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DESIGN OF FLEXTENSIONAL PIEZOELECTRIC ACTUATORS USING TOPOLOGY OPTIMIZATION

Emílio C. N. Silva
Department of Mechanical Engineering,
Escola Politécnica da Universidade de São Paulo,
São Paulo, SP, 05508-900, Brazil
email:ecnsilva@usp.br

Abstract. *Flextensional piezoelectric actuators consist of a piezoceramic (or a stack of piezoceramics) connected to a flexible mechanical structure that converts and amplifies the output displacement of the piezoceramic. The actuator performance depends on the distribution of stiffness and flexibility in the coupling structure domain, which is related to the coupling structure topology. By designing other types of coupling structures connected to the piezoceramic, we can obtain novel types of flextensional transducers with enhanced performance. In this work, we propose a method for designing flextensional actuators by applying topology optimization technique based on the homogenization design method, which consists of finding the optimal material distribution in a perforated design domain with infinite microscale voids. The problem is posed as the design of a flexible structure coupled to the piezoceramic that maximizes the output displacements in a specified direction and point of the domain. Since complex topologies are expected, the finite element method is used for transducer modeling. Only static and low-frequency applications are considered. As a result, designs of flextensional actuators are presented.*

Keywords: *Topology optimization, Homogenization design method, Flextensional actuators, Piezoelectric actuators, Finite element*

1. INTRODUCTION

Flextensional actuators consist of a piezoceramic (or a stack of piezoceramics) connected to a flexible mechanical structure that converts and amplifies the output displacement of the piezoceramic. A well–known type of flextensional actuator is the "moonie" transducer, which consists of a piezoceramic disk sandwiched between two metal endcaps (Xu *et al.*, 1991). The performance of the flextensional actuator is measured in terms of output displacement and generative (or "blocking") force (Dogan *et al.*, 1997), that is, the maximum force supported by the transducer without deforming, for a certain applied voltage.

Flextensional actuators have been developed by using simple analytical models and experimental techniques (Xu *et al.*, 1991 e Dogan *et al.*, 1994), and the finite element method (Dogan *et al.*, 1997). However, the design is limited to the optimization of some dimension of a specific topology chosen for the coupling structure. These studies showed that the performance depends on the distribution of stiffness and flexibility in the coupling structure domain, which is related to the coupling structure topology. Therefore, the design of the coupling structure can be achieved by using topology optimization. By designing other types of flexible structures connected to the piezoceramic, we can obtain novel types of flextensional actuators that produce high output displacements (or generative forces) in different directions, according to a specific application of the actuator.

Based on this idea, in this work, we propose a method for designing flextensional actuators by applying topology optimization techniques. The problem is posed as the design of a flexible structure coupled to the piezoceramic that maximizes the output displacement and generative force in some specified direction. Only static and low-frequency applications (inertia effects are negligible) are considered. The topology optimization method (Bendsøe, 1995) applied is based on the homogenization design method developed by Bendsøe and Kikuchi (1988). FEM is applied to the structural analysis in the optimization procedure (Lerch, 1990). Even though two-dimensional (plane strain) topologies of flextensional actuators are presented to illustrate the implementation, the method can be extended to three-dimensional topologies.

2. TOPOLOGY OPTIMIZATION PROCEDURE

Topology optimization is a computational design method that combines optimization algorithms (usually sequential linear programming) and finite element method to find the optimum topology of mechanical parts considering a desired objective function and some constraints. Essentially, it allows us to design structures with holes optimally placed to maximize (or minimize) a defined structure cost function. It is a method more general than the parametric and shape optimization methods, where only some dimensions or the shape of the structure are optimized, respectively. The main advantage of topology optimization is that allows us to find new holes in the structure and, therefore, the weight reduction obtained is much larger than the reduction obtained using the other methods. Topology optimization has been widely applied in the automotive and aeronautic industries to design structures and mechanical elements with high stiffness and low weight (Bendsøe, 1995). The most general topology optimization formulation is based on the so-called homogenization design method developed by Bendsøe and Kikuchi (1988).

2.1 Homogenization design method

The topology optimization method applied is based on two main concepts (Bendsøe and Kikuchi, 1988): the extended fixed domain method and the relaxation of the design domain.

The extended domain is a large fixed domain bounded by supports and applied loads that must contain the unknown structure. The objective of topology optimization is to find the optimal distribution of material properties in this domain that maximizes some structure cost function. Therefore, the finite element model does not change during the optimization process which makes easy the calculation of the derivatives of any function defined over the extended domain.

The second concept is related to the relaxation of the design domain. A material model that relates the properties in each point of the domain to some design variable must be defined, and

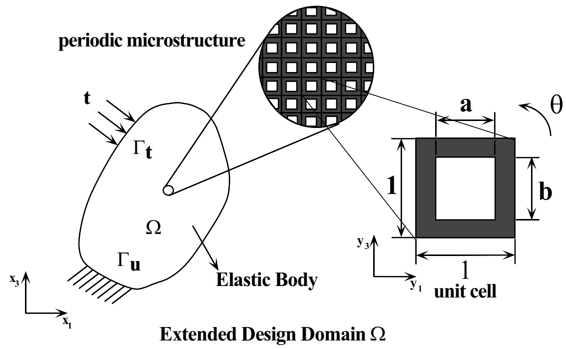

Figure 1: Microstructure used for relaxation of the optimization problem.

this model must allow materials with intermediate properties and not only zero or full material. This concept is called relaxation of the design domain.

A high relaxation of the optimization problem can be obtained using a material model based on the microstructure described in the Fig. 1 which is defined in each point of the domain. This microstructure was proposed by Bendsøe and Kikuchi (1988) and consists of a unit cell with a rectangular hole inside whose dimensions are defined by the design variables a and b, and the orientation θ. Therefore, in each point of the domain there is a composite material defined by the periodic repetition of the microstructure corresponding to that point. The composite material in each point of the domain can vary from void ($a = b = 1$ or a maximum value) to full material ($a = b = 0$), also assuming intermediate materials. In this sense, the problem consists of optimizing the material distribution in a perforated domain with infinite microscale voids. The homogenized (or effective) elasticity properties of this composite material in each point of the design domain is obtained using the homogenization method described by Guedes and Kikuchi (1990).

When the unit cell hole is rotated by the angle θ, as shown in Fig. 1, the new homogenized elasticity tensor $\mathbf{c^G}$ is given by the following equation:

$$\mathbf{c^G} = \mathbf{R}(\theta)^{\mathbf{t}}\mathbf{c^H}\mathbf{R}(\theta) \tag{1}$$

where $\mathbf{R}$ is the rotation matrix defined by

$$\mathbf{R}(\theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \tag{2}$$

The gradients of the elastic properties in relation to a and b are necessary for the optimization method. They are calculated by building a table that contains the values of homogenized properties for combinations of discrete values of a and b. The orientation θ is determined during the optimization procedure by considering the local principal stresses direction in each point of the domain (Pedersen, 1989).

2.2 Formulation of optimization problem

The design of flextensional actuators requires two different objective functions (Silva, 1998): *mean transduction* and *mean compliance*. Mean transduction is related to the electromechanical conversion between two regions, Γ_{d_1} (electrical one) and $\Gamma_{\boldsymbol{t}_2}$ (mechanical one) (see Fig. 2), of the design domain. The larger this function, the larger the displacement generated in a certain direction and in region $\Gamma_{\boldsymbol{t}_2}$ due to an input electrical charge in region Γ_{d_1}. The concept of mean

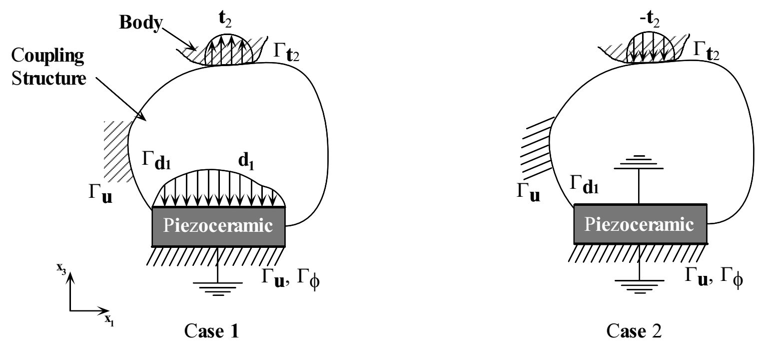

Figure 2: Load cases for calculation of mean transduction (case 1) and mean compliance (case 2).

transduction is obtained by extending the reciprocal theorem in elasticity to the piezoelectric medium. The mean transduction is calculated considering two load cases described in case 1 of Fig. 2. Therefore, the maximization of the output displacement is obtained by maximizing the mean transduction which is given by the expression (Silva, 1998):

$$\begin{aligned} L_2(\mathbf{u}_1, \phi_1) &= L_t(\mathbf{t}_2, \mathbf{u}_1) = L_d(d_1, \phi_2) = A(\mathbf{u}_1, \mathbf{u}_2) + \\ &+ B(\phi_1, \mathbf{u}_2) + B(\phi_2, \mathbf{u}_1) - C(\phi_1, \phi_2) = \\ &= B(\phi_2, \mathbf{u}_1) - C(\phi_1, \phi_2) \end{aligned} \tag{3}$$

where the following operators were defined to make the notation compact:

$$\begin{aligned} A(\boldsymbol{u}, \boldsymbol{v}) &= \int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{u})^t \boldsymbol{c}^E \boldsymbol{\varepsilon}(\boldsymbol{v}) d\Omega \qquad B(\phi, \boldsymbol{v}) = \int_\Omega (\boldsymbol{\nabla}\phi)^t \boldsymbol{e}^t \boldsymbol{\varepsilon}(\boldsymbol{v}) d\Omega \\ C(\phi, \varphi) &= \int_\Omega (\boldsymbol{\nabla}\phi)^t \boldsymbol{\epsilon}^S \boldsymbol{\nabla}\varphi d\Omega \qquad L_t(\boldsymbol{t}_i, \boldsymbol{v}_i) = \int_{\Gamma_{\boldsymbol{t}_i}} \boldsymbol{t}_i \cdot \boldsymbol{v}_i d\Gamma \\ L_d(d_i, \varphi_i) &= \int_{\Gamma_{d_i}} d_i \varphi_i d\Gamma \end{aligned} \tag{4}$$

and Ω is the fixed domain considered for design (it can also contain non-piezoelectric materials), $\boldsymbol{\nabla}$ is the gradient operator, $\boldsymbol{\varepsilon}(.)$ is the strain operator, $\boldsymbol{t}_i$ is a traction vector, d_i is the surface electrical charge, and $\boldsymbol{c}^E$, $\boldsymbol{e}$, and $\boldsymbol{\epsilon}^S$ are the elastic, piezoelectric, and dielectric properties, respectively, of the medium. $\boldsymbol{u}$ and $\boldsymbol{v}$ are displacements, and ϕ and φ are electric potentials.

If only the electromechanical function is considered, a structure with no stiffness at all, may be obtained. Therefore, a structural function must also be considered to provide sufficient stiffness in the coupling structure between the regions $\Gamma_{\boldsymbol{t}_2}$ and Γ_{d_1}. The coupling structure stiffness is also related to the generative force. The larger the stiffness, the larger the generative force. This stiffness can be obtained by minimizing the mean compliance ($L_3(\boldsymbol{u}_3, \phi_3)$) in the contact region actuator/body, which is given by the expression (Silva, 1998):

$$\begin{aligned} L_3(\mathbf{u}_3, \phi_3) &= \int_{\Gamma_{\mathbf{t}_2}} \mathbf{t}_3 \cdot \mathbf{u}_3 d\Gamma = A(\mathbf{u}_3, \mathbf{u}_3) + 2B(\phi_3, \mathbf{u}_3) + \\ -C(\phi_3, \phi_3) &= A(\mathbf{u}_3, \mathbf{u}_3) + B(\phi_3, \mathbf{u}_3) \end{aligned} \tag{5}$$

To combine both optimization problems the following objective function is proposed:

$$F(\mathbf{x}) = w * ln\,(L_2(\mathbf{u}_1, \phi_1)) - (1-w)\,ln(L_3(\mathbf{u}_3, \phi_3)) \tag{6}$$

where $0 \le w \le 1$ is a weight coefficient. This objective function allows us to control the contributions of the mean transduction (Eq. 3) and the mean compliance (Eq. 5) in the design. Therefore, the new optimization problem is stated as:

$$\begin{array}{ll}
Maximize: & F(\mathbf{x}) \\
a, b, \text{ and } \theta & \\
subject\ to: & \mathbf{t}_3 = -\mathbf{t}_2 \qquad (\Gamma_{\mathbf{t}_3} = \Gamma_{\mathbf{t}_2}) \\
& A(\mathbf{u}_1, \mathbf{v}_1) + B(\phi_1, \mathbf{v}_1) = 0 \\
& B(\varphi_1, \mathbf{u}_1) - C(\phi_1, \varphi_1) = L_d(d_1, \varphi_1) \\
& \quad \text{for } \mathbf{u}_1, \phi_1 \in V_a \text{ and } \forall \mathbf{v}_1, \forall \varphi_1 \in V_a \\
& A(\mathbf{u}_2, \mathbf{v}_2) + B(\phi_2, \mathbf{v}_2) = L_t(\mathbf{t}_2, \mathbf{v}_2) \\
& B(\varphi_2, \mathbf{u}_2) - C(\phi_2, \varphi_2) = 0 \\
& \quad \text{for } \mathbf{u}_2, \phi_2 \in V_a \text{ and } \forall \mathbf{v}_2, \forall \varphi_2 \in V_a \\
& A(\mathbf{u}_3, \mathbf{v}_3) + B(\phi_3, \mathbf{v}_3) = L_t(\mathbf{t}_3, \mathbf{v}_3) \\
& B(\varphi_3, \mathbf{u}_3) - C(\phi_3, \varphi_3) = 0 \\
& \quad \text{for } \mathbf{u}_3, \phi_3 \in V_b \text{ and } \forall \mathbf{v}_3, \forall \varphi_3 \in V_b \\
& 0 \le a \le a_{sup} < 1 \\
& 0 \le b \le b_{sup} < 1 \\
& \Theta(a, b) = \int_S (1 - ab) dS - \Theta_S \le 0
\end{array}$$

where:

$V_a = \{\mathbf{v} = v_i \overline{\mathbf{e}}_i, \varphi : v_i, \varphi \in H^1(\Omega) \quad \text{with } \mathbf{v} = 0 \quad \text{on } \Gamma_{\mathbf{u}} \quad \text{and } \varphi = 0 \quad \text{on } \Gamma_\phi, i = 1 \text{ or } 3\}$

$V_b = \{\mathbf{v} = v_i \overline{\mathbf{e}}_i, \varphi : v_i, \varphi \in H^1(\Omega) \quad \text{with } \mathbf{v} = 0 \quad \text{on } \Gamma_{\mathbf{u}}, \text{ and } \varphi = 0 \quad \text{on } \Gamma_\phi \text{ and } \Gamma_{d_1}, i = 1 \text{ or } 3\}$

S is the design domain Ω without including the piezoceramic, Θ is the volume of this design domain, and Θ_S is an upper bound volume constraint to control the maximum amount of material used to build the coupling structure. The index i assumes value 1 or 3 because the problem is considered in the plane 1-3. The piezoceramic is polarized in the #3 direction.

The above optimization problem was defined in a continuous form, however since the domain is discretized in finite elements, the above definitions must be substituted by their equivalent discretized ones using FEM. In addition, the variables a, b, and θ which theoretically are a continuous function of $\mathbf{x}$, became sets of continuous design variables a_n, b_n, and θ_n defined for the n finite element subdomain in the numerical problem. The upper bounds a_{sup} and b_{sup} specified for a and b, respectively, are necessary to avoid numerical problems such as singularity of the stiffness matrix in the finite element formulation. In this work, the upper bounds a_{sup} and b_{sup} were chosen to be 0.995. Numerically, regions with $a = b = 0.995$ have practically no structural significance and can be considered void regions.

Considering the discussion above, the final objective function is composed of three load cases described in Fig. 2. These load cases are solved separately, and their solution is used in the mean transduction and mean compliance calculations.

2.3 Numerical implementation

The optimization problem is solved using Sequential Linear Programming (SLP) which consists of the sequential solution of linearized problems defined by writing a Taylor series expansion for the objective and constraint functions around the current design points a_n and b_n in each iteration step. θ_n is obtained by considering the local principal stresses direction in each finite element after each optimization step. The sensitivities of the objective function necessary

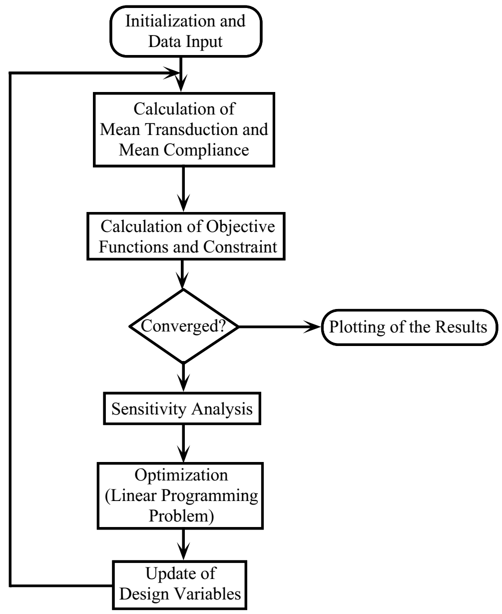

Figure 3: Flow chart of the optimization procedure.

for the linearization of the problem were derived by Silva (1998). In each iteration, moving limits are defined for the design variables. After optimization, a new set of design variables a_n and b_n is obtained and updated in the design domain. A flow chart of the optimization algorithm describing the steps involved is shown in Fig. 3.

3. RESULTS

Two examples will be presented to illustrate the design of flextensional actuators using the proposed method. The design domains used for the examples below are described in Fig. 4a and b (40×20 mesh - 800 finite elements). They consist of a domain of piezoceramic that remains unchanged during the optimization and a domain of brass where the optimization is conducted. The mechanical and electrical boundary conditions for both domains are described in the same figures. Electrical degrees of freedom are considered only in the ceramic domain. For each example, the correspondent interpretation of the topology is obtained through a threshold of the topology optimization image.

Table 1 describes the piezoelectric material properties used in the simulations. The Young's modulus and Poisson's ratio of the brass are equal to $106\ GPa$ and 0.3, respectively. Two-dimensional elements under plane strain assumption are used in the finite element analysis. For all these examples, the total volume constraint of the material Θ_S is considered to be 30% of the volume of the whole domain Ω without piezoceramic (domain S). The initial value of the microscopic design variables a_n and b_n is 0.9, and that of θ is 0.0 in all elements. The amount of electrical charge applied to the piezoceramic electrode is 4 $\mu C/m^2$. Any value can be applied since the problem is linear.

3.1 Example 1

The optimization problem is defined as the maximization of the deflection at point A in the direction shown when electrical charges d_1 are applied to the piezoceramic at electrode

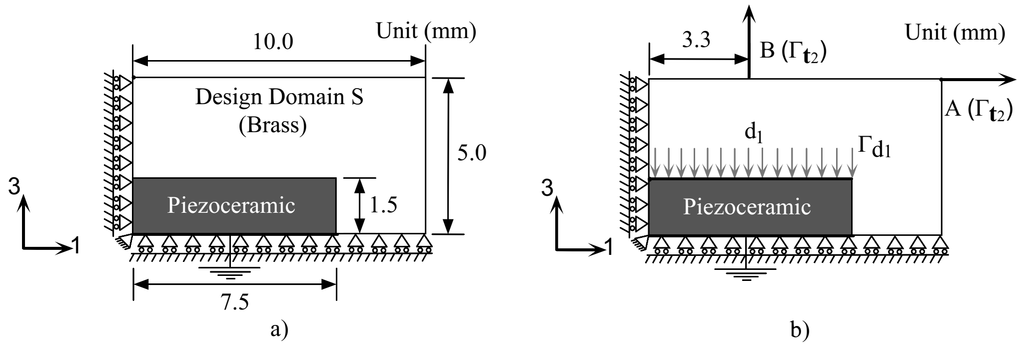

Figure 4: Design domain considered.

Table 1: Material Properties of PZT5.

c_{11}^E (10^{10} N/m^2)	12.1		e_{13} (C/m^2)	-5.4
c_{12}^E (10^{10} N/m^2)	7.54		e_{33} (C/m^2)	15.8
c_{13}^E (10^{10} N/m^2)	7.52		e_{15} (C/m^2)	12.3
c_{33}^E (10^{10} N/m^2)	11.1		$\epsilon_{11}^S/\epsilon_0$	1650
c_{44}^E (10^{10} N/m^2)	2.30		$\epsilon_{33}^S/\epsilon_0$	1700
c_{66}^E (10^{10} N/m^2)	2.10			

Γ_{d_1} (see Fig. 4b), while the mean compliance at point A is to be minimized since the actuator is supposed to have contact with a body at point A. The coefficient w was considered equal to 0.5. Figure 5a shows the topology optimization result. Figure 5b shows the image of the final actuator obtained by reflecting the interpreted image of Fig. 5a to the symmetry axis. The corresponding deformed shape obtained using FEM is described in Fig. 5c.

3.2 Example 2

The objective is to maximize the deflecton at point B in the direction shown when electrical charges d_1 are applied to the piezoceramic at electrode Γ_{d_1} (see Fig. 4b), while the mean compliance at point B is also to be minimized for the reason stated before. The coefficient w was considered equal to 0.8. The topology optimization result for this different type of actuator

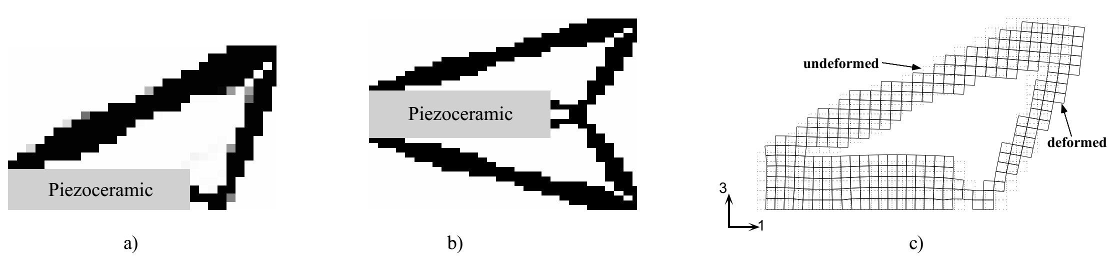

Figure 5: a) Topology result ($w = 0.5$); b) Final actuator image; c) Corresponding deformed structure obtained using FEM.

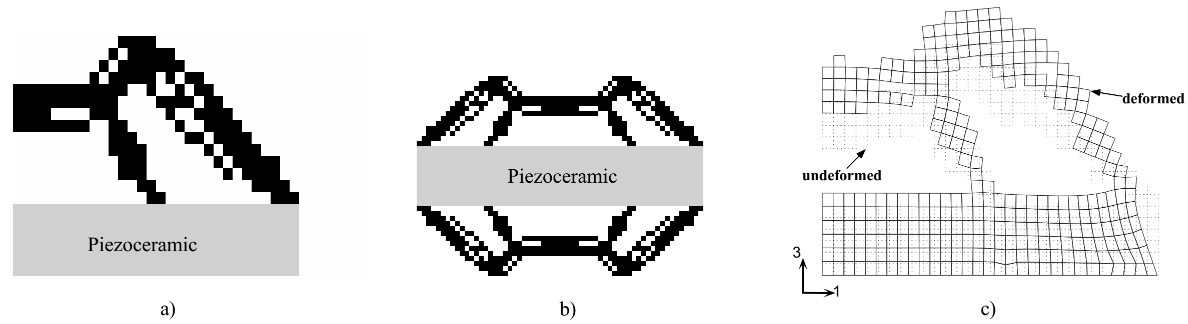

Figure 6: a) Topology result ($w = 0.8$); b) Final actuator image; c) Corresponding deformed structure obtained using FEM.

is presented in Fig. 6a. Figure 6b shows the image of the final actuator obtained by reflecting Fig. 6a to both symmetry axes. The corresponding deformed shape obtained using FEM is shown in Fig. 6c. Due to the output displacement of the coupling structure, this result could be used to design a low-frequency sonar with high directivity, for example.

4. CONCLUSIONS

A method for designing flextensional piezoelectric actuators for static and low-frequency applications has been proposed. This method is based upon topology optimization using the homogenization design method. The method consists of designing a flexible structure (coupling structure) connected to a piezoceramic (or stack of piezoceramics) that amplifies and converts the output piezoceramic displacement, depending on the actuator task. Therefore, novel types of actuators for different tasks can be designed. The complex topologies obtained can be manufactured using rapid prototyping techniques.

In future work, we intend to manufacture some prototypes and measure their output displacements by using laser interferometry techniques. The method will be also extended for designing sonars and hydrophones with specified directivity.

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