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STRUCTURAL DYNAMICS ANALYSIS USING MODE SYNTHESIS TECHNIQUES

Maher N. Bismarck-Nasr
Instituto Tecnológico de Aeronáutica, ITA,
12228-900, São José dos Campos, SP, Brazil

Richard Rigobert Lucht
Instituto Tecnológico de Aeronáutica, ITA,
12228-900, São José dos Campos, SP, Brazil

ABSTRACT. *Structural dynamic analysis using the classical finite element method applied to complex aeronautical structures, such as wings with external stores, tip tanks and side winders, presents huge models with many degrees of freedom, that render the analysis extremely costly. Further, in many circumstances the parts are produced by different design teams and/or organizations, and sometimes in different countries, this fact render their classical finite element models integration extremely difficult and time consuming. Moreover, in many cases the builders of these structural parts furnish their structural dynamic properties only as modal characteristics obtained from ground vibration tests or classical engineering analysis. The modal synthesis techniques present an effective alternative for the structural dynamic analysis of such complex structures and their integration, at the same time represent a computation resources saving. The present work address several aspects involved in the formulation of the modal synthesis techniques, and discuss their relative merits, accuracy and computational efficiency. The application of the method to complex structures is presented and the results obtained are discussed and compared with the solution of full finite element models.*

1. INTRODUCTION

The integration of external stores to a clean aircraft configuration is a problem that requires detailed studies in order to predict its airframe structural dynamic behavior after the modification. This question, which has pluged engineers since the earliest days of flying, gained the recognition in World War I. Since that time, regardness of the size or the speed of the aircraft, or whether the stores were carried externaly or internaly, weapons compatibilities have been a continuous problem, despite the stagerring advances in technology during the past fifty years. However, with the advent of high speed jet aircraft, those problems become of significant magnitude. The speed and complexibility of modern bomber-fighter aircraft have made the solution of aircraft/store compatibilty problems a necessity from both tactical and flight safety standpoints.

The finite element method of analysis, which is the state-of-art representation of complex structural dynamic configurations, is nowadays extensively used in the aeronautical facilities. However, structural dynamic analysis using the classical finite element method applied to complex aeronautical structures, such as wings with external stores, tip tanks and side winders, presents huge models with many degrees of freedom, that render the analysis extremely costly. Further, in many circumstances the parts are produced by different design teams and/or organizations, and sometimes in different countries, this fact render their classical finite element models integration extremely difficult and time consuming. Moreover, in many cases the builders of these structural parts furnish their structural dynamic properties only as modal characteristics obtained from ground vibration tests or classical engineering analysis.

The modal synthesis techniques present an effective alternative for the structural dynamic analysis of such complex structures and their integration, at the same time represent a computation resources saving. The present work address several aspects involved in the formulation of the modal synthesis techniques, and discuss their relative merits, accuracy and computational efficiency. The application of the method to complex structures is presented and the results obtained are discussed and compared with the solution of full finite element models.

2. PROBLEM FORMULATION

Hamilton's principle forms the basis of the variational methods applied to structural dynamic problems. It is a generalization of the minimum total potential energy principle to include the dynamic effect. The related principle reads,

$$\delta\pi_L = \delta\int_{t_0}^{t_1} (T - U - V)\, dt = 0 \,, \tag{1}$$

where T (kinetic energy functional), U (strain energy functional) and V (work done by the applied loads) are given by

$$T = \frac{1}{2}\int_V \rho \left\{\dot{q}\right\}^T \left\{\dot{q}\right\} dv \,, \tag{2}$$

$$U = \frac{1}{2}\int_V \{\varepsilon\}^T [\,C\,] \{\varepsilon\} dv \,, \tag{3}$$

and,

$$V = -\int_V \{q\}^T \{\underline{b}\} dv - \int_{S_\sigma} \{q\}^T \{\underline{p}\} ds \, . \quad (4)$$

Performing the variational operation, one gets

$$\int_{t_0}^{t_1} \delta\pi_L \, dt = \int_{t_0}^{t_1} [- \int_V \rho \{\delta q\}^T \{\ddot{q}\} dv - \int_V \{\delta q\}^T [b]^T [C][b] \{q\} dv + \int_V \{\delta q\}^T \{\underline{b}\} dv + \int_{S_\sigma} \{\delta q\}^T \{\underline{p}\} ds \,] \, dt = 0. \quad (5)$$

Then, considering a typical finite element, the field variable {q}, can be interpolated as

$$\{q\} = [N]\{q_i\}, \quad (6)$$

Using Eqs. (6) into Eq. (5), one obtains for each element the following equation

$$\int \rho \, [N]^T [N] \, dv \, \{q''_i\} + \int [B]^T [C] \, [B] \, dv \, \{q_i\} = \int [N]^T \, \{b\} \, dv + \int [N]^T \{p\} \, ds \quad (7)$$

Or,

$$[m]\{\ddot{q}\} + [k]\{q\} = \{f_i\}, \quad (8)$$

Now, using the standard assembly technique of finite element method and applying the appropriate forced boundary conditions for a free vibration problems, one obtains, for the whole structure, the following matrix equation,

$$[M]\{\ddot{q}\} + [K]\{q\} = \{F\}. \quad (9)$$

Applying a modal coordinate transformation defined by $\{q\} = [\,\phi\,]\,\{\xi\}$, and for free vibration we can write,

$$[\mu]\,\{\xi''\} + [\gamma]\,\{\xi\} = \{0\} \quad (10)$$

- At this stage we can now apply the mode synthesis technique, to this end consider two different structures ("α" and "β"), a representation of each of them is made in their modal base. The procedure involves the following essential considerations without that the analysis will not guide to the correct results.
- The choice of the DOF's and modes is not a randomic process. On the contrary, it must be very careful so that the selected ones could best represent the dynamic structure's behavior. As a rule of thumb, the first modes are the most representatives.
- When selecting modes from a free-free structure, it is mandatory to include in the reduced free vibration mode matrices, $[\,\phi\,]_r$, <u>all</u> the rigid body modes.
- In the selecting process of DOF's and modes, one has to check if the "new reduced modes" are linearly independent. If not, $[\,\phi\,]_r$ will be singular and the mode synthesis method impossible to be applied.

Following the method, for each free vibration mode chosen, there are counterparts on both the generalized mass and stiffness matrices. From these, two reduced generalized mass matrices and two reduced generalized stiffness matrices are obtained. Then, a set of equations related to the system can be written as

$$[\mu]_r^\alpha \left\{ \ddot{\xi}_r^{(\alpha)} \right\} + [\gamma]_r^\alpha \left\{ \xi_r^{(\alpha)} \right\} = \{0\}, \tag{11}$$

and

$$[\mu]_r^\beta \left\{ \ddot{\xi}_r^{(\beta)} \right\} + [\gamma]_r^\beta \left\{ \xi_r^{(\beta)} \right\} = \{0\}. \tag{12}$$

In order to recover the physical coordinate {q}, one applies the transformations defined by

$$\{\xi r\} = [\ \phi\ ]_r^{-1}\ \{q_r\}\ , \tag{13}$$

One can now join the structures "α" and "β" applying all the compatibility conditions. Thus, rewriting the above equations results in

$$\begin{bmatrix} m_{aa}^\alpha & m_{ab}^\alpha & 0 \\ m_{ba}^\alpha & (m_{bb}^\alpha + m_{bb}^\beta) & m_{ba}^\beta \\ 0 & m_{ab}^\beta & m_{aa}^\beta \end{bmatrix} \begin{Bmatrix} \ddot{q}_{r_a}^{(\alpha)} \\ \ddot{q}_{r_b} \\ \ddot{q}_{r_b}^{(\beta)} \end{Bmatrix} + \begin{bmatrix} k_{aa}^\alpha & k_{ab}^\alpha & 0 \\ k_{ba}^\alpha & (k_{bb}^\alpha + k_{bb}^\beta) & k_{ba}^\beta \\ 0 & k_{ab}^\beta & k_{aa}^\beta \end{bmatrix} \begin{Bmatrix} q_{r_a}^{(\alpha)} \\ q_{r_b} \\ q_{r_b}^{(\beta)} \end{Bmatrix} = \{\ 0\ \}, \tag{14}$$

which represents the whole system.

3. APPLICATION

The structural set adopted to develop this work is based on an air-combat and interception configuration employed by fighter aircraft. This set, composed with an air-to-air missile installed in a wing station, where an F-5E "Tiger II" aircraft is loaded with a "Python III" missile. The wing and the missile have been modeled using MSC/NASTRAN® software [1994]. The detailed description of the wing and missile models are presented in Lucht [1998]. In the present work, the structural dynamic coupling simulation includes three different steps. The first is characterized by a finite element modeling of the complete structure, henceforth called "standard model". From this model, one can obtain its natural frequencies and free vibration modes. The second phase consists in selecting a set of modes that could best dynamically represent the structure. Within the scope of this step, three different wing-missile models are considered in this work and described as follows.

- First Test Model
  Dynamic model formed by all missile nodal points (24 degrees of freedom) and all nodes located in the bottom surface of the wing (150 degrees of freedom). With respect to free vibration modes, one selects the first hundred and fifty of the wing and all twenty four of the missile.
- Second Test Model
  Dynamic model formed by all missile nodal points (24 degrees of freedom) and the nodes in the bottom surface of wing stations 2, 4, 6, 8 and 10 (75 degrees of

freedom). With respect to free vibration modes, one selects the first seventy five of the wing and all twenty four of the missile.

- Third Test Model
  Dynamic model formed by all missile nodal points (24 degrees of freedom) and just the nodes in the bottom surface of wing station 6 (15 degrees of freedom). With respect to free vibration modes, one selected the first fifteen of the wing and all twenty four of the missile.

Finally, the third and last step establishes the application of the Mode Synthesis Method using the finite element models. Therefore, with the aid of MATLAB® software [1994], the foreseen sequence of calculus is conducted in order to get the new natural frequencies and free vibration modes of the considered structural set.

An estimation of the error has been calculated using the following formula,

$$Error = \left(\frac{f_{i\ (standard)} - f_{i\ (test)}}{f_{i\ (standard)}} \right) \cdot 100\% \, , \qquad (15)$$

where $f_{i\ (standard)}$ stands for the exact natural frequency and $f_{i\ (test)}$ stands for the approximate solution, both for i^{th} free vibration mode. The final results obtained are summarized in tables 1 and 2, in given in figure 1.

Table 1 – Comparison of exact and test models natural frequencies.

Mode of Vibration	**Natural Frequency Standard Model (Hz)**	**Natural Frequency 1st Test Model (Hz)**	**Natural Frequency 2nd Test Model (Hz)**	**Natural Frequency 3rd Test Model (Hz)**
1	1.489	1.489	1.502	1.492
2	6.534	6.536	6.530	6.579
3	10.155	10.187	10.203	11.179
4	11.638	11.673	11.679	12.525
5	12.115	12.262	12.450	12.662
6	13.312	13.335	13.347	13.384
7	14.010	14.015	14.017	14.136
8	16.841	16.847	16.849	16.998
9	22.090	22.095	22.097	22.166
10	22.630	22.649	22.658	23.138
11	27.359	27.387	27.398	27.741
12	27.671	27.715	27.720	29.134
13	30.367	30.557	30.836	31.206
14	31.715	31.740	31.770	32.273
15	33.718	33.862	34.057	34.482

Table 2 – Errors associated with the test models natural frequencies.

Mode of Vibration	Error 1st Test Model (%)	Error 2nd Test Model (%)	Error 3rd Test Model (%)
1	0	-0.87	-0.20
2	-0.03	0.06	-0.69
3	-0.32	-0.47	-10.08
4	-0.30	-0.35	-7.62
5	-1.21	-2.77	-4.52
6	-0.17	-0.26	-0.54
7	-0.04	-0.05	-0.90
8	-0.04	-0.05	-0.93
9	-0.02	-0.03	-0.34
10	-0.08	-0.12	-2.24
11	-0.10	-0.14	-1.40
12	-0.16	-0.18	-5.29
13	-0.63	-1.54	-2.76
14	-0.08	-0.17	-1.76
15	-0.43	-1.01	-2.27

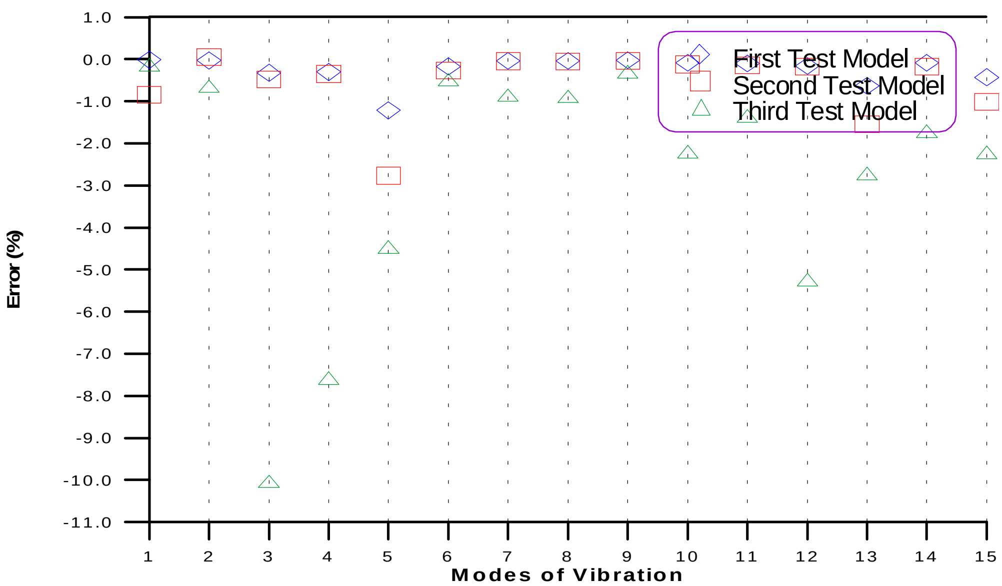

Figure 1 – Natural frequency errors of the adopted test models.

4. CONCLUSIONS

In this work, the objective of studying the dynamic coupling of two representatives aeronautical structures by applying the Mode Synthesis method presented before is accomplished. Numerical simulation are done, starting from three different models of the same structural set, and the obtained results are compared with the complete system's known natural frequencies.

In this way, one can verify that the methodology provides good results, since the calculated natural frequencies are very close to that admitted as exact. Also, the errors are small for most part of the cases analyzed which, in other words, assures a good convergence of the method presented.

It can be observed that the precision obtained is a direct consequence from selecting appropriate degrees of freedom and modes of vibration that remain in the model. Although the dynamical studies are, normally, restricted in a frequency band and, thus, the modal model is in a practical way truncated, one can easily understand that this is a critical point in the method. In this step, the engineer's skills in analyzing and separating important data are indispensable, under the penalty of getting incoherent values, or worse, results where potentially important frequencies are omitted. Within the scope of modeling, it is relevant to check the points below.

- The method produces satisfactory results despite the simplicity of the structural model. In the numerical simulations performed, the adopted formulation consider the nodal translations as degrees of freedom (no rotations) and lumped mass.
- The quality of the involved substructures imposes a considerable influence on the calculations. An inconsistent modeling introduces inaccuracy in the numerical calculation and produces deviations that can result in useless information. This observation reinforces that a pre-adjustment in the structures model is needed before applying the mode synthesis routine, mainly when the dynamic parameters are experimentally obtained from ground vibration tests.
- The interface between the involved substructures must have a sufficient number of degrees of freedom in order to guarantee consistent results.

Finally, all the numerical results of this trend study make clear the potential of the Mode Synthesis Method application on the dynamic coupling of aeronautical structures. The good accuracy of the approximated solutions and the potential savings on computers' processing time justify the application of the technique in large and complex real cases, as part of preliminary studies, to certificate external stores in military aircraft.

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