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STRESS ANALYSIS AND STABILITY OF COMPOSITE HOLLOW BARS OF VARIABLE DIAMETER

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Abstract . *The aim of this numerical investigation was to analyze the mechanical behavior of perfect, carbon / epoxy and E-glass / epoxy, filament wound shells of revolution subjected to uniaxial, static, compressive loads. The shell wall of the models was an angle - ply laminate in which the orientation of the fibers varied from 15^o to 65^o. The structure analyzed was composed of three cylindrical segments (two of identical diameters at both extremities and another of larger diameter in the center) connected by cones. The simulations were carried out using a non-linear finite element code, and both, material failure of the layers as well as critical buckling loads were calculated. In all the simulations of this study, material failure of the composite layers always took place before buckling.*

Keywords: *Composite materials, Stress analysis, Buckling*

1. INTRODUCTION

With the advent of fibers such as carbon and E-glass, which present high stiffness, high tensile strength, and low weight, many industries started to use composites, based on epoxy resin reinforced with fibers, in structural components like beams, columns, plates and shells. Such components are able to present many advantages in comparison with comparable metallic structures, but, on the other hand, they normally do not exhibit any kind of plasticity, and are typically brittle. As a consequence, even when a composite structure is very slender, its design needs to incorporate both: stability and stress analysis (Levy Neto, 1991: Gonçalves, 1997).

In the present study, shells of revolution of thin wall, clamped at the base, with total length *L* = 700 mm, and composed of cylindrical and conical segments, assembled as a composed bar, as shown in Fig. 1, were simulated under the action of static uniaxial load, applied at the free extremity which was covered with a steel circular plate. Before this work, an experimental and theoretical investigation, in which 47 prismatic composite cylinders covered with a steel plate were tested to failure, was carried out by Gonçalves (1997). The present work is a continuation of such study.

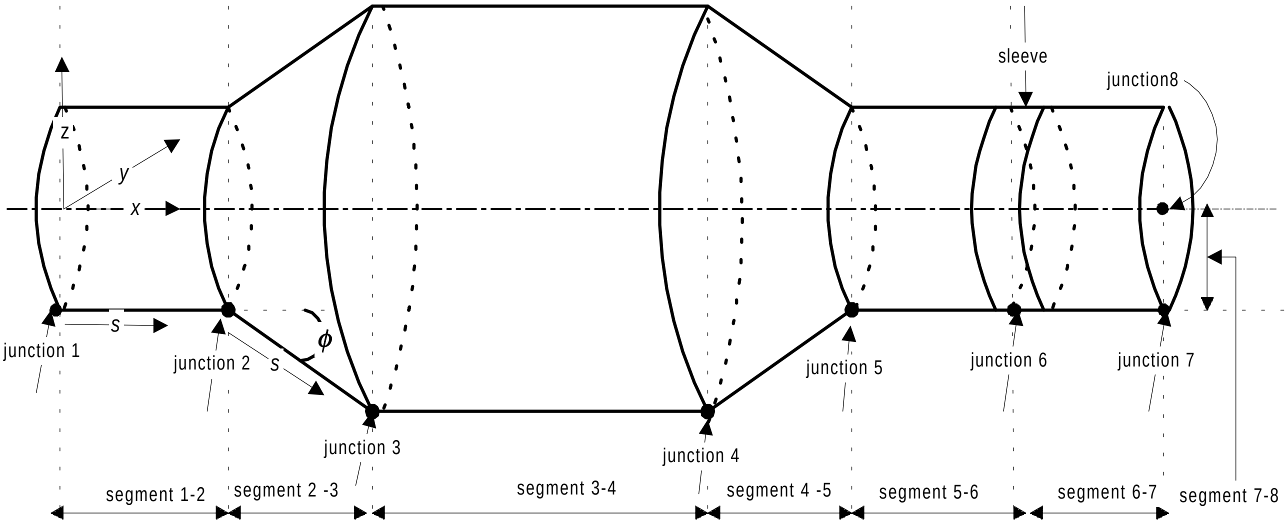

Figure 1 - General view of the hollow composite bars investigated

The structure in focus has a potential to be used as a low weight vehicle steering wheel segment. And, one aspect that needs to be carefully analyzed in the design of such component, is its collapse strength in a situation in which the vehicle crashes in a frontal collision, and its conductor hits the steering wheel column in a violent way.

In such analysis, many failure mechanisms can take place (Levy Neto, 1991), such as:

(i) axisymmetric collapse;
(ii) bifurcation buckling; and
(iii) material failure.

The main purpose of this study is to estimate an equivalent static load for the collision of the conductor with the steering wheel (Lau et al., 1991), and simulate the mechanical behavior of the composite bar using a finite element code.

2. MATHEMATICAL MODEL

2.1 Static stress analysis

The finite element modeling of the composite structure simulated in this study was based on the shell theory of Novozhilov, for thin axisymmetric shells, and the Classical Laminate Theory. More details concerned with the formulation can be found in the work of Baldoino (1998).

The differential equation that governs the mechanical behavior of the structure, in matrix form (Baldoino, 1998) is given by:

$$[[S_E] + [G_E]]\{\xi\} = \{Q_E\} - \{N_E\}, \qquad (1)$$

where:
[ξ] = vector of all degrees of freedom,
[S_E] = linear stiffness matrix,
[G_E] = geometric stiffness matrix,
[Q_E] = vector of external loads,
[N_E] = vector of nodal prestress

2.2 Elastic stability analysis

The stability analysis was carried out by transformation of Eq. (1) into an eigenvalue problem, obtaining:

$$[[S_E] + \lambda_o [G_E]]\{\xi\} = \{Q_E\} - \lambda_o \{N_E\} \qquad (2)$$

Where λ_o 's are eigenvalues associated with the critical loads, and $\{\xi\}$ are the eigenfunctions .
According to Bushnell (1982), as the external compressive load increases, but the stability determinant remains positive, the load deflection points of the structure follows a fundamental path (from point **O** towards point **B** in Figure 2) and the number of circumferential waves of the shell, ***n***, is zero (i.e. it remains axisymmetric for ***n*** = 0). If point **B** is reached during the loading, and the stability determinant changes its sign, bifurcation buckling takes place, and the shell assumes a new geometric shape, associated with a given number of circunferential waves, ***n*** ≠ 0 (i.e. there is a bifurcation to the alternative path), and a critical load λ_o. If the sign of the determinant does not change, from **O** to point **D**, but, eventually, the tangential stiffness of the structure vanishes (point **D**), non linear axisymmetric collapse (or snap buckling) takes place. And, in this case the shell remains axisymmetric (i.e. ***n*** = 0).

The modes of failure discussed above can be also described by the limit points *B* (bifurcation buckling) and *D* (axisymmetric collapse along the load path diagram) as illustrated in Fig. 2 (Bushnell, 1982).

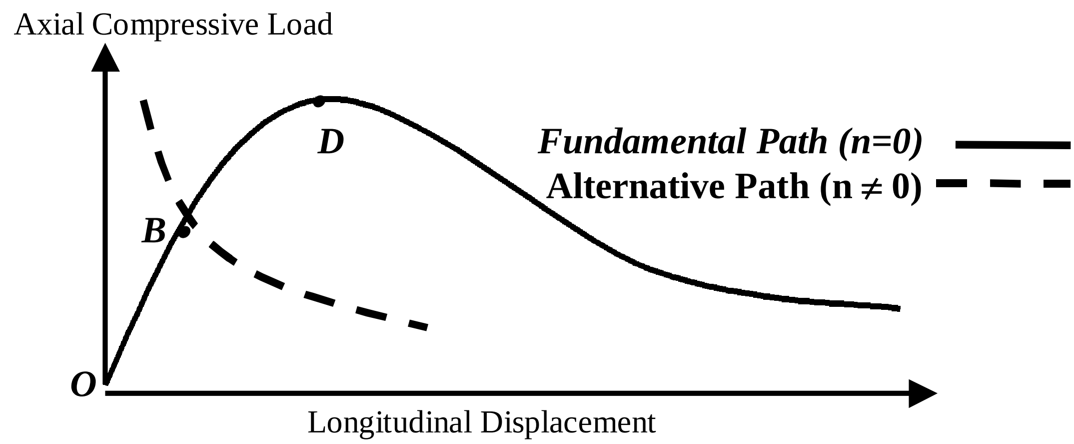

Figure 2 - Load path diagram for perfect cylinders under uniaxial compression

3. FAILURE CRITERIA FOR ORTHOTROPIC LAMINAE

In the present analysis, according to the Classical Laminate Theory (Jones, 1975; Gibson, 1994), the composite layers were assumed to be subjected to in -plane stresses, which can be calculated either: (i) along the main coordinate system of the axisymmetric shell (i.e. the meridional, s, and the circunferential, θ , coordinate); or (ii) along the fibers, 1, and perpendicularly to the fibers, 2. In each layer of reinforced plastic, the system (1,2), according to the orientation of the fibers, can be rotated about the system (s,θ) by an angle, δ, as illustrated in Fig 3.

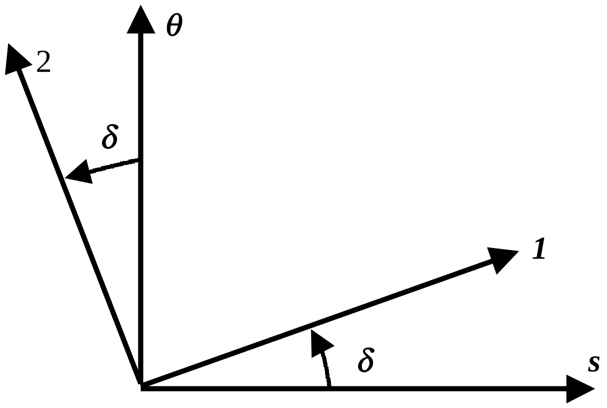

Figure 3 - Orientation of the fibers relatively to the shell coordinate system (s, θ)

In order to find out if a set of in - plane stresses (σ_1 , σ_2 , γ_{12}) can cause the failure of any particular layer of reinforced plastic, along all the elements of a finite element mesh, a suitable failure criterion, specific for orthotropic materials, can be employed. Among many options, the failure theory of Tsai - Hill has been widely adopted in many studies (Jones, 1975; Levy Neto, 1991; Baldoino, 1998). Such criterion is based on a quadratic equation and the threshold of failure for an orthotropic ply, in the system of coordinate (1, 2) is given by:

$$\left(\frac{\sigma_1}{X_R}\right)^2 - \frac{\sigma_1\,\sigma_2}{X_R^2} + \left(\frac{\sigma_2}{Y_R}\right)^2 + \left(\frac{\tau_{12}}{S_{12}}\right)^2 = 1 \qquad (3)$$

where the strength properties of the material are :

X_R = X_{1T} or X_{1C} depending on the sign of $\sigma_1^{(*)}$;
Y_R = X_{2T} or X_{2C}, depending on the sign of σ_2 $^{(*)}$; and
S_{12} is the in - plane shear stress.

(*) when the normal stress σ is positive one has to use the tensile strength, and, when it is negative the compressive strength.

4. MATERIALS AND SIMULATION METHODS

4.1 Mechanical properties of the materials

The elastic (E_{11}, E_{22}, G_{12}, ν_{12} and ν_{21}) and strength properties (X_{1T}, X_{2T}, X_{1C}, X_{2C} and S_{12}) of the carbon / epoxy plies in the coordinate system (1, 2), as well as its density (ρ), are given in Table 1. The subscripts 1 and 2 refer to the directions along the fibers (1) and perpendicular to the fibers (2).

Table 1. Mechanical Properties of the Carbon / epoxy Plies (Baldoino, 1998)

Property	Value	Unity
Elasticity modulus, E	170	GPa
Elasticity modulus, E	12,4	GPa
Shear Modulus G	7	GPa
Poisson Rations ν_{12} and ν_{21}	0,27 and 0,02	dimensionless
Maximum Tensile Stress, X_{1T} (direction 1)	1009	MPa
Maximum Tensile Stress, X_{2T} (direction 2)	16	MPa
Maximum Compressive Stress, X_{1C} (direction 1)	661	MPa
Maximum Compressive Stress, X_{2C} (direction 2)	126	MPa
Maximum Shear Stress, S_{12} (plane 1 - 2)	40	MPa
Density ρ	0.0016	g /mm^3

The mechanical properties of the E - glass / epoxy plies are given in Table 2 (Baldoino, 1998).

Table 2. Mechanical Properties of the E - glass / epoxy plies (Baldoino, 1998)

Property	Value	Unity
Elasticity modulus, E	41,3	GPa
Elasticity modulus, E	8,65	GPa
Shear Modulus G	4,10	GPa
Poisson Rations ν_{12} and ν_{21}	0,31 and 0,07	dimensionless
Maximum Tensile Stress, X_{1T} (direction 1)	1000	MPa
Maximum Tensile Stress, X_{2T} (direction 2)	30	MPa
Maximum Compressive Stress, X_{1C} (direction)	600	MPa
Maximum Compressive Stress, X_{2C} (direction	110	MPa
Maximum Shear Stress, S_{12} (plane 1 - 2)	40	MPa
Density ρ	0.0019	g /mm^3

The models investigated in this study were composed of cylindrical and conical segments of thin wall, as shown in Fig. 1. The first joint (x = 0) was assumed to be fully clamped, and the last joint (x = 700 mm) was covered by a steel circular plate, which was free to move along the meridional direction x. The Mechanical properties of the steel (SAE 4340), according to Baldoino (1998), are given in Table 3.

Table 3. Mechanical Properties of SAE 4340 Steel

Property	Value	Unity
Elasticity modulus, E = E	200	GPa
Shear Modulus G	77,0	GPa
Poisson Ration $\nu_{12} = \nu_{21}$	0,30	Dimensionless
Tensile Yield Strength, $X_{1T} = X_{2T}$	1000	MPa
Compressive Yield Strength $X_{1C} = X_{2C}$	1000	MPa
Shear Strength, S	500	MPa
Density ρ	0,0078	g / mm^3

Although the failure criterion of Tsai Hill is suitable for brittle materials (Jones, 1975), and predicts the fracture of carbon / epoxy and E - glass / epoxy plies, it was based on the Von Mises failure theory for ductile metals, in which the onset of failure is controlled by the yield stress of the material. Due to this fact, if mechanical properties of an isotropic steel are used in the criterion of Tsai Hill, and the yield strengths are given in place of the ultimate stresses, its failure index becomes identical of that predicted by the failure theory of Von Mises.

4.2 Simulation of the behavior of the composite models

The basic geometric parameters of the simulated models, including the global coordinate system (*X*, *Y*, *Z*); a ring element with its nodal points; the segments; the meridional coordinate *s*; and the cone angle ϕ, is illustrated in Fig. 4.

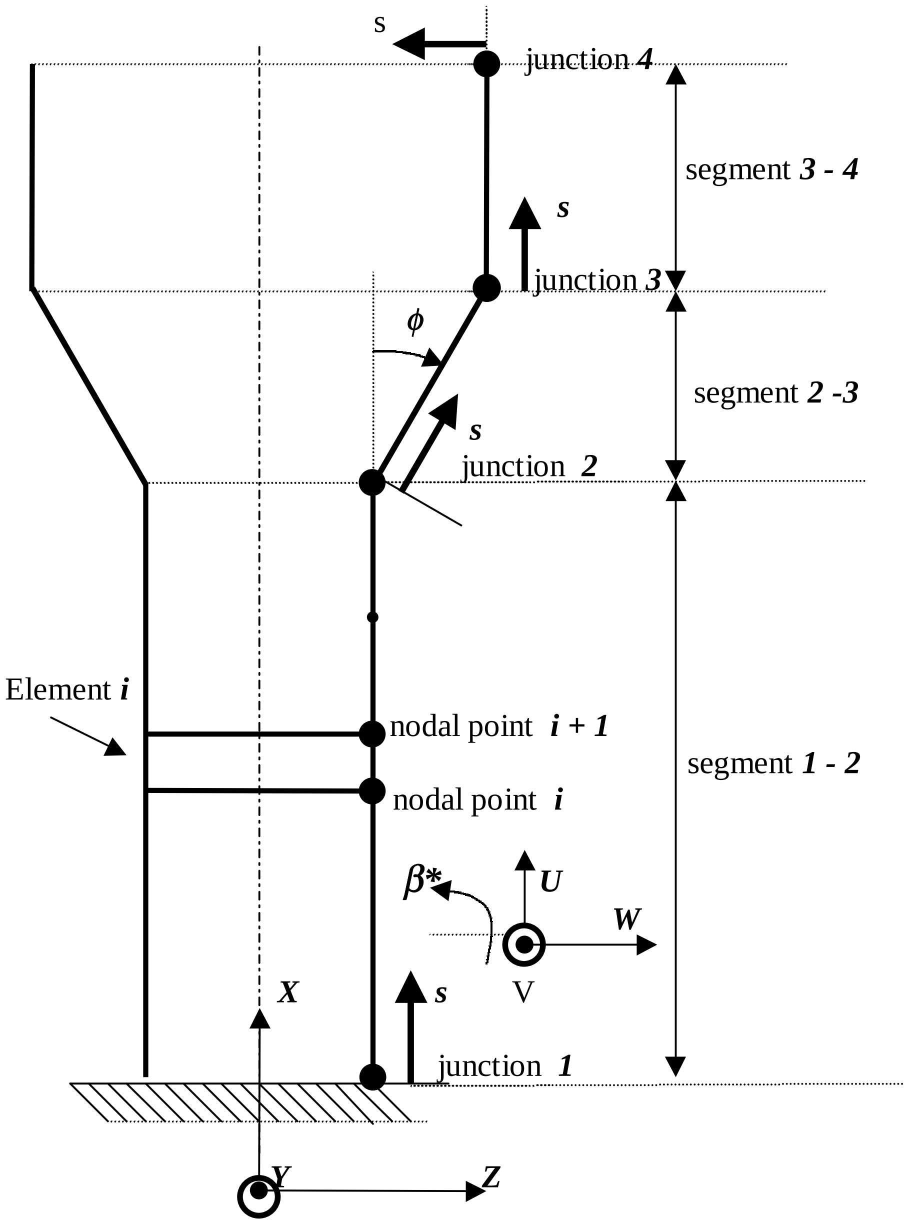

Figure 4 - Some details concerned with the geometry of the simulated models

In the present study it was assumed ϕ = 16.7^o for the cone angle, D = 100 mm and d = 40 mm for the larger and smaller diameters, respectively, and L = 700 mm for the total length of the structure. Using the geometrically non-linear finite element code COMPSHELL (Levy Neto, 1991; Baldoino, 1998), both, static stress analysis and elastic stability were carried out. The program COMPSHELL takes into account moderate deformations, and is based on a two-node ring in which each node has four degrees of freedom. Such degrees of freedom, relative to the global coordinates (X, Y, Z), as shown in Fig 4, are: the meridional, U, the circunferential, V, and the radial, W, displacements, as well as the meridional rotation β^* .

The external boundary conditions of the structure, as shown in Figs 1 and 4, were :

- junction 1, $X = 0$, $U = W = \beta^* = 0$; and V is free;
- junction 6, $X = 600$ mm $W = 0$; and U, V and β^* are free;
- junction 8, $X = 700$ mm, $V = W = \beta^* = 0$, and U is free.

The boundary condition at junction 6 represents a frictionless short sleeve which allows the structure to slide along the longitudinal direction X. The segment 7 - 8 is a 10 mm thick circular plate of steel, in which a longitudinal, uniform, compressive pressure (p) is applied on the structure. This pressure simulates a equivalent static load that takes place when the conductor of the vehicle hits the steering wheel in an eventual frontal collision. Its magnitude can be estimated from the mass of the conductor, and the velocity of the vehicle, following a procedure described in Lau et al. (1991). In the present study, the values of p that can cause : (i) axisymmetric collapse (AC) or bifurcation buckling (BB); and (ii) First - Ply -Failure (FPF) of the composite layers (i.e. material failure) of the structure will be calculated using the program COMPSHELL. The program has capabilities to perform stability analysis, and obtain the critical value of p that causes either AC or BB, and can also carry out stress analysis to calculate the ultimate value of p that causes FPF.

5. RESULTS

5.1 Structure reinforced with carbon fibers

The shell wall of the structure was composed by 8 unidirectional layers of carbon epoxy, having thickness tc = 0.1 mm each, and the mechanical properties given in Table 1. The total thickness of the shell wall is t = 0.8 mm constant in all the segments of the structure. The structure represented in Fig. 1, is composed of 3 cylindrical segments of length l = 100 mm each and smaller diameter, d = 40 mm; 1central cylindrical segment of large diameter, D = 100 mm, and length l' = 200 mm; and 2 conical segments of cone angle ϕ = 16.7^o, in which the length of the axial projection is l'' = 100 mm each. The average diameter of the structure is Dm = 65.715 mm, and its ratio of total length over average diameter (L / Dm) = 10.652`.

Using the program COMPSHELL, both stability and stress analysis were performed. The results, in which the orientation of the fibers (± δ, the winding angle) varied from 15^o to 55^o (in absolute values) are in Fig. 5. The FPF (First - Ply - Failure) and buckling pressures refer to the compressive, uniaxial, uniform pressure applied on the steel circular plate.

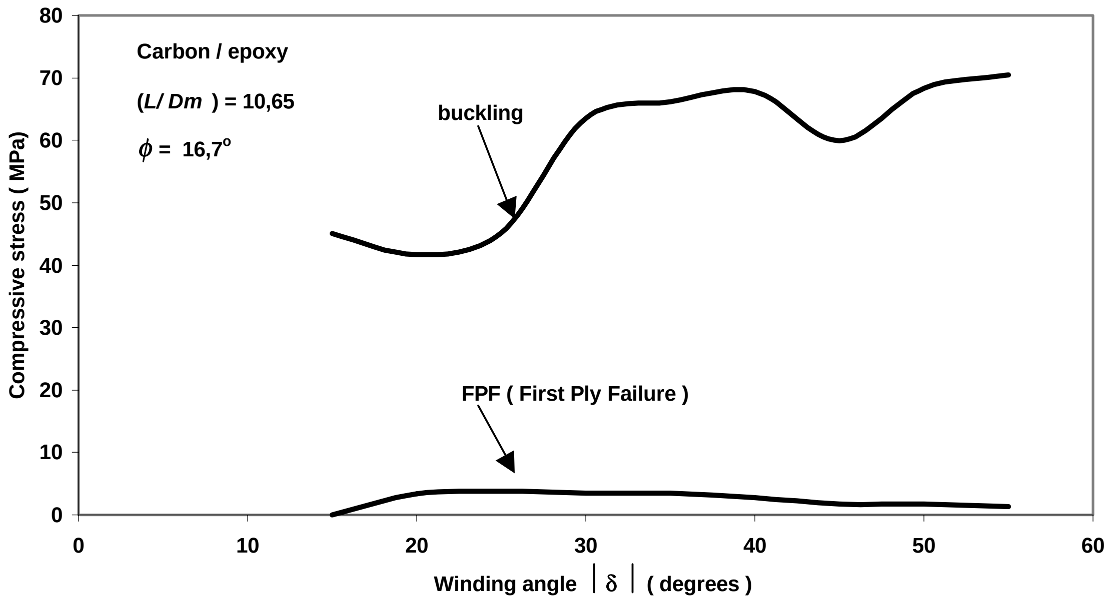

Figure 5 - Comparison between the buckling and FPF pressures

5.2 Structure reinforced with E - glass fibers

The structure reinforced with E -glass fibers, with mechanical properties given in Table 2, had the same geometry as the one reinforced with carbon fibers (see section 5.1). The only difference is on the number of plies of the larger cylindrical segment (D = 100 mm), which, in this case, was reinforced, with only 4 plies of E - glass / epoxy. All the other segments, as in section 5.1, were reinforced with 8 composite layers. The results in which the absolute values of the winding angles varied from 15^{o} to 65^{o}, are shown in Fig. 6

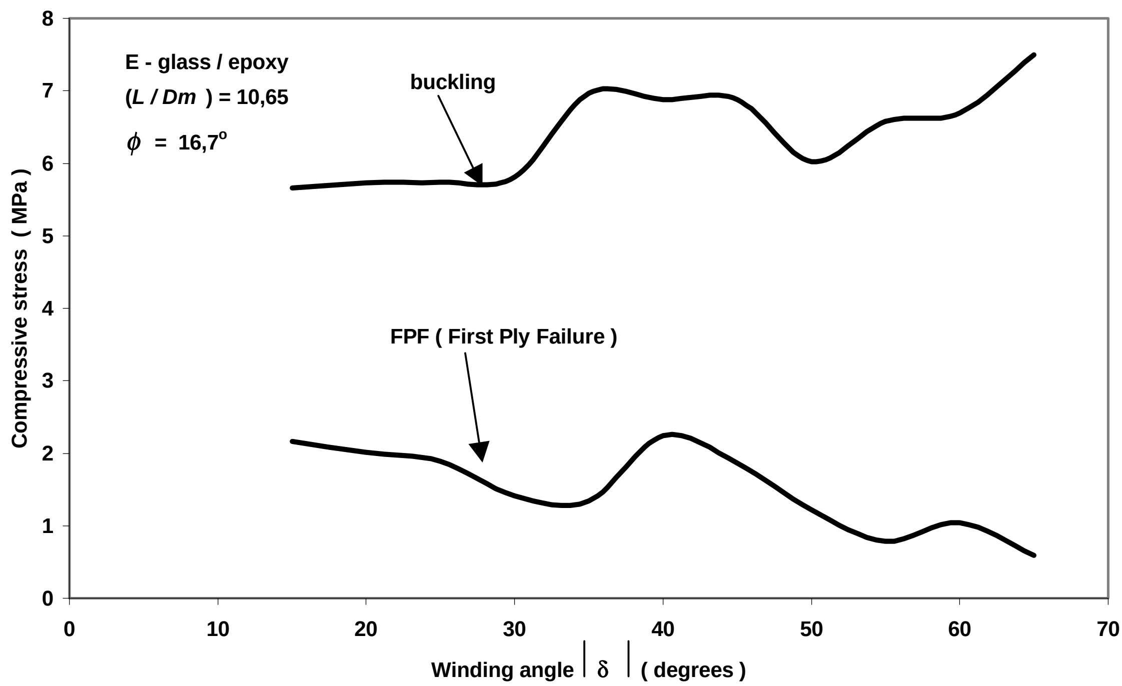

Figure 6 - Comparison between the buckling and FPF pressures

6. DISCUSSION OF RESULTS AND CONCLUSIONS

The results obtained so far indicated that, by a significant margin, FPF (i.e. material failure) always takes place before buckling / axisymmetric collapse (i.e. stability failure). For the carbon / epoxy models (section 5.1), the ratio of the minimum stability failure pressure over the maximum FPF pressure (STABILITY / FPF) was about 12. In absolute values, the threshold of minimum stability pressure is higher than the maximum FRF pressure by more than 40 MPa, which is about of 400 bars (or atmospheres). In addition, the locations of the FPF (i.e. layer, in an element, in which the failure index is the highest) were always at the junctions 3, 4, or 8 as shown in Figure 4.

In section 5.2, the models were reinforced with E - glass fibers, which is approximately 3 times less rigid than the carbon fibers (Gibson, 1994). In addition, the models of section 5.2 were reinforced with only 4 layers of plies in the larger cylinder segment (instead of 8 plies, as in section 5.1). Despite of these reductions in stiffness ($\cong$ 3 times overall) and thickness ($\cong$ 2 times, in the central segment), the FPF pressures were still below the buckling pressures by a factor of 3 times (i.e. (stability / FPF) $\cong$ 3). In absolute terms the minimum stabily pressure was higher than the FPF pressure by approximately 3 MPa (about 30 bars). Again, the locations of the FPF were at the junctions 3,4, or 8, as shown in Figure 4.

The results of both sets of simulations also indicated that the stability failure pressures are more sensitive to the winding angle, δ, than to the FPF pressures, as shown in Figs 5 and 6. The average diameter over the average thickness (Dm / t_{avg}) of the models analyzed in this work varied in the range $82 \leq (Dm / t_{avg}) \leq 95$.

An experimental and theoretical investigation carried out by Gonçalves (1997) in which 47 carbon / epoxy cylinders of diameter 40 mm were tested to failure, obtained the following results.

For cylinders with D / t = 66,7 and 2,5 ≤ L/ D ≤ 7,5. FPF took place well before buckling, confirming the results of the present work.

For cylinders with D/ t = 200; and L/D = 11,25. Experimental buckling took place before FPF, but theoretical FPF prediction was slightly below buckling .

So, for the geometries and materials investigated in the present work, there is a strong indication that material failure will also take place before buckling or axisymmetric collapse, in particular for structures in which the imperfections are reduced and do not knock down the failure loads.

Apart from the geometries analyzed in the present investigation, some additional simulations were carried out by Baldoino (1998), in which the cone angle, ϕ , varied from 5.7^o to 21.8^o, and the ratio of length over average diameter, (L / D_m), varied from 4.8 to 14.4. Basically, in such simulations, the results were similar to those reported in this paper, and the failures controlled by FPF were still taking place before buckling. Again, the critical points were the FPF took place were the junctions 2,3,4,5, or 8. Since the junctions, in particular the cylinder/cone transitions, are critical points, it is the intent of the authors to carried out simulations in which torispherical segments will be adopted instead of the cones, in order to reduce the stress levels at such junctions.

Finally, it needs to be emphasized that all the calculations carried out in the present study is only concerned with perfect shells. Since it is well known in the literature that geometric imperfections can knock down the stability loads of slender metallic and composite structures by significant values (Bushnell, 1984; Galletly et al., 1990; and Blachut et al., 1990), the results presented in this work have to regarded more as reference values rather than design values.

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