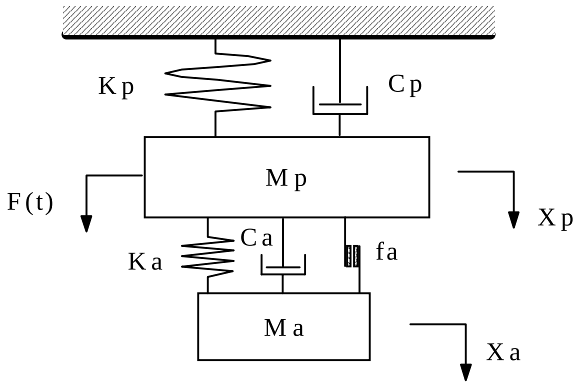

Figure 1 - Theoretical model with two degrees of freedom.

Note: **p** - main system; **a** - absorption system; **k** - stiffness; **c** - viscous damping coefficient; **x** - displacement; **fa** - friction force; **M** - mass; **F(t)** - force; **t** - time.

System of differential equations “Eq.(1)” (Nashif et al. ,1985) is the mathematical model that represents the system illustrated in “Fig.1”. It is known that the friction force is opposite to the displacement, because it always resists any movement, and its sign will always be opposite to that of the velocity. In system of equations “Eq.(1)”, the term $\mathrm{sgn}(\dot{x}_p - \dot{x}_a)$ represents the sign of the relative velocity between the main and the absorption systems. The period over the variable ***x*** means derivative with respect to time ***t***.

$$m_p.\ddot{x}_p + k_p.x_p + c_p.\dot{x}_p + k_a.(x_p - x_a) + c_a.(\dot{x}_p - \dot{x}_a) + f_a.sgn(\dot{x}_p - \dot{x}_a) = F(t)$$

$$m_a.\ddot{x}_a + k_a.(x_a - x_p) + c_a.(\dot{x}_a - \dot{x}_p) - f_a.sgn(\dot{x}_p - \dot{x}_a) = 0 \qquad (1)$$

It is important to observe that depending on the values of the parameters involved, the absorption system might not work, remaining static relative to the main system, and performing like a mass attached to the main system. It can also happen that the absorption system acquires a non-linear movement, therefore it stays sticking and slipping during the excitation. Coulomb damping always leads to "piecewise" linear motion, which is a non-linear motion (see Nashif et al., 1985, pp.57-61). In this paper, "non-linear" means the movement that embraces "piecewise" linear motion. So, the solution of system of equations "Eq.(1)" is linked to two conditions that have to be verified at each instant. The first one consists in determining the sign of the relative velocity between the main and the absorption systems at each instant. The second condition is to determine whether the friction element of absorption system is unlocked (two degrees of freedom - 2df) or locked (one degree of freedom - 1df). This condition can be verified by "Eq.(2)".

$$fa_e \geq k_a(x_a - x_p) + c_a(\dot{x}_a - \dot{x}_p) + m_a\ddot{x}_a \tag{2}$$

If "Eq.(2)" is satisfied, it means that the system is locked (1df) and its movement could be expressed by "Eq.(3)".

$$(m_p + m_a).\ddot{x}_p + k_p.x_p + c_p.\dot{x}_p = F(t) \tag{3}$$

It was not possible to find an analytical solution to system of equations "Eq.(1)", and to face this problem it was decided to perform a numerical integration using the Runge-Kutta method (Press et al. , 1986).

When the absorption system is unlocked (2df), the equations to be integrated are "Eq.(4)", "Eq.(5)", "Eq.(6)" and "Eq.(7)" obtained from system of equations "Eq.(1)". To integrate these equations numerically, some changes in variables have to be performed like: $U = \dot{x}_p$; $\dot{U} = \ddot{x}_p$; $V = \dot{x}_a$ and $\dot{V} = \ddot{x}_a$.

$$\dot{U}_{(t+dt)} = \left[\frac{F(t) - \left[k_p x_p + c_p\dot{x}_p - k_a(x_p - x_a) - c_a(\dot{x}_p - \dot{x}_a) - f_a sgn(\dot{x}_p - \dot{x}_a)\right]}{m_p}\right] \tag{4}$$

$$U_{(t+dt)} = \dot{x}_p(t) \tag{5}$$

$$\dot{V}_{(t+dt)} = -\left[\frac{k_a(x_a - x_p) + c_a(\dot{x}_a - \dot{x}_p) - f_a sgn(\dot{x}_p - \dot{x}_a)}{m_a}\right] \tag{6}$$

$$V_{(t+dt)} = \dot{x}_a(t) \tag{7}$$

When the absorption system is locked, the equations to be integrated are "Eq.(8)" and "Eq.(9)" and they originate from "Eq.(3)". The changes in variables to be performed are: $G = \dot{x}_p$ and $\dot{G} = \ddot{x}_p$.

$$\dot{G}_{(t+dt)} = \left[\frac{F(t) - k_p x_p - c_p\dot{x}_p}{m_p + m_a}\right] \tag{8}$$

$$G_{(t+dt)} = \dot{x}_p \tag{9}$$

3. EXPERIMENTAL TESTS

The experimental model representing the main system is a metallic cantilever beam, constituted of an **I** profile (0.30mx67kgf/m), as showed in "Fig. 2". This beam is fixed by two bolts to the slab reaction of the Structures Laboratory. The main system is also composed of a metallic cylinder placed at the edge of the beam and a jump platform. This platform is made up of two metallic plates with 6 load-cells, placed between the plates, to measure the applied force (Louroza, Roitman, Magluta, 1997).

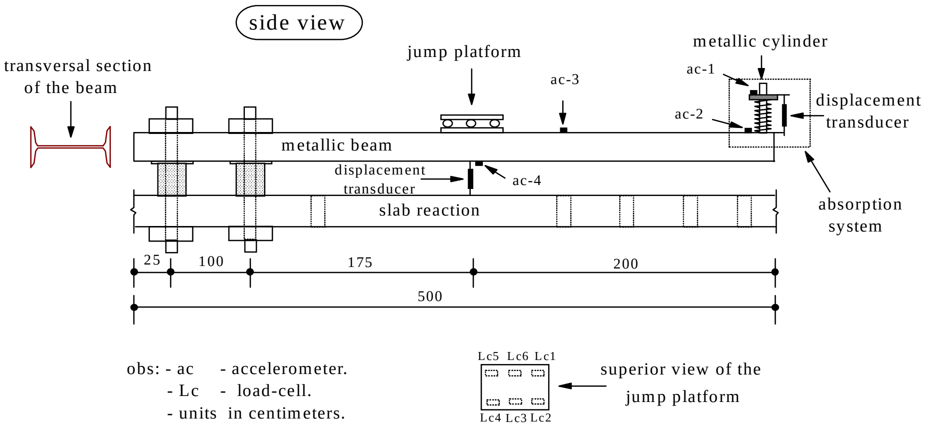

Figure 2 - Complete experimental model with the utilized instrumentation.

The absorption system experimental model is showed in "Fig. 3", and it is composed of a spring around the metallic cylinder that belongs to the main system, two metallic plates (the first two from top to bottom) with the friction device on the top. The spring is welded to two metallic plates and the absorption system mass is approximately 10 kg. In the higher plate, a little bronze block (friction element) is installed near the cylinder. The distance between the friction element and the absorption system axis is variable, so that the friction force can be controlled. In "Fig. 2" the absorption system is placed at the free end of the main system.

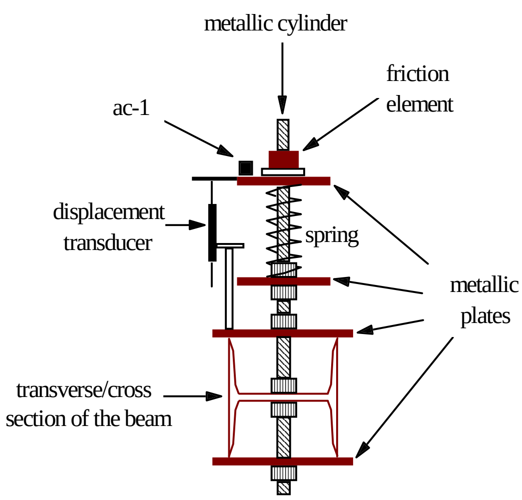

Figure 3 - Absorption system with instrumentation (frontal view).

The sensors utilized to response measurement are piezo-resistive accelerometers and displacement transducers, and their lay-out is showed in "Fig. 2".

Experiments performed

Many tests were developed with three different loading types: impacts, harmonic excitation and people jumping. All the analyses were performed around the first mode.

The free vibration tests (impact loading on jump platform) were performed to determine dynamic characteristics like natural frequencies and damping ratios of: the main system, the absorption system, and the complete structure (main plus absorption systems). The force and response signals were transferred to a dynamic signal analyzer by which the natural frequencies, and later the viscous damping ratio was obtained. The damping ratio was calculated through the free vibration decay method.

"Figure 4" presents a typical frequency response function (FRF) for the structure without the absorption system, with viscous damping and with Coulomb damping. It is important to emphasize that the ideal calibration ("Eq.10" - damping ratio) from Den Hartog's theory (1956) was not reached in these experiments. So, it was not possible to get the maximum efficiency from the viscous model.

$$c_a / c_c = \sqrt{\frac{3\mu}{8(1+\mu)^3}} \tag{10}$$

note: c_c - critical viscous damping coefficient;

c_a - absorption system viscous damping coeficient;

μ - mass ratio between the absorption and the main systems.

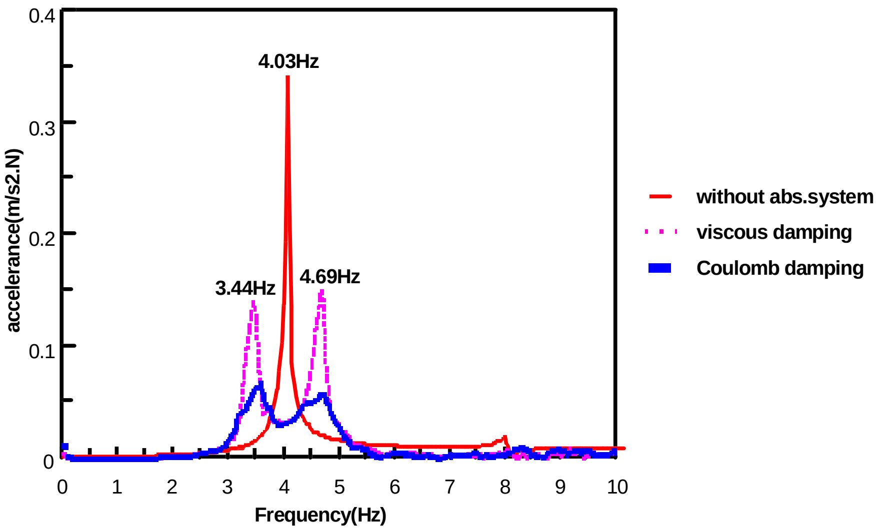

Figure 4 - Comparison of frequency response functions.

The harmonic excitation was used to simulate some loading types that are common in some industries. For instance, some rotary machines and motors that transfer harmonic signals to the structure. The excitation frequencies of the harmonic forces were set equal to the two structure's natural frequencies (3.44 and 4.69 Hz). This test was performed to the following structural configurations: without absorption system, with viscous damping, and with Coulomb damping for many different friction forces. The measured responses were utilized to adjust the developed computational model.

The jump loading was performed due to some structures that have vibration problems of this type, such as stadiums, gymnasiums and dance clubs. In this test, people were asked to jump with a frequency close to the first natural frequency of the structure (3.44 Hz). This experiment, like the harmonic loading, was developed to the structure: without the absorption system, with viscous damping, and with Coulomb damping for many different friction forces. The obtained responses were also used to adjust the computational model.

4. COMPARISON BETWEEN COMPUTATIONAL AND EXPERIMENTAL RESULTS

In the following, some comparative charts in the time domain are showed for some situations. The responses were obtained by the accelerometer placed at the end of the structure (ac-2 in “Fig. 2”). The loading utilized in the following charts was people jumping.

The structure’s dynamic characteristics and the loading measured in the experiments were introduced into the numerical model and the results were compared with the experimental results.

From “Fig. 5 ”, it can be seen that the numerical model fits very well to the experimental results, indicating a good adjustment of the numerical model with the introduction of friction force to the absorption system.

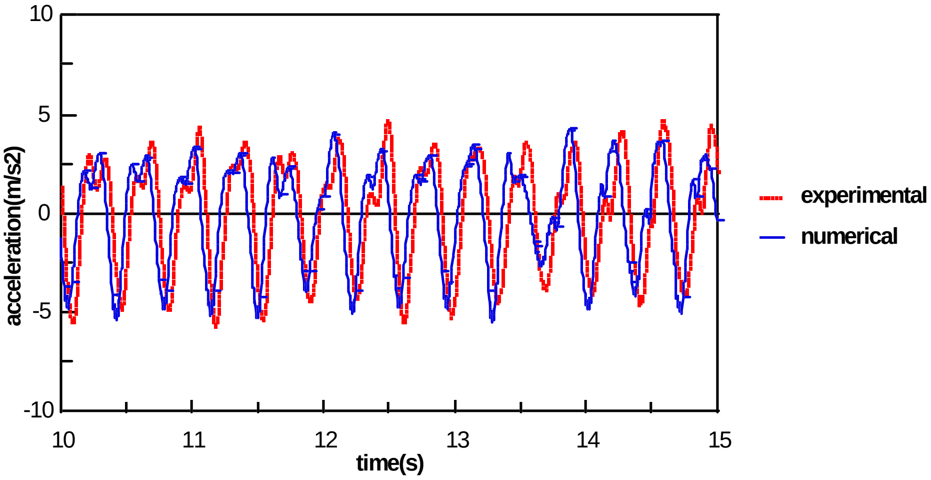

Figure 5 - Comparison between numerical and experimental responses of the structure with Coulomb damping.

“Figure 6” shows the structure’s response with the absorption system working when the “stick-slip” process occurs. The used loading was the harmonic excitation. In the higher part of this chart, one can observe the system oscillating between one (1df - imaginary straight line for which acceleration is constant and equals to 4) and two degrees of freedom (2df - acceleration equals to 7).

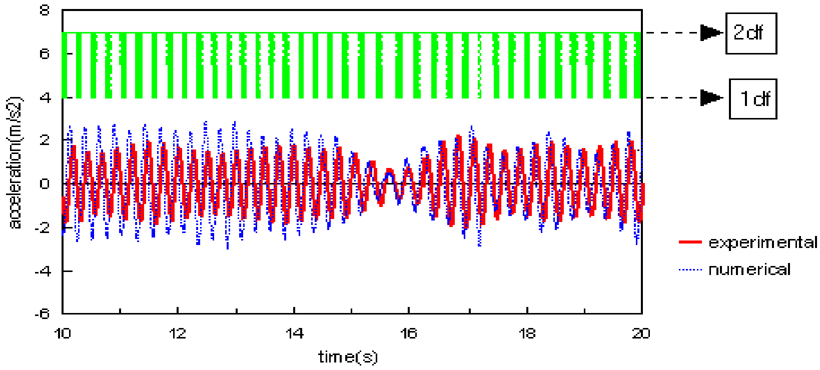

Figure 6 - Comparison between numerical and experimental responses of the structure with Coulomb absorption system with “non-linear” behavior.

After all the performed analyses, it can be affirmed that the numerical model is well adjusted to the experimental model for all the analyzed situations.

5. PARAMETRIC ANALYSIS

After adjusting the numerical model, a parametric analysis was performed. The used load was composed of three harmonics and its frequencies were, respectively, the structure’s natural frequencies with the absorption system working (2df) and with the absorption system locked (1df). This load was chosen in order to keep the structure in resonance state.

Response analysis to fixed mass ratio

The mass ratio is the rate between the absorption system mass and the main system mass. The results obtained from the numerical model can be seen in “Fig. 7”, which shows the maximum displacement of the structure with absorption system (X), normalized to the maximum displacement of the structure without the absorption system (X_{vo}), versus the friction force (f_a), normalized to the maximum amplitude of the excitation force (F). It has to be emphasized that all the structure’s maximum amplitudes were obtained near the resonance state.

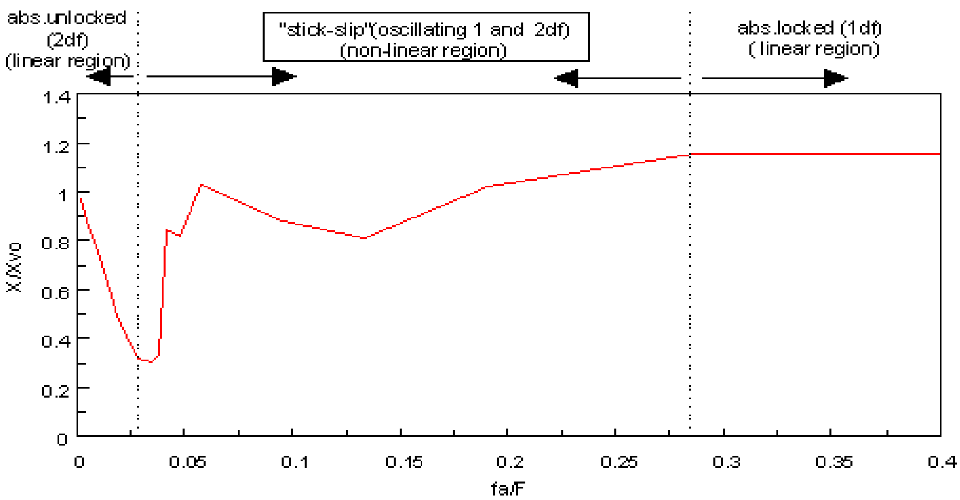

Figure 7 - Structure’s response versus friction force normalized with 2% of mass ratio .

"Figure 7" shows three different regions: the first one (from the left) represents the structure with the absorption system acting (2df - linear movement); the middle region reproduces the "stick-slip" process (see "Fig. 6"); finally the last region shows the response of the structure with absorption system locked.

Still in "Fig. 7", it can be observed that the greatest reductions in response amplitude are associated to linear movement, where it is possible to control the response. This does not occur in the "non-linear" movement, where great amplitudes are reached and movement stays out of control.

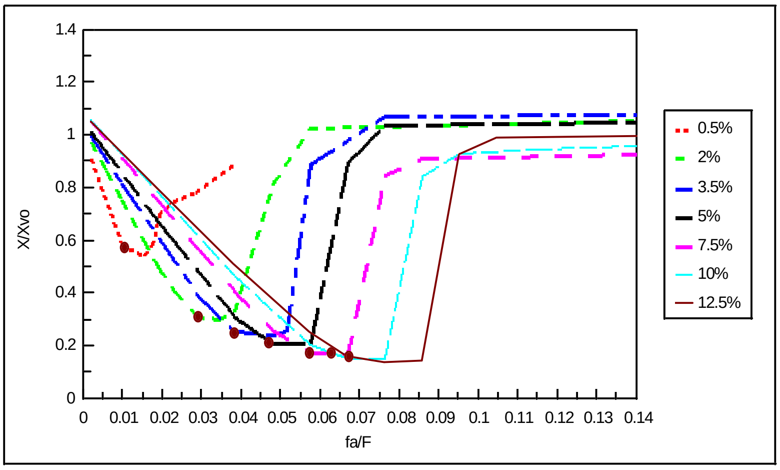

Figure 8 - Structure's response versus friction force normalized to different mass ratios.

When the mass ratio increase, greater amplitude reductions are obtained in the linear region. It can be observed that each mass ratio has a different **f_a/F** limit, represented in "Fig.8" by dots. Above this limit, the system behavior can be "non-linear" (oscillating between 1df or 2df) or linear (1df - locked). This situation can result in response amplitudes as great as the responses obtained from the structure without the absorption system around the resonance state.

Some ideas to design an absorption system

As observed in previous items, for the utilized loading, the absorption system is efficient until a limit. Beyond this limit (f_a/F limit), the system begins the "stick-slip" movement, assuming a "non-linear" behavior, which results in greater values for the response amplitude. Therefore, the "non-linear" movement has to be avoided. When the force amplitude has small oscillations, it is possible to avoid the "non-linear" movement. However, when there are great oscillations, a usual situation in real cases, it is very difficult to guarantee the response control. Therefore, for great oscillations of force amplitude, besides verifying the response caused by the maximum force amplitude, the structure's response for intermediate amplitudes for which the "non-linear" movement occurs must be analyzed too. "Figure 9" helps to clarify this statement.

To design an absorption system one must know the maximum force amplitude (F_P) and the allowed maximum displacement amplitude (X_P). With these data, one can determine the friction force required to maintain the displacement below or equal to limit $\mathbf{X_P}$, when force $\mathbf{F_P}$ acts on the structure. "Figure 9" shows the structure's displacement (X) normalized to the maximum displacement of the structure without the absorption system due to F_P (X_{vomax}), versus the force amplitude (F), normalized to the amplitude of design force $\mathbf{F_P}$.

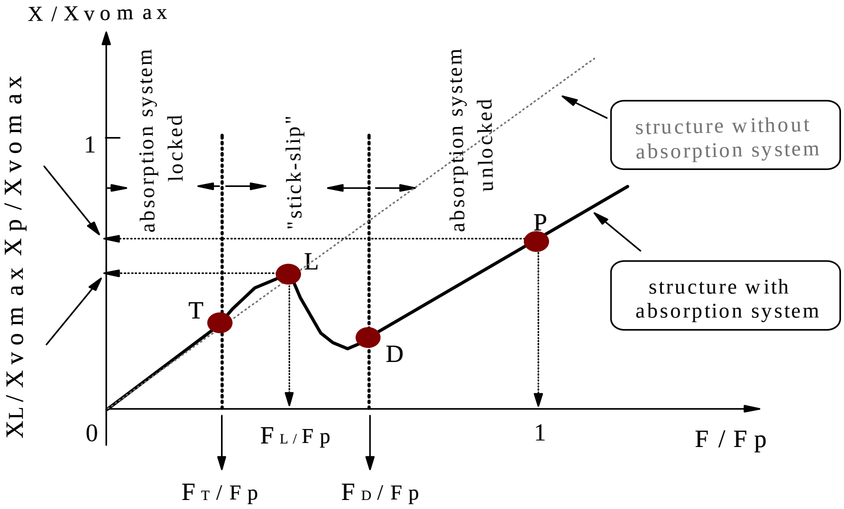

Figure 9 - Standard chart: displacement versus normalized force with constant friction force.

In “Fig. 9”, the friction force is constant, the excitation force amplitude (F) ranges from zero to $\mathbf{F_P}$. Note that from zero to $\mathbf{F_T}$, the absorption system remains locked to main system with linear behavior. Between points **T** and **D**, the structure’s response is “non-linear” and, point **L** is the maximum response amplitude in this middle region. Between **D** and **P**, the absorption system begins to act keeping the displacement levels under the established limit (X_P). “Figure 10” (numerical simulation example) illustrates the response reduction (in time domain) reached for the absorption system with linear behavior, in the region between **D** and **P** in “Fig.9”.

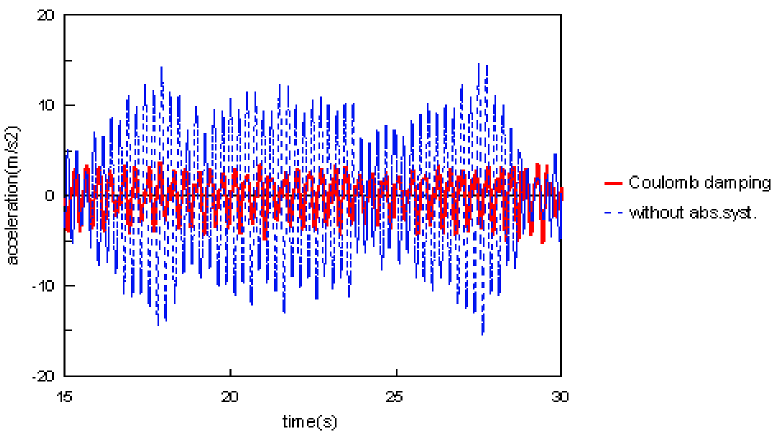

Figure 10 - Comparison between structure’s response with and without the Coulomb damping absorption system in the linear region.

In “Fig. 9”, the displacement amplitudes of point **L** is under the established limit (X_P), but there are some cases they can surpass the limit. This will depend on: main structure and absorption system features (mass ratio, stiffness, damping ratio), established design displacement (X_P), design loading amplitude $\mathbf{F_P}$ and applied friction force.

“Figure 11” (results from a numerical simulation) presents a chart similar to “Fig. 9”, showing the structure’s response for three different friction forces, where $\mathbf{f_{a1}<f_{a2}<f_{a3}}$. It can be seen that as the friction force increases, the displacement amplitude associated to point **L** increases too, while the response amplitude associated to point **P** decreases. Therefore, there is

a value for the friction force that keeps this amplitude in the same level of the established limit. This value is called *optimum* friction force. It can be seen that to reduce the response amplitude to 50%, only friction force $\mathbf{f_{a2}}$ solves the problem, as for $\mathbf{f_{a1}}$, point $\mathbf{P_1}$ exceeds the desired value, and for $\mathbf{f_{a3}}$, point $\mathbf{L_3}$ is above the established limit.

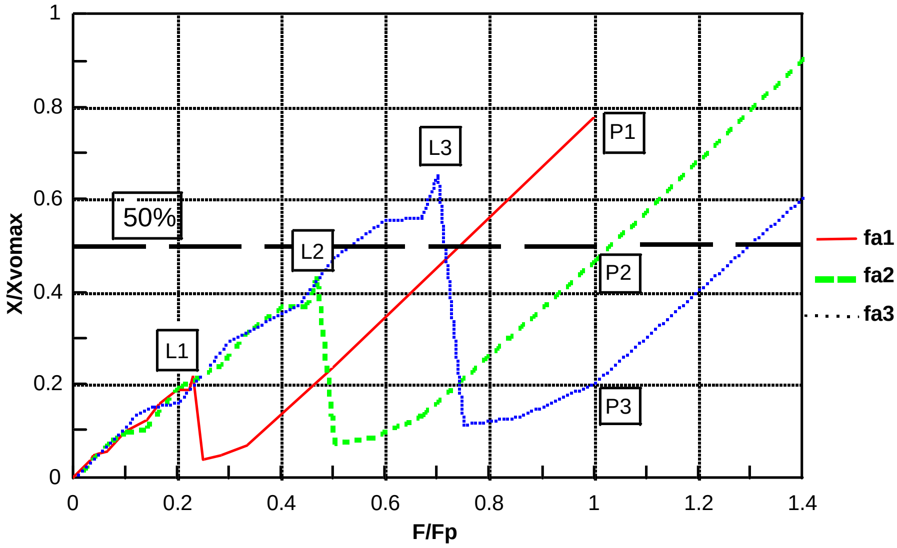

Figure 11 - Displacement versus force normalized for three different friction forces ($f_{a1}<f_{a2}<f_{a3}$).

6. FINAL COMMENTS

The experimental and numerical models mentioned in this paper show that the absorption system with Coulomb damping is indicated for cases in which the excitation force amplitudes oscillate in the region with linear behavior (between points **D** and **P** - “Fig. 9”).

To design this absorption system it is not sufficient to verify the structure’s response associated to the maximum force amplitude, the response for force amplitudes to which the absorption system is moving with “non-linear” behavior must be checked too. In order to guarantee that the established amplitude response limit should not be exceeded.

Acknowledgments

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