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FINITE ELEMENT MODELING AND EXPERIMENTAL INVESTIGATION OF A LINEAR VIBRATORY CONVEYOR

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Abstract. *The efficiency of vibratory conveyors is determined by the velocity of transportation of the material along its track, which is primarily determined by the track slope, the friction between the track and the transported material and the excitation characteristics (amplitude, frequency and wave form). Although many studies have already been performed, due to the complexity of the interactions between the components of the mechanical system, it is not possible, to date, to predict, in the design phase, the transportation velocity with the required accuracy. In this context, this work addresses the use of a finite element model to characterizing the actual distribution of displacements, velocities and accelerations along the track. The ultimate interest is to use the model in a general methodology intended for the prediction of the transportation velocity as a function of various design parameters. It is considered a linear conveyor excited by prescribed displacements introduced by a rod-crank mechanism. The model is adjusted and validated by comparing its previsions to experimental data obtained through static and dynamic tests performed on a prototype. Both the finite element modeling and experimental procedures are described. The distributions of steady-state harmonic responses, provided by finite element simulations are compared to the experimental counterparts, demonstrating the accuracy of the model.*

Keywords: *Vibratory conveyors, Finite elements, Dynamic tests, Automation, Handling materials.*

1. INTRODUCTION

Compared to other devices utilized in automated production systems, vibratory conveyors have the advantage of being simple to construct and operate, allowing low-cost maintenance. Moreover, they can be associated with various kinds of operations such as washing, drying, cooling, etc.

Vibration conveyors utilize the oscillatory motion of the track for the transportation. In this process, the friction between the track and the transported material is responsible for the transmission of the motion to the material. The overall efficiency of the conveyors is directly related to the velocity of the material over the track. This parameter depends mainly on the coefficient of friction, the track slope and the characteristics of the excitation function, in terms of amplitude, frequency and wave form.

Although a significant number of practical and theoretical studies have been performed since the beginning of the 19th century aiming at identifying the effect of different parameters on the transportation velocity (Povydaylo, 1959), (Povydaylo, 1960), (Both, 1964), (Gaberson,1972), the available data and knowledge is still insufficient to predict this velocity during the early design phase.

Many researchers have adopted simplified theoretical models in order to circumvent difficulties such as those related to: a) the description of the motion of the transported material during its return to the track after the flight phase; b) the modeling of the coupled bending-torsion dynamic behavior of the springs; c) the transition between static and dynamic coefficient of friction; d) the relation between the vibration modes of the system and the motion of the material. As a consequence, according to some industrial manufactures, design and construction procedures are generally based on empirical knowledge.

In the works developed by Carvalho (1991), various problems related to the validity of simplified mathematical models were detected. Among them, the most remarkable one is that a general analytical relation between the excitation wave form and the motion of the track does not exist. Experimental tests performed by Martins (1998) on industrial magnetically excited conveyors have shown discrepancies between the mathematical model usually utilized to represent the track motion and the actual behavior. In the usual modeling procedures, a purely sinusoidal excitation is assumed. This hypothesis has not been confirmed by the experimental investigation.

In the case of conveyors excited by imposed displacements, for which the amplitude, frequency and wave form are known, it is usually assumed that motion of the track is the same as the imposed motion. This hypothesis disregards the dynamics of the track as an elastic body which becomes more significant as the flexibility is increased.

As a part of a more comprehensive study of the dynamics the system conveyor-transported material, the present work describes a procedure of finite element (FE) modeling and numerical simulation of the dynamic behavior of a linear vibratory conveyor excited by imposed displacements. Particular emphasis is given to the process of model adjustment by using experimental data obtained from static and dynamic tests performed on a prototype. Evaluation of the accuracy of the adjusted model is made by comparing the simulated steady-state harmonic responses predicted by the model to the actual responses measured on the prototype during normal operation.

2. DESCRIPTION OF THE PROTOTYPE

In order to perform the experimental investigation, a prototype of a linear conveyor, excited by a rod-crank mechanism driven by an electric motor was constructed. The main features of the prototype, illustrated in Fig. 1, are the following:

- **driving motor**: cc, with speed control (maximum speed: 2.200 rpm).
- **track**: material: aluminum, length: 340 mm.
- **eccentric**: 0,38 mm.
- **springs**: formed by sets of 8 parallel thin steel blades, each one having dimensions 0,5×25,0×175,0 mm, angle of inclination: $\beta=40^o$.

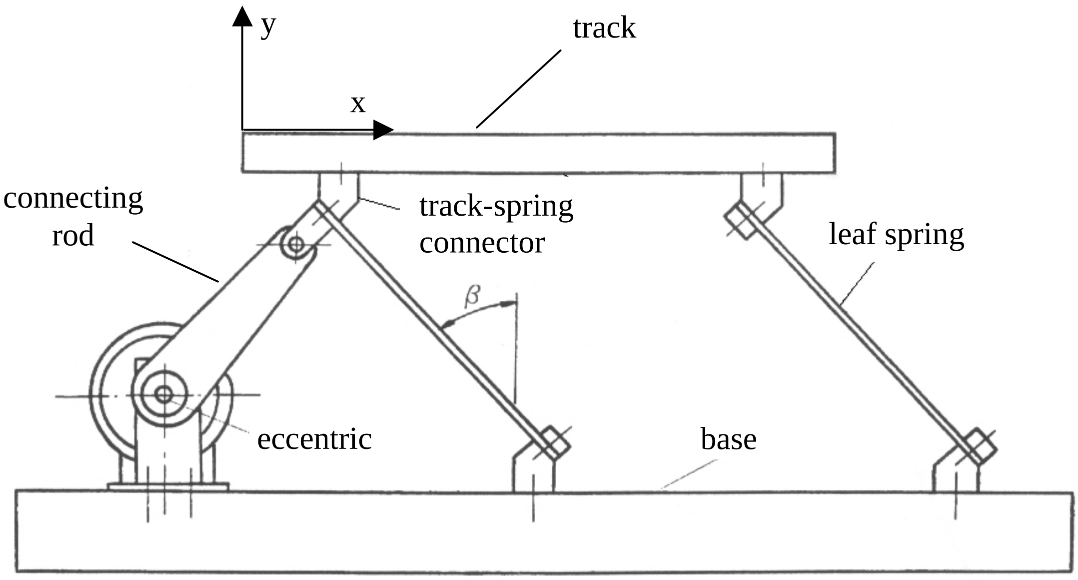

Figure 1 - Sketch of the prototype of the linear vibratory conveyor

3. THE FINITE ELEMENT MODEL

Finite element modeling was carried out by using the commercial FE analysis package ANSYS® v5.2 for operating system UNIX®. The goal was to produce a bidimensional model as simple as possible, being capable of representing the behavior of the prototype described in the previous section, in both static and low frequency dynamic domains. The components of the mechanical system were modeled as follows:

a) **Track**: the element BEAM3 (bidimensional Euler-Bernoulli beam element, with 2 nodes and 3 degrees-of-freedom per node) was chosen. The track was discretized with 69 nodes and 68 elements of equal length of 5mm.
b) **Leaf springs**: each of the 8 blades was discretized with 10 BEAM3 elements. They were assumed to be clamped to the fixed base and rigidly connect to the track through their endmost nodes.
c) **Track-spring connectors**: their inertia was modeled using lumped mass element MASS21;
d) **Connecting rod**: since this component was found to be very stiff and massive, it was modeled by using a single BEAM54 element (bidimensional tapered beam element, with 2 nodes and 3 degrees-of-freedom per node).

The values of the physical properties used for generating the model are given in Table 1. The total number of degrees-of-freedom of the model is 694.

It should be noted that the major modeling simplification was made in the connections of the springs with the track and the fixed base. In the prototype, these connections were made through steel bolted blocks between which the leafs were clamped, forming a "sandwich", whereas in the model the connections were represented by single-point rigid attachments. This way, it was foreseen that model refinement might be necessary to match the bending stiffness of the springs in the model to that of the prototype. The adjustment procedure will be described later.

Table 1. Values of the physical properties used in the FE model

Component	Material	Characteristics	Value/units
track	aluminum	Young modulus density Poisson ratio	$7{,}1\times10^{10}$N/m^2 2770 Kg/m^3 0,3
spring	steel	Young modulus density Poisson ratio	1,91x10^{11} N/m^2 7750 Kg/m^3 0,3
Connecting rod	aluminum	Young modulus density Poisson ratio	$7{,}1\times10^{10}$N/m^2 2770 Kg/m^3 0,3
track-spring connector	steel	mass	0,14 Kg

4. COMPUTATION OF THE TIME-DOMAIN HARMONIC RESPONSES

After the generation of the FE model, by using the ANSYS® package, the element mass and stiffness matrices were retrieved for later computation of the steady-state harmonic responses in MATLAB® environment. This procedure was adopted for the sake of flexibility in manipulating computation parameters and ease of visualization and analysis of the results. The formulation employed is presented in the following.

Letting $[K]$ and $[M]$ be the global stiffness and mass matrices of the FE, the governing equation for steady-state harmonic motion is written:

$$\left([K]-\omega^2[M]\right)\{X(\omega)\}=\{F\}, \tag{1}$$

where ω is the forcing frequency, $\{X(\omega)\}$ designates the vector of harmonic displacement responses and $\{F\}$ stands for the vector of amplitudes of the excitation forces.

The following partitioning of coordinates is introduced into Eq. (1):

$$\left(\begin{bmatrix}[K_{aa}] & [K_{ab}]\\ [K_{ba}] & [K_{bb}]\end{bmatrix}-\omega^2\begin{bmatrix}[M_{aa}] & [M_{ab}]\\ [M_{ba}] & [M_{bb}]\end{bmatrix}\right)\begin{Bmatrix}X_a(\omega)\\ X_b(\omega)\end{Bmatrix}=\begin{Bmatrix}F_a\\ F_b\end{Bmatrix}, \tag{2}$$

where indices "*a*" indicate the "free" coordinates while indices "*b*" indicate the coordinates at which the displacements are prescribed. Taking into account the partition of coordinates and assuming that coordinates "*a*" are free of external forces, the following equations can be obtained from Eq. (2):

$$\left(\left[[K_{aa}]-\omega^2[M_{aa}]\right]\right)\{X_a(\omega)\}+\left(\left[[K_{ab}]-\omega^2[M_{ab}]\right]\right)\{X_b(\omega)\}=\{0\} \tag{3}$$

$$\left(\left[[K_{ba}]-\omega^2[M_{ba}]\right]\right)\{X_a(\omega)\}+\left(\left[[K_{bb}]-\omega^2[M_{bb}]\right]\right)\{X_b(\omega)\}=\{F_b\} \tag{4}$$

From Eq. (3), the responses at the free coordinates can be calculated in terms of the prescribed displacement as follows:

$$\{X_a(\omega)\} = -\left([K_{aa}] - \omega^2 [M_{aa}]\right)^{-1} \{F_a\} - \left([K_{ab}] - \omega^2 [M_{ab}]\right)^{-1} \{X_b(\omega)\} \quad (5)$$

Once vector $\{X_a(\omega)\}$ has been calculated from Eq. (5), the reaction forces at the constrained coordinates, forming $\{F_b\}$, can be obtained by using Eq. (4).

5. EXPERIMENTAL TESTS

The experimental tests were performed in three phases:

• **static tests**, aiming at the adjustment of the spring bending stiffness.

• **dynamic tests**, for the determination of the first natural frequency of vibration of the track-spring set. The value of this frequency was later used for a first validation of the FE model.

• **dynamic tests**, for obtaining the harmonic responses in terms of displacement, velocities and accelerations along the track, to be compared to the FE simulation results using the formulation presented in the preceding section.

The experimental procedure and main results are presented in the following.

5.1. Static tests of the track-spring set

Static tests were carried out on the prototype without the connecting rod, according to the sketch depicted in Fig. 2. For each value of the load *P*, applied with the aid of calibrated masses, the resulting horizontal displacement of the track Δ was measured by using a linear position transducer (LVDT). The same loads were applied to the FE model and the corresponding static displacements were computed by inverting the global stiffness matrix. It was immediately noticed that the model was more compliant than the prototype. In average, the displacements of the model were 1,2 times greater than those of the prototype. This value was then used as a correction factor for the bending stiffness of the elements used in the discretization of the leaf springs. The obtained results are presented in Fig. 3, which enables to compare the experimental values to those of the numerical simulations based on both the initial and adjusted model.

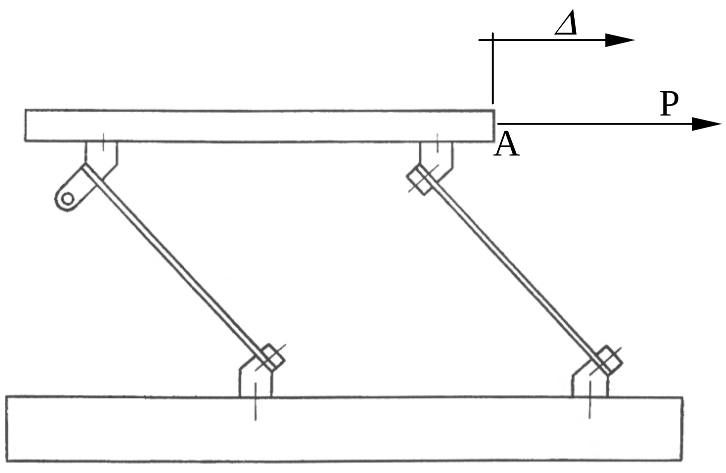

Figure 2 - Sketch of the static tests

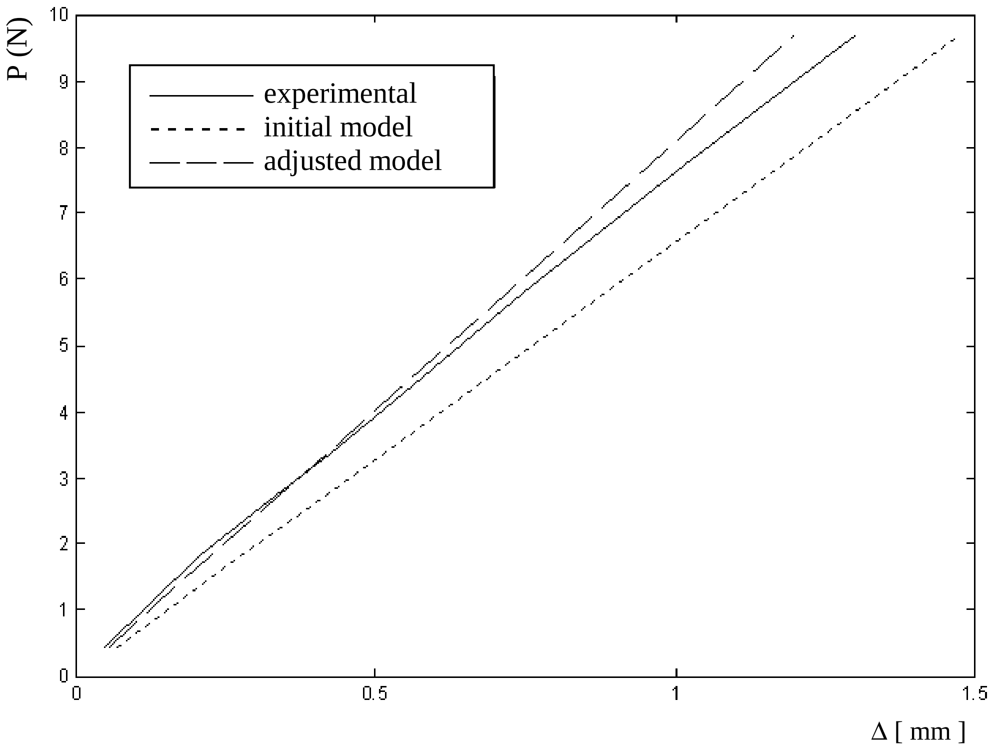

Figure 3 - Experimental and simulated results of the static tests.

5.2. Dynamic tests for the determination of the first natural frequency of the track-spring set

In addition to the static tests, dynamic experiments were performed aiming at verifying the accuracy of the track-spring set with respect to its inertia distribution. The static tests were considered to be sufficient to fully characterize the stiffness properties of the system.

Using a modal hammer instrumented with a piezoelectric force transducer, the prototype without the connecting rod was excited by impact forces and the corresponding acceleration responses were measured through a piezoelectric accelerometer. A 2-channel spectrum analyzer was used for estimating the frequency response functions (FRF's), relating the spectra of excitation and response. Each FRF was calculated as the mean of 50 samples, with a frequency resolution Δf = 0,0625 Hz. Several tests were performed considering different excitation and measurement locations. As an example, Fig. 4 shows the amplitudes of the driving point FRF pertaining to motion of point A (indicated in Fig. 2) in the horizontal direction. The existence of the first vibration mode with a natural frequency of 9,37 Hz can be clearly noticed.

Figure 5 illustrates the first mode shape of the FE model of the track-spring system, corresponding to a natural frequency of 9,61 Hz. The difference between the FE and experimentally identified natural frequencies is approximately 2,5 %, which is considered to be acceptable, indicating that the FE model is fairly accurate in terms of both the stiffness and inertia distributions.

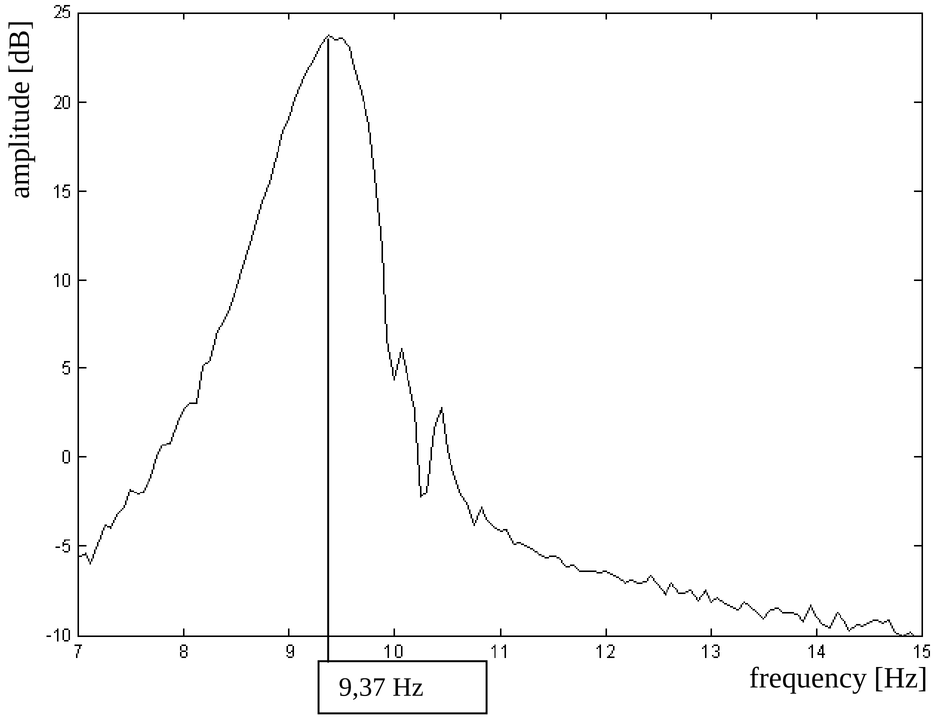

Figure 4 - Amplitudes of the driving point FRF pertaining to point A.

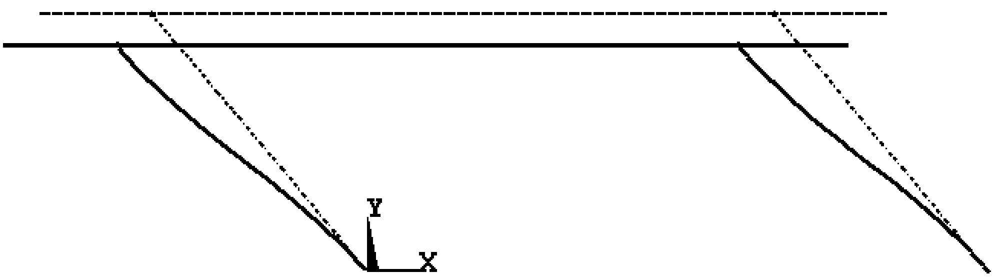

Figure 5 - Illustration of the first FE mode shape

5.3. Dynamic test for the determination of the responses along the track

Once the FE model of the track-spring system has been adjusted and validated, a series of dynamic tests were performed with the conveyor under operation at a driving frequency of 30 Hz, for obtaining the steady-state distributions of displacements, velocities and accelerations along the track for a later comparison to the response predicted by the FE model of the complete conveyor (including the connecting rod). The acceleration time responses in both directions: longitudinal (direction x) and normal to the track (direction y), were simultaneously acquired by using a 2-channel digital oscilloscope, two piezoelectric accelerometers, and two signal conditioners. The signals were acquired with a time step of 5×10^{-4} s and the oscilloscope was set up to compute the mean responses over 128 samples. Velocity and displacement histories were obtained by analogical successive integrations of the acceleration responses. Measurements were made at 11 points, with a regular step of 34 mm along the track.

Figures 6 and 7 present the amplitudes of the FE and measured displacements in direction x and y, respectively. The distances indicated in the abscissae were measured at the leftmost extremity of the track (see Fig. 1). Based on the numerical values, it was verified that the maximum deviations found were of 5,1% for displacements in x direction, at the rightmost point of the track, and of 9,1% for displacements in y direction, at the leftmost measurement station. As can be seen, the amplitudes change very little along the track.

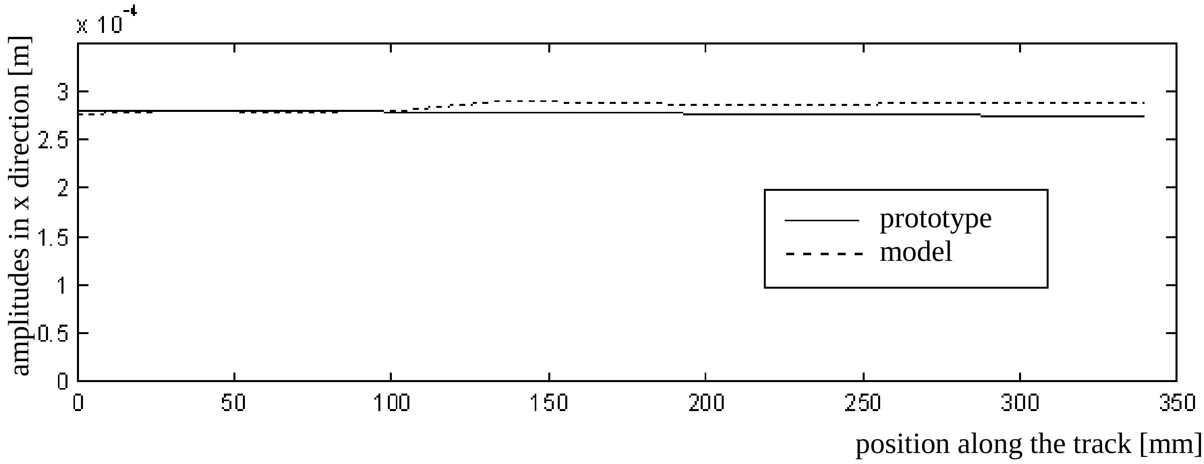

Figure 6- Amplitudes of harmonic displacements in x direction along the track.

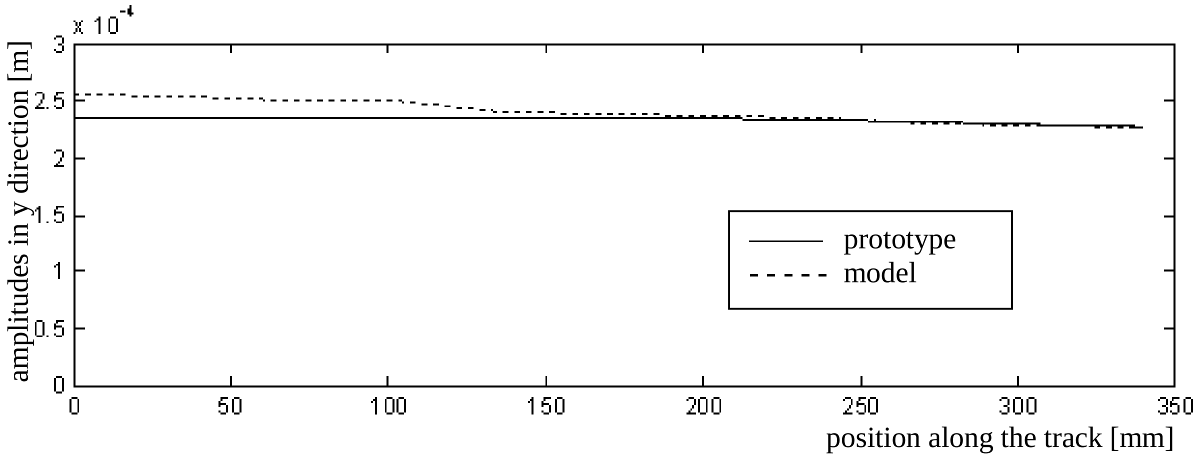

Figure 7- Amplitudes of harmonic displacements in y direction along the track.

Figures 8 and 9 enable to compare the displacement time histories for two points: the first located on the left extremity (x = 0 mm) and the second located at the mid-length of the track (x = 170 mm). A good correlation between experimental and simulation results can be noticed.

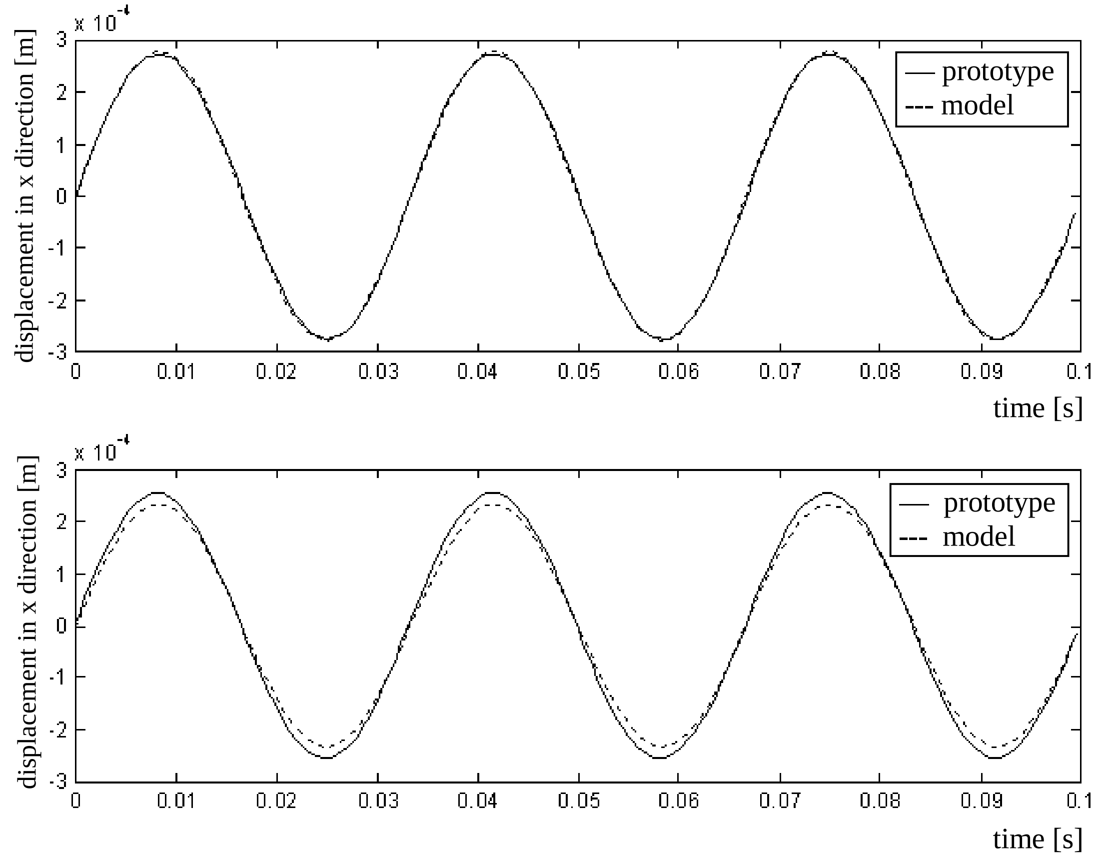

Figure 8 - Displacement time responses for the leftmost measurement point

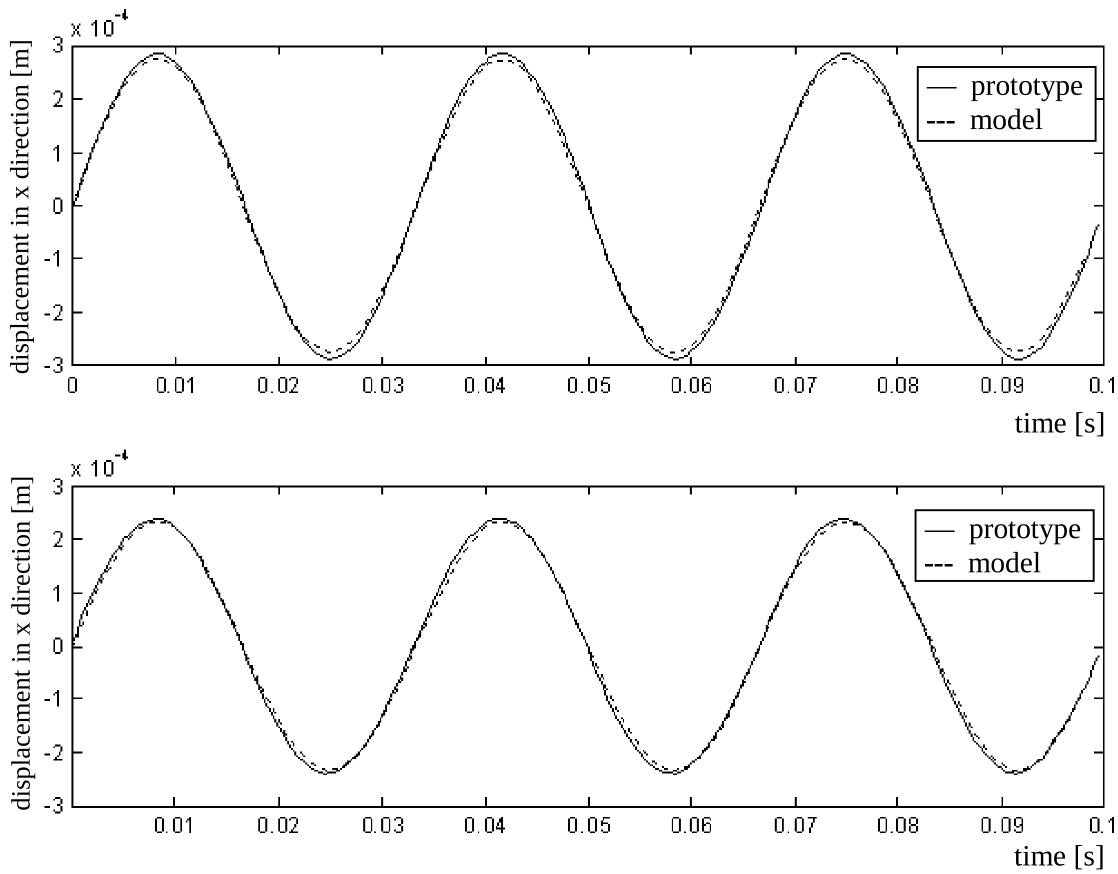

Figure 9 - Displacement time responses for the mid-length measurement point

6. CONCLUSIONS

A methodology was presented concerning the finite element modeling and experimental investigation of a linear vibratory conveyor driven by imposed displacements. The experimental results were used for refinement and validation of the model derived. After a first adjustment of the stiffness characteristics from static tests, a good correlation between measured harmonic responses and numerical simulations could be obtained.

The adjusted model is intended to be used in subsequent phases of the research work in an attempt to completely characterize the motion of the transported material. For this, it is planned to incorporate the simulated responses of the track into a general mathematical model for the motion of the transported material, using the equations of motion written for this latter. It is expected that this methodology will enable to provide an efficient tool for the analysis of the effects of various parameters such as frequency, amplitude and form of excitation, stiffness of the track and springs and track slope on the final velocity of the transported material. Eventually, applications to the optimum design of vibratory conveyors will be investigated.

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